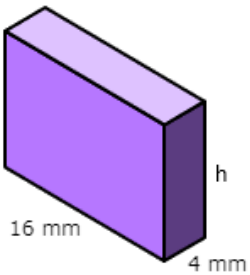
"Math is Cool" Masters -- 2023-24

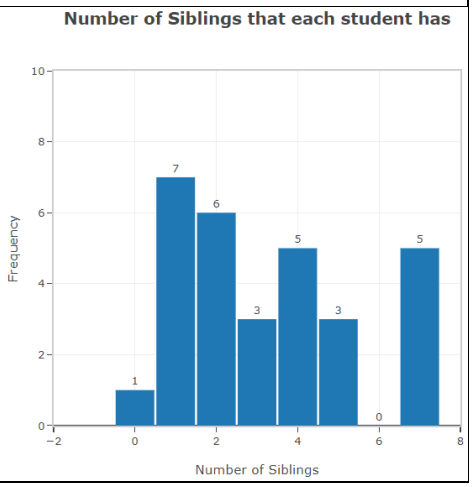
High School

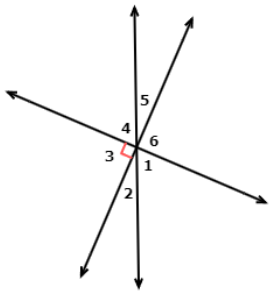
Mental Math Solutions

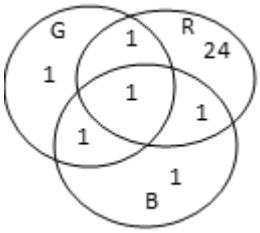
	Answer	Solution
1	789	What is the sum of three hundred sixty-eight and four hundred twenty-one? 368 + 421 = 789
2	1,000,000	How many thousands are in one billion? $10^3 \times 10^6 = 10^9$
3	4047 [= a – c]	If 'a' minus 'b' equals two thousand twenty-three, and 'b' minus 'c' equals two thousand twenty-four, what is the value of 'a' minus 'c'? a – b = 2023 b – c = 2024 a – c = 2023 + 2024 = 4047
4	40 [%]	As a percentage, what is the value of five elevenths divided by twenty-five twenty-seconds? (5/11)/(25/22) = (5/11)*(22/25) = 2/5 = 40%
5	25	Yessenia added together some perfect squares, all of which were different. Their sum was fifty-five. What was the largest perfect square that was added? 1 + 4 + 9 + 16 + 25 = 55
6	128 [square units]	In square units, what is the maximum area of a rectangle with a diagonal that has a length of sixteen units? The maximum area will be a square. If the diagonal is 16, then the side length will be $\frac{16}{\sqrt{2}}$. Therefore the area will be $16^2/2 = 256/2 = 128$.
7	58 [terms]	In the sequence of odd integers, beginning with one, three, five and so on, how many terms are there after the term 91, but before the term two hundred nine? Count the terms from 93 to 207. (207-91)/2 = 116/2 = 58
8	50 [%]	When a circuit containing blinking lights A and B is turned on, lights A and B blink together. After that, A blinks once every five seconds and B blinks once every eleven seconds. Asher looks at the lights just in time to see A blink alone. As a percent, what is the probability that the next light to blink will be A blinking alone? A will always blink alone twice before B blinks. Therefore, since Asher saw A blink once, there is a 50/50 chance that A will blink alone again.

"Math is Cool" Masters -- 2023-24
High School
Individual Test Solutions

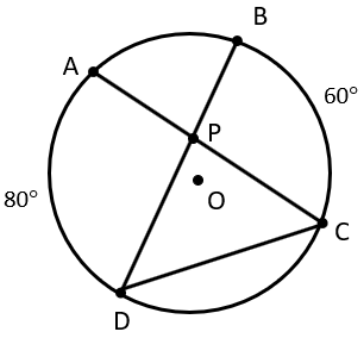
	Answer	Solution
1	21	Evaluate the expression for x = –1: 8 – 13x 8 – 13(-1) = 8 + 13 = 21
2	700 [cm/week]	Gregg is knitting a scarf at a rate of 1 meter per day. How many centimeters per week is he knitting? 1 meter/day x (100 cm/1 m)x (7 days/week) = 700 cm/week
3	12 [mm]	The surface area of the following rectangular prism is 608 mm^2. What is the height of the prism, in millimeters? SA = 608 = 16x4x2 + 16xhx2 + 4xhx2 480 = 40h h = 12 
4	-27 [=x]	What is the value of x in the following sequence? -34, x, -20, -13, -6, 1, 8 Pattern is +7
5	68 [%]	The following table summarizes Mr. Orr's Statistics students at Columbia Basin College by their gender and whether they are from Washington state or some other location. What is the probability in percent that a randomly selected student is Male or from Washington state? There are a total of 4 + 1 + 12 = 17 students who are male or from WA. 17/25 = 68/100 = 68% | | From WA | Not from WA | | Male | 4 | 1 | | Female | 12 | 8 |
6	-21	What is the value of: 1 – 2 + 4 – 8 + 16 – 32 1 – 2 + 4 – 8 + 16 – 32 = -21
7	7 [prime numbers]	How many prime numbers are between 20 and 50? 23, 29, 31, 37, 41, 43, 47

8	58 [years]	Diego asked his grandmother how old she was. She replied by telling him that she has 6 children, and each was born 4 years apart. Her first child was born when she was 19 years old, and her youngest child is now 19 years old. In years, how old is Diego's grandmother? The ages of her children are: 19, 23, 27, 31, 35 and 39. 39+19 = 58.
9	-19 [= f(4)]	Find f(4) for the piecewise-defined function f(x): $$f(x) = \begin{cases} -3x - 7 \; if \; x < 6 \\ -4x + 3 \; if \; x \geq 6 \end{cases}$$ x = 4 which is < 6 f(4) = -3(4) – 7 = -12 – 7 = -19
10	8 [donuts]	Raj and Howard went to the bakery and bought boxes of donuts. Raj bought 40 donuts and Howard bought 24. Each box contains the same number of donuts. What is the largest possible number of donuts in each box? In other words, what is the GCD of 40 and 24. $40 = 2^3 x5$ $24 = 2^3 x3$ Therefore, GCD = $2^3 = 8$
11	1440	As an integer, evaluate the following product: $(2.4 \times 10^5)(6.0 \times 10^{-3})$ 240000 x 0.006 = 1440
12	35 [ways]	Yareli has seven favorite movies. In how many ways can she choose three of them to watch at her upcoming slumber party? The order is not important. 7C3 = $\frac{7!}{3!4!} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1}$ = 35
13	1 [sibling]	The following histogram shows the number of siblings that each student in Ms. Lutrell's class has. What is the mode of this data, in number of siblings? The most responses (7) were for one sibling. 

14	2 [angle]	Which angle is vertical to angle 5? Vertical angles are the angles opposite each other when two lines intersect. 
15	7 [= x + y]	The solution to the following system of equations is the ordered pair (x, y). What is the value of x + y? $4x + 3y = 10$ $y = -1.5(x - 1)$ (1) $4x + 3y = 10$ $y = -1.5(x - 1)$ ➔ $2y = -3(x - 1) = -3x + 3$ (2) $-3x - 2y = -3$ Multiply (1) by 3, and (2) by 4 $12x + 9y = 30$ $-12x - 8y = -12$ $y = 18$, therefore $x = -11$ $18 - 11 = 7$
16	2514	As an integer, what is the value of the following Roman numeral? MMDXIV M = 1000 D = 500 X = 50 IV = 4
17	-9	Triangle ABC has vertices A (3, -8), B (6, -8) and C (0, -3). After the triangle is rotated 180° counterclockwise around the origin to new vertices A', B' and C', what is the sum of the x-coordinates of A', B' and C'? $-3 + (-6) + 0 = -9$

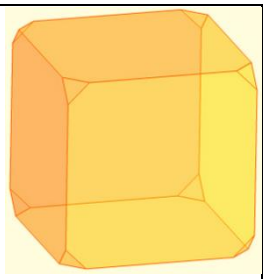
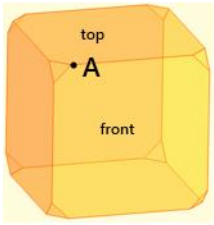
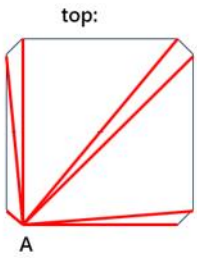
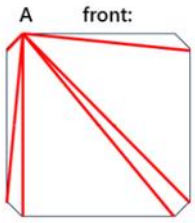
18	24 [respondents]	In a survey, people were asked whether they like red, green, blue, or two or more of the colors. Thirty people were surveyed, all of whom gave one, two, or all three colors as colors they like. At least one respondent gave all three as colors they like, and at least one gave each possible pair of colors (but not the third) as colors they like, and at least one gave only red, green, or blue as a color they like. What is the largest possible number of respondents who could have said they only like red? In the diagram below there is a 1 in each of the 6 sections, not including the section labeled red that does not overlap with any other section. 30 – (1 + 1 + 1 + 1 + 1 + 1) = 24 
19	4 [cm]	Grace has a cylindrical glass which is half full of water. The glass has a radius of 3 cm and height of 16 cm. She drinks one-half of the water she has and then pours the remaining water into Lily's cylindrical glass. Lily's glass has a height of 9 cm. What is the radius of Lily's glass in cm if the water fills up a quarter of the glass? Grace starts with 8 cm of water, then drinks half of it so ends up with 4 cm of water. $V = 9\pi*4 = 36\pi$ Lily starts with a height of 9/4 cm of water. $36\pi = \pi r^2(9/4)$ $r^2 = 16$ $r = 4$
20	6 [30-year olds]	A group of 15 students have an average age of 16 years. How many thirty-year olds need to be added to the group to raise the average age to 20 years old? current sum / 15 = 16 current sum = 240 (240 + 30x)/(x + 15) = 20 240 + 30x = 20x + 300 10x = 60 x = 6

21	0	What is the sum of the sine and cosine of an angle measuring $\frac{7\pi}{4}$ radians? $\frac{7\pi}{4}$ radians = 315°, which is equivalent to -45°. From the unit circle, or the special 45-45-90 triangle, sin + cos = $-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}$ = 0
22	24440 [= sum]	Find the sum of the 40 terms in the finite arithmetic sequence which begins as follows: -13, 19, 51, 83, … The common difference d = 32 a_{40} = -13 + 39(32) = 1235 S_{40} = (40/2)(-13 + 1235) = 24440
23	4253	Aditya and Sahil are sitting opposite each other playing cards. Sahil is holding 4 cards numbered 2, 3, 4, and 5 (not in that order). Sahil wants the cards in ascending order from his perspective, from left to right. Aditya watches Sahil take the leftmost card (from Aditya's perspective) and put it last. He finally watches Sahil take the third card from the right (from Aditya's perspective) and put it last. From Sahil's perspective, what was the original order of the cards, as a 4-digit integer? Call the 4 cards abcd, from Aditya's perspective. First move results in: bcda Second move results in: bdac From Aditya's perspective, the numbers are in reverse order: 5432, therefore: b = 5 d = 4 a = 3 c = 2 From Sahil's perspective, the original order was: dcba, which equals 4253.
24	9120 [teams]	Twenty schools have each sent a 2-member math team to a competition. For the "fun sum" round, how many different 3-member teams can be formed that do not contain any students from the same school? The total number of 3-member teams is 40C3 = 9880. Have to subtract off any that contain 2 students from the same school. There are 20 "pairs": A1A2, B1B2, … For each pair, there are 19x2 = 38 ways to pick the 3rd. 38 ways x 20 pairs = 760 options that have to be subtracted. 9880 – 760 = 9120 Alternatively: (40)(38)(36)/3! = 9120 Because each selection of a student removes two options instead of just one. The multiplication accounts for different orders, so divide by 3!.

25	45	What is the next number in the sequence that begins: 0, 1, 4, 11, 24, … Difference: 1, 3, 7, 13, 21 difference: 2, 4, 6 next is 8, 13 +8 = 21, 24 + 21 = 45
26	2 [integer solutions]	How many integer solutions does this absolute value inequality have? $\lvert 3x-2\rvert < 4$ -4 < 3x – 2 < 4 -2 < 3x < 6 -2/3 < x < 2 Integer solutions are x = 0, 1
27	12	The totient function, $\Phi(m)$, is the number of integers between 1 and m (inclusive) that are relatively prime to m. What is $\Phi(21)$? There are 12 integers that are relatively prime to 21: 1, 2, 4, 5, 8, 10, 11, 13, 16, 17, 19, 20.
28	100 [°]	In circle O shown here, arc BC measures 60°, and arc AD measures 80°. Find the value of ∠APD + ∠BDC, in degrees.  ∠BDC is the inscribed angle for the 60° arc, so it measures 30°. ∠ACD is the inscribed angle for the 80° arc, so it measures 40°. Therefore in triangle PDC, ∠DPC measures 100°, and therefore ∠APD measures 70°. The sum of the two angles is 100°.
29	11	Four integers are added to the set {3, 4, 5, 5, 8}, which increases the mean, median and mode by 1 each. What is the greatest integer in the new set? The mean, median and mode of the current set all equal 5, therefore they all equal 6 in the new set. We must add three 6's to get a mode of 6. The sum of the new set must = 54 to give a mean of 6, therefore the 4th integer must be 11, which also makes the median = 6.

30	9 [=y]	Let x equal the determinant of the following matrix. If the value of y is reduced by 13, the determinant of the modified matrix is equal to 1 – x. What is y? $\begin{bmatrix} 7 & 4 & -6 \\ 1 & y & 2 \\ 0 & 2 & 3 \end{bmatrix}$ $\begin{vmatrix} 7 & 4 & -6 \\ 1 & y & 2 \\ 0 & 2 & 3 \end{vmatrix}$ = x 7(3y – 4) – 12 – 12 = x x = 21y - 52 $\begin{vmatrix} 7 & 4 & -6 \\ 1 & y-13 & 2 \\ 0 & 2 & 3 \end{vmatrix}$ = 1 – x 7(3y – 39 – 4) – 12 – 12 = 1 – x x = -21y + 326 21y – 52 = -21y + 326 42y = 378 y = 9
31	42 [= 50th term]	A geometric sequence has 99 terms, with first term 12 and last term 147. What is the 50th term in the sequence? The 50th term is the median. By definition, this is the geometric mean of the first and last terms. median = $\sqrt{(12)(147)} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 7 \cdot 7}$ = (2)(3)(7) = 42
32	2307	The year 1978 had an unusual property. When the number is split in half into the two numbers 19 and 78, the sum of the two numbers 19 + 78 equals 97, which are the two digits in the middle of the year 1978. The current year, 2023, does not have this property, because 20 + 23 does not equal 02. What is the next year after 1978 that has this same property? 23 + 07 = 30
33	131 [test scores]	In October 2023, there were 192 high school students that competed in the Tri-Cities Math Is Cool competition. The individual scores for these students are in a bell-shaped or normal distribution. According to the Empirical Rule, how many of the individual scores lie within one standard deviation of the mean? Round to the nearest integer. The Empirical Rule states that approximately 68% of the data will lie within one standard deviation of the mean. (0.68)(192) = 130.56 Round to 131

34	10848 [= A + B]	Five cards are randomly dealt from a standard 52-card deck of playing cards. As a reduced common fraction, the probability that the 5-card hand contains at least three Kings is A/B. What is A + B? At least three Kings could be 3 or 4. There are 48 ways to get all 4 Kings + 1 other card. To get KKKOO (O = other), there are 4C3 = 4 ways to get the Kings. There are 48C2 = 1128 ways to get the 2 Others. Therefore 4x1128 = 4512 ways. P(at least 3 K) = (4512 + 48)/52C5 = 4560/2598960 = 19/10829 19 + 10829 = 10848
35	9,486,720 [millions of km]	Light travels at a speed of approximately 3×10^8 meters per second. At this speed, how far will light travel in the year 2024, in millions of kilometers? $\frac{3\times10^8 m}{sec} \cdot \frac{1km}{1000m} \cdot \frac{million\ km}{1000000\ km}$ = 0.3 million km/sec $\frac{0.3\ m-km}{sec} \cdot \frac{3600sec}{hr} \cdot \frac{24h}{d} \cdot \frac{366d}{yr}$ = 9,486,720 millions of km

36	120 [line segments]	The eight corners of a cube are cut off, leaving a truncated cube as shown here. The solid has eight triangular faces with a total of 24 vertices. If each of the 24 vertices is connected to every other vertex with a line segment, how many of those line segments will pass through the interior of the truncated cube?  Take vertex A for example. It is on both the top face and the front face of the solid. On the top face, vertex A can be connected to 7 other vertices. On the front face, vertex A can additionally be connected to 6 more vertices, not double-counting the one that has already been counted. Therefore, 13 line segments out of 23 are on the surface, and 23 – 13 = 10 must be in the interior. 10 interior line segments x 24 vertices = 240. That counts each one twice, so the final number is 240/2 = 120.   
37	89 [pieces]	Angie buys three different kinds of candy, which cost 40 cents, 10 cents and 1 cent each. The total cost is 259 cents for 100 pieces of candy. How many pieces of the 1 cent candy did Angie buy? A + B + C = 100 40A + 10B + C = 259 By inspection, A must be between 1 and 6: $1 \le A \le 6$. Additionally, C must end in a 9, such as 9, 19, 29, etc. Therefore, A + B must end in a 1, such as 11, 21, etc. Some trial and error will result in the solution: A = 2, B = 9, C = 89.

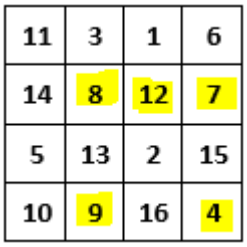
38

7

In the following grid, which number is two places away from itself plus 2, one place away from itself plus 5, two places away from itself less 3, and two places away from itself plus 1? One place away indicates adjacent, either horizontally, vertically or diagonally. Two places away indicates two steps in the same direction, either horizontally, vertically or diagonally.

11	3	1	6
14	8	12	7
5	13	2	15
10	9	16	4

The number 7 is two places away from 9, one place away from 12, two places away from 4 and two places away from 8.



39

5 [triples]

How many ordered triples of positive integers exist, (x, y, z), where x < y < z, whose product is four times their sum?

Solutions are (1, 5, 24), (1, 6, 14), (1, 8, 9), (2, 3, 10), (2, 4, 6).

$xyz = 4(x + y + z) > 4z$, ➔ $xy > 4$

$xyz = 4(x + y + z) < 12z$, ➔ $xy < 12$

$5 \le xy \le 11$

For x = 1:

$yz = 1 + y + z$, $z = 4(y + 1)/(y - 4)$

$1 \le y \le 11$

The values of y that work are 5, 6, 8.

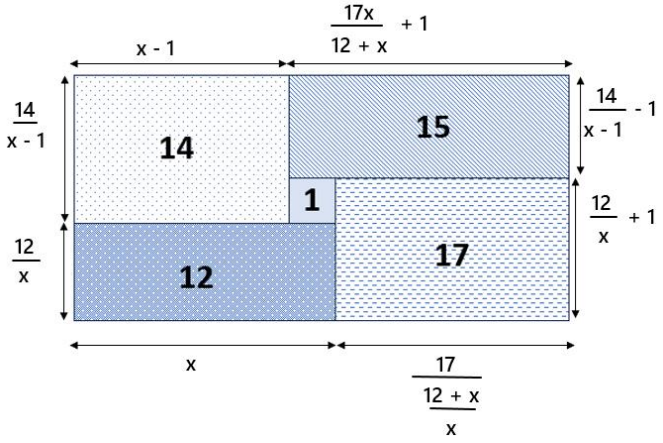
For x = 2:

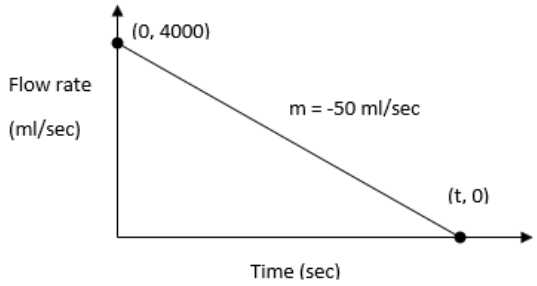
$2yz = 4(2 + y + z)$, $z = 2(y + 2)/(y - 2)$

y = 3 or 4

For x = 3:

$y \ge 4$, but this is impossible since $xy < 12$.

40	318 [units] [= 10 times perimeter]	The following figure shows a large rectangle composed of four smaller rectangles and a central square with side length equal to 1. The area of each rectangle and square is given, in square units. What is the value of 10 times the perimeter of the large rectangle, in units?  Each dimension of each rectangle can be written in terms of a single variable 'x', starting with the length of the lowest-left rectangle. The total area of the rectangle is the sum of the individual areas, which equals 59. The total length times the total height = 59: $\left(x + \frac{17x}{12+x}\right)\left(\frac{12}{x} + \frac{14}{x-1}\right) = 59$ Simplify to: $(x^2 + 29x)(26x - 12) = 59(x + 12)(x^2 - x)$ Simplify to: $33x^2 - 93x - 360 = 0$ $11x^2 - 31x - 120 = 0$ $(11x + 24)(x - 5) = 0$ X = -24/11 (not possible), or x = 5 Using x = 5, the length of the rectangle = 10, and the height = 5.9 Perimeter = 2(10) + 2(5.9) = 31.8 10 times perimeter = 318.
41	-25	Simplify: $i^2(3 + 4i)(3 - 4i)$ where $i = \sqrt{-1}$ $i^2(3 + 4i)(3 - 4i) = -1(9 - 12i + 12i - 16i^2)$ $= -1(9 + 16) = -25$
42	0	What is the minimum number of real solutions that a 6^{th} degree polynomial function can have? An even polynomial can be entirely above or below the x-axis (example: $x^6 + 1$), therefore having no real solutions.

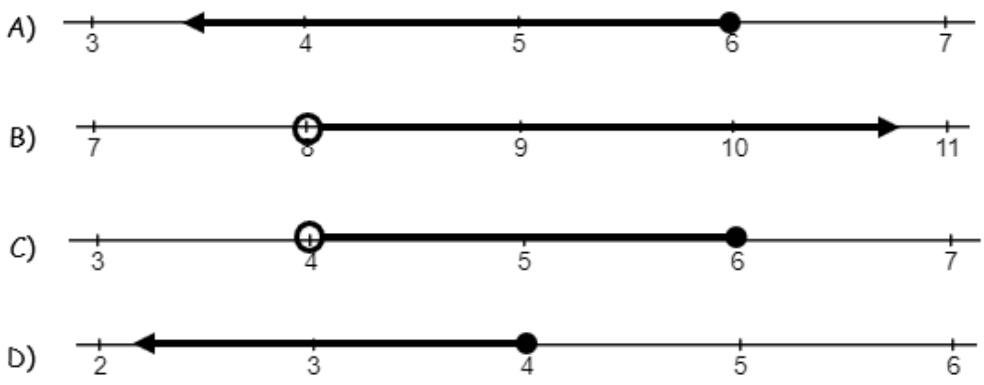
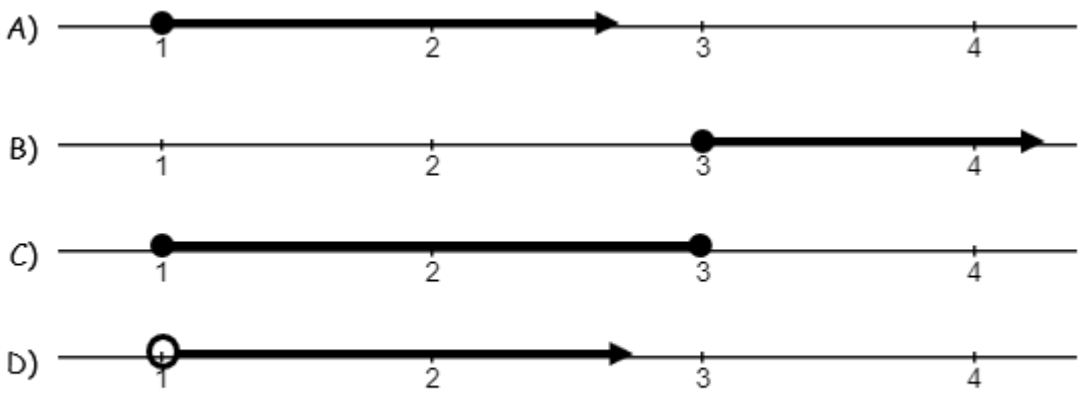
43	-64	De Moivre's Theorem states that if: z = x + iy = $re^{i\theta}$, and n is a natural number, then: $z^n = (x+iy)^n = \left(re^{i\theta}\right)^n = r^n e^{n\theta i}$ Find the value of: $\left(-\sqrt{3}+i\right)^6$ $\left(-\sqrt{3}+i\right)^6 = \left(2e^{150°i}\right)^6 = 2^6\left(e^{6\cdot 150°i}\right)$ $= 64e^{900°i} = 64(cos900° + isin900°)$ = 64(-1 + i0) = -64 Note: can also be solved using the binomial theorem to expand the powers.
44	160 [liters]	Sir Amon Gus is filling his swimming pool with a hose, but he hasn't paid his water bill. The hose starts filling the pool at a rate of 4 liters per second but immediately slows down at a constant rate of 50 mL per second, until it reaches 0 liters per second. What is the volume of the water in the pool, in liters, once Gus's water supply runs dry? 4 liters/sec = 4000 ml/sec Think of a linear function with time on the x-axis in seconds, and filling rate in ml/sec on the y-axis.  m = -50 = -4000/t t = 80 sec The total volume will be the area of the triangle. A = ½ (80)(4000) = 160,000 ml = 160 liters
45	1 [= slope]	For the given function, find the slope of the graph of the function at the point (1, 0). $f(x) = \frac{2x^2 - 3x + 1}{x}$ $f'(x) = 2 - \frac{1}{x^2}$ $f'(xa) = 2 - \frac{1}{1^2} = 1$

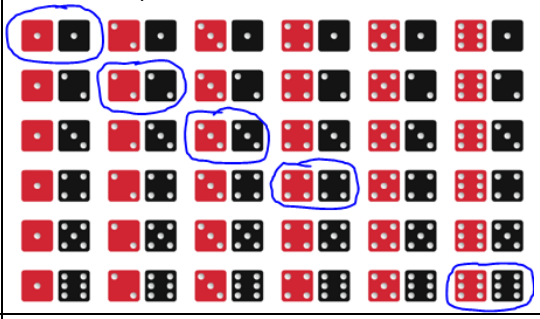
"Math is Cool" Masters -- 2023-24

High School

Multiple Choice Solutions

9/ 10th	11/ 12th	Answer	Solution
1	1	A	What is the lateral surface area of a cylinder that has a radius of 7 centimeters and a height of 21 centimeters? A) 294π cm^2 B) 392π cm^2 C) 454π cm^2 D) 1029π cm^2 E) Answer not given. LA = Circumference * height = $2\pi r*h = 2\pi(7)(21) = 294\pi$
2	2	D	Evaluate the expression: $\left(\frac{1}{2}+\frac{1}{4}\div\frac{1}{3}\right)\div\frac{3}{16}$ A) 5/6 B) 5/16 C) 12 D) 20/3 E) Answer not given. $\left(\frac{1}{2}+\frac{1}{4}\div\frac{1}{3}\right)\div\frac{3}{16}$ = $\left(\frac{1}{2}+\frac{3}{4}\right)\div\frac{3}{16}$ = $\left(\frac{5}{4}\right)\times\frac{16}{3}=\frac{20}{3}$

3		E	Which of the following graphs shows the solution to the following inequality: $2(x-6)-3 \leq 1$  E) Answer not given. $2(x-6)-3 \leq 1$ $2x - 12 - 3 \leq 1$ $2x \leq 16$ $x \leq 8$ 
	3	B	Which of the following graphs shows the solution to the following compound inequality: $2x - 5 \geq -3 \quad and \quad 5x + 2 \geq 17$  E) Answer not given. $2x \geq 2, x \geq 1$ And $5x \geq 15, x \geq 3$ Both must be true, so the solution is the overlap, which is $x \geq 3$.

4		B	In the game of Yahtzee, players roll 5 standard dice and try to get various number combinations. They have a total of 3 opportunities to roll each turn. A Large Straight consists of 5 consecutive numbers: 1, 2, 3, 4, 5, OR 2, 3, 4, 5, 6. After two rolls, a player has 2, 3, 4, 5, 5. They pick up one of the 5's, and roll it again. What is the probability that they get a Large Straight? A) 1/6 B) 1/3 C) 2/3 D) 1 E) Answer not given. Have: 2 3 4 5 Either a 1 or 6 will give a Large Straight. P = 2/6 = 1/3
	4	C	In the game of Yahtzee, players roll 5 standard dice and try to get various number combinations. They have a total of 3 opportunities to roll each turn. A Full House consists of 3 dice that are the same plus two dice that are the same, but different from the first three, such as: 3, 3, 3, 2, 2. A Yahtzee consists of 5 dice that are the same, such as 3, 3, 3, 3, 3. After two rolls, a player has 3, 4, 5, 5, 5. Assume they pick up the 3 and 4, and roll them again. What is the probability of getting a Full House that is NOT a Yahtzee? A) 1/6 B) 2/18 C) 5/36 D) 7/36 E) Answer not given. Have: 5 5 5. Need a pair, but NOT 5 5, to get a Full House. There are 5 ways to do this, out of 36 outcomes. P = 5/36. 
5		D	Mr. Gardener collects data from his Statistics students at Columbia Basin College, with one data value being their age in years. For the Fall 2023 quarter, his 30 students have reported the following ages, sorted in order: Seven students are younger than 20 years old, 20, 20, 20, 21, 21, 22, 22, 22, 23, 23, 24, 25, 25, 25, 26, 26, 27, 30, five students are older than 30 years old. What is the mean age of his students, in years? A) 22 years B) 22.5 years C) 23 years D) Cannot determine. E) Answer not given. Cannot determine the mean without having all of the data values, since the first step is to add all of the data values.

	5	B	Mr. Gardener collects data from his Statistics students at Columbia Basin College, with one data value being their age in years. For the Fall 2023 quarter, his 30 students have reported the following ages, sorted in order: Seven students are younger than 20 years old, 20, 20, 20, 21, 21, 22, 22, 22, 23, 23, 24, 25, 25, 25, 26, 26, 27, 30, five students are older than 30 years old. What is the median age of his students, in years? A) 22 years B) 22.5 years C) 23 years D) Cannot determine. E) Answer not given. The median is the middle value. Since there are 30 values, it will be the mean of values #15 and #16. (22+23)/2 = 22.5
6	6	B	Which of the following equations will produce the graph of $y = 4^x$ after being reflected over the y-axis? A) $y = (-4)^x$ B) $y = (0.25)^x$ C) $y = (0.4)^x$ D) $y = 2^{2x}$ E) Answer not given. $Y = (0.25)^x$ To reflect over y-axis, replace x with -x: $Y = (0.25)^{-x} = (1/4)^{-x} = 4^x$
7		C	Which of these answers is equivalent to the following equation? $$\left(\frac{1}{x+1}\right)^2 + \left(\frac{1}{x-1}\right)^2 = 9$$ A) $3x^4 - 18x^2 + 5 = 0$ B) $9x^4 + 18x^2 + 5 = 0$ C) $9x^4 - 20x^2 + 7 = 0$ D) $3x^4 - 20x^2 + 7 = 0$ E) Answer not given. $$\frac{1}{(x+1)^2} + \frac{1}{(x-1)^2} = 9$$ $(x-1)^2 + (x+1)^2 = 9(x+1)^2(x-1)^2$ $x^2 - 2x + 1 + x^2 + 2x + 1 = 9(x^2 + 2x + 1)(x^2 - 2x + 1)$ $2x^2 + 2 = 9(x^4 + 2x^3 + x^2 - 2x^3 - 4x^2 - 2x + x^2 + 2x + 1)$ $2x^2 + 2 = = 9x^4 - 18x^2 + 9$ $9x^4 - 20x^2 + 7 = 0$
	7	D	If $\log_2(10) = a$, what is the value of $\log_{16}(10)$? A) 4a B) a^4 C) $\sqrt[4]{a}$ D) 0.25a E) Answer not given. $\log_2(10) = a \rightarrow 10 = 2^a$ $\log_{16}(2^a) = a \cdot \log_{16}(2)$ Let $\log_{16}(2) = x$ $2 = 16^x = 16^{1/4}$, $x = 1/4$ Therefore, $\log_{16}(10) = a \cdot x = 0.25a$

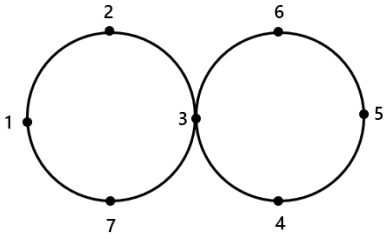
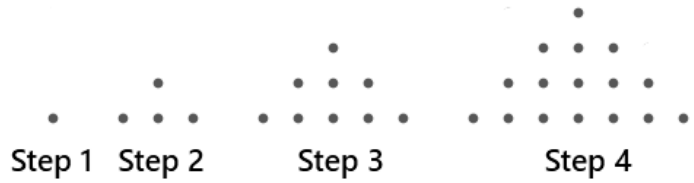
8		B	Express the following decimal number as a reduced fraction: $0.2\overline{09}$ A) 19/90 B) 23/110 C) 209/900 D) 211/990 E) Answer not given. x = 0.2090909… 1000x = 209.090909… 10x = 2.09090909 990x = 207 x = 207/990 = 23/110
	8	C	Express the following base-4 number as a reduced fraction in base-10: 1.32_4 A) 7/8 B) 19/16 C) 15/8 D) 21/8 E) Answer not given. 1.32_4 = $1x4^0 + 3x4^{-1} + 2x4^{-2} = 1 + \frac{3}{4} + 2/16 = 30/16 = 15/8$
9	9	A	A sequence is defined as follows: $a_n = 1 + a_{n-1} - a_{n-2}\ , for\ n \geq 3$ In other words, starting with the third term, each term is equal to 1 plus the previous term minus the term before that. The first term a_1 equals x, and the second term a_2 equals y. What is the sum of the first 2023 terms in the sequence, as an expression in terms of x and y? A) x + 2022 B) 2023 – y C) x D) x – y + 2023 E) Answer not given. By writing out the terms, can discover that the sequence repeats itself every 6 terms. The 6 are: x y 1 + y – x 2 – x 2 – y 1 – y + x 2023/6 = 337 r1 The sum of 1 group of the 6 terms is 6. Therefore, the sum of 337 groups = 337x6 = 2022. The remainder of 1 means to add the next term which is x, making the sum x + 2022.

10	**10**	***A***	Four standard 6-sided dice are rolled, and the lowest number of the four is discarded. What is the probability that the sum of the remaining three numbers is at least 17? A) 25/432 B) 7/108 C) 61/1296 D) 1/54 E) None of the above. 6666 – 1 way 666 + 1 other, 20 ways 21 ways to get 3 6's, which gives a sum of 18. The only other way to have at least 17 is to get 665 + one other. For each way to arrange the two 6's, there are 9 ways, for example: 6655 6654 6653 6652 6651 6645 6635 6625 6615 And, there are a total of 6 ways to arrange the two 6's, for a total of 6x9 = 54 ways. 54 + 21 = 75 favorable outcomes. Total number of outcomes = 6^4 = 1296. P = 75/1296 = 25/432

"Math is Cool" Masters -- 2023-24

High School

Team Test Solutions

9/ 10th	11/ 12th	Answer	Solution
1	**1**	6	 There are two tangent circles, each with radius 2 cm, with four numbers evenly spaced around the circumferences of each circle. An ant starts at point number 1 and walks a 'figure 8' path along the circumferences, going in order through points: 1, 2, 3, 4, 5, 6, 3, 7, 1 (one full circuit). The ant repeats this pattern until it has walked a distance of 165π, where it stops on a numbered dot. What number is the dot? The circumference of one circle = $\pi(2)^2 = 4\pi$. So one circuit = 8π. 165/8 = 20 r5. Therefore the ant makes 20 complete circuits, with 5 segments left to go. Five segments away from '1' will bring the ant to '6'.
2		25 [= x]	What is the value of 'x' in the sequence that begins as follows: 15, 20, 10, x, 5, 30, 0, … Pattern is: +5, -10, +15, -20, +25, etc. 10 + 15 = 25 = x
	2	104 [more dots]	The first four steps of a sequence are shown. How  many more dots are in Step 15 than are in Step 11? The sequence of dots begins: 1, 4, 9, 16, …, the squares of the positive integers. Therefore, there will be $15^2 = 225$ dots in Step 15, and 11^2 dots in Step 11. 225 – 121 = 104.

3		19 [multiple choice questions]	Ashwin scored 78 points on a biology test that had four essay questions worth 7 points each and 24 multiple-choice questions worth 3 points each. He answered all of the questions, and was incorrect on one of the essay questions. How many multiple-choice questions did he answer correctly? Y = no. of correct multiple choice (3)(7) + 3y = 78 3y = 57 Y = 19
	3	30 [1st hour classes]	There are 780 students that have a 1st hour class at Maryam Mirzakhani High School. The number of 1st hour classes is four more than the mean number of students in each 1st hour class. How many 1st hour classes are there? Let x = no. of 1st hour classes 780/x + 4 = x X^2 – 4x – 780 = 0 (x + 26)(x – 30) = 0 X = -26 or x = 30
4		2210	Multiplying the two largest prime numbers that are less than 50 results in a 4-digit integer. What is the largest possible number that can be obtained by rearranging the digits in that 4-digit integer? 47*43 = 2021 rearrange → 2210
	4	8773	Multiplying the two largest prime numbers that are less than 90 results in a 4-digit integer. What is the largest possible number that can be obtained by rearranging the digits in that 4-digit integer? 89*83 = 7387 rearrange → 8773
5		114	Evaluate f(x) when x = 4: $f(x) = 7(2)^x + 2$ = 7(16) + 2 = 114
	5	2 [=x]	What is the minimum value in the domain of the following function? $f(x) = 4 + \sqrt{x-2}$ x – 2 ≥ 0 x ≥ 2

<table>
<tr>
<td>6</td>
<td>6</td>
<td>4 [marmots]</td>
<td>A ‘madness’ of unusual marmots have just awoken from their winter hibernation on Mt. Rainier. Some of them have red fur, some have yellow fur, some have green fur, and the rest have blue fur. Some have long tails, and the rest have short tails. Additionally:

Four of them have red fur and long tails.
Eighteen of them have short tails.
Twelve of them have both short tails and either green or blue fur.
Six of them have blue fur and long tails.
Eight of them have both long tails and either yellow or green fur.
Thirty of them have fur that is either yellow, green or blue.

How many of the marmots have both yellow fur and short tails?
Create a two-way table with the fur color and tail status. Use logic to determine that there are 4 with yellow fur and short tails.
<table>
<tr><td></td><td>Red</td><td>Yellow</td><td>Green</td><td>Blue</td><td>Total</td></tr>
<tr><td>Long</td><td>4</td><td colspan="2">Sum of these two is 8</td><td>6</td><td>Must = 18</td></tr>
<tr><td>Short</td><td>Must = 2</td><td>Must = 4</td><td colspan="2">Sum of these two is 12</td><td>18</td></tr>
<tr><td>Total</td><td>Must = 6</td><td colspan="3">Sum of these three is 30</td><td>Must equal 36</td></tr>
</table>
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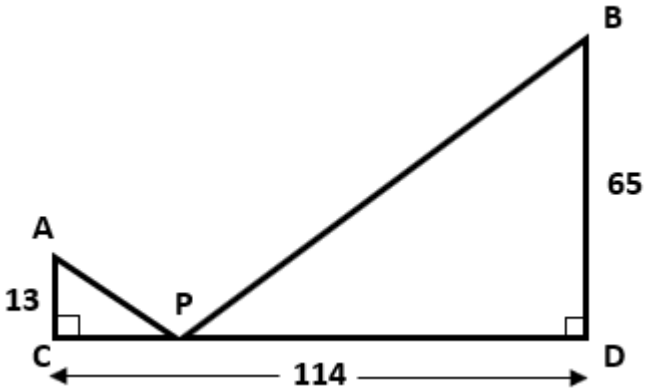
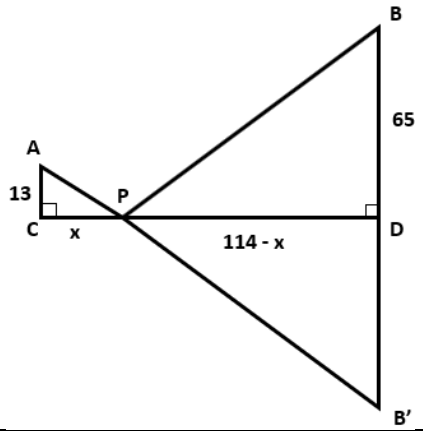
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>75 [%]</td>
<td>Two 8-sided dice, with sides numbered from 1 to 8, are rolled. What is the probability in percent that at least one of them shows a prime number?
Out of the 8 numbers, 4 are prime (2, 3, 5, 7).
P(at least 1 prime) = 1 – P(no prime) = 1 – (1/2)(1/2) = $\frac{3}{4}$ = 75%.</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>7 [= A + B]</td>
<td>Two circles, P and Q, are defined as follows. As a reduced common fraction, the slope of the line that passes through the points of intersection of the two circles can be written as A/B. What is A + B?
P: $(x - 5)^2 + (y - 3)^2 = 36$
Q: $(x - 2)^2 + (y - 7)^2 = 64$
Since the two shapes are circles, the line between the points of intersection will be perpendicular to the line between the centers. This follows from both points of intersection being equidistant from either center. By inspection, we can see the centers are (5, 3) and (2, 7), which define a line of slope m = -4/3. Therefore, a perpendicular line will have slope m = ¾.</td>
</tr>
</table>

	8	95 [units]	Point P is the point on $\overline{CD}$ for which the length $\overline{AP} + \overline{PB}$ is a minimum. Find the length of $\overline{PD}$ in units. All lengths shown on the graph are given in units.  Reflect B across CD to B'. Therefore PB = PB', and AP + PB' is also a minimum, making A, P and B' collinear. Therefore, triangle ACP is similar to triangle B'DP with a scale factor of 13/65 = 1/5. Therefore, CP/PD = x/(114 – x) = 1/5. Solve for x = 19, therefore PD = 114 – 19 = 95. 

9	**9**	1351 [points]	Find the number of integer grid points (x, y) in the region between $y = x^2$ and y = 100, including those lying on $y = x^2$ and y = 100. From: (1, 1) ➔ (1, 100) = 100 points (2, 4) ➔ (2, 100) = 97 points (3, 9) ➔ (3, 100) = 92 points (4, 16) ➔ (4, 100) = 85 points (5, 25) ➔ (5, 100) = 76 points (6, 36) ➔ (6, 100) = 65 points (7, 49) ➔ (7, 100) = 52 points (8, 64) ➔ (8, 100) = 37 points (9, 81) ➔ (9, 100) = 20 points (10, 100) ➔ = (10, 100) = 1 point sum = 625 Multiply by 2 to account for negative side. On the y-axis: (0, 0) ➔ (0, 100) = 101 points 625x2 + 101 = 1351
10		1878 [= median]	Consider the set of all 4-digit positive integers less than 2000, whose digits have a sum of 24. What is the median of this set of integers? The integers will be: 1 _ _ _ Therefore, the remaining three digits must sum to 23. The other 3 digits can be 995 (3 ways), 986 (6 ways), 977 (3 ways) or 887 (3 ways), for a total of 15 ways. The integers in order are: 1599 1689 1698 1779 1788 1797 1869 1878 1887 1896 1959 1968 1977 1986 1995 1878 is exactly in the middle, thus it is the median.

	10	44 [3-digit integers]	The 'digit sum' of a positive integer is found by summing its digits. For example, the digit sum of 274 is 2 + 7 + 4 = 13. The 'digital root' of a positive integer is found by repeatedly calculating the digit sum until a single digit is achieved. For example, the digit root of 274 is 1 + 3 = 4. How many positive 3-digit integers that are less than 500 have a digital root of 5? To get a digital root of 5 in one step, the 3 digits must be: 4, 0, 1 (4 ways), 3, 1, 1 (3 ways), 3, 2, 0 (4 ways) or 2, 2, 1 (3 ways), for a total of 14 ways. To get a digital root of 5 in two steps, the digital sum of the initial 3-digit number must be 14, then the digital root after the 2nd step will be 5. The three digits can be: 9, 4, 1 (4 ways), 9, 3, 2 (4 ways), 8, 5, 1 (2 ways), 8, 4, 2 (4 ways), 8, 3, 3, (2 ways), 7, 6, 1 (2 ways), 7, 5, 2 (2 ways), 7, 4, 3 (4 ways), 6, 6, 2 (1 way), 6, 5, 3 (2 ways), 6, 4, 4 (2 ways), 5, 5, 4 (1 way), for a total of 30 ways. It is impossible to get a digital root of 5 in three steps. The maximum sum of the digits is 4+9+9 = 22. The only number from 10 – 22 to have a 2-digit 'digit sum' is 19, 1+9 = 10, 1+0 = 1, not 5. Therefore the total is 14+30 = 44.

"Math is Cool" Masters -- 2023-24

High School

Pressure Round Solutions

9/ 10th	11/ 12th	Answer	Solution
1	1	10 [pairs]	A data set consisting of ten distinct positive integers has a mean of 23. The smallest 8 members of the set are: 12, 15, 16, 19, 20, 22, 24, and 27. How many possible pairs of integers could be the 9th and 10th members of the set? The order of the integers in the pairs does not matter. (10)(23) = 230, the sum of the 10 integers. The sum of the 8 given integers is 155. 230 – 155 = 75, which is the sum of the two remaining integers. The first possible pair is 28 and 47, through (37, 38), for a total of 10 pairs.
2		8 [mph]	Raven takes 20 minutes to ride her bike at an average rate of 6 miles per hour to school and then rides twice as fast along the same route back home. In miles per hour, what is Raven's overall average rate for the round trip? Average speed = the harmonic mean = 2ab/(a + b) a = 6 mph, b = 12 mph Average speed = (6)(12)/(6 + 12) = 8
	2	2 [= a + b]	The following expression can be simplified to a binomial in the form ax + b, given that x ≠ - 2. What is the value of a + b? $\frac{3x^2+5x-2}{x+2}$ $3x^2 + 5x - 2 = (x + 2)(3x - 1)$ a = 3, b = -1, a + b = 2
3	3	5 [side lengths]	Priya has a rope that is 200 cm long. She cuts the rope into four pieces, and uses one piece to form a square, and the remaining three pieces to form three identical equilateral triangles. All of the rope lengths are used exactly, with no overlap or gaps. All four shapes have integral side lengths, in cm. How many different side lengths are possible for the square? If a = square side length (cm) and b = triangle side length (cm), then: 4a + 9b = 200 a = (200 – 9b)/4 Therefore 200 – 9b must be divisible by 4. Can use some logic and trial and error to discover that b must equal 4, 8, 12, 16 or 20. Therefore, there are 5 possibilities.

4		90	What is the sum of the positive integral divisors of 360 that are not divisible by 3? $360 = 2^3 3^2 5^1$ Sum of divisors not divisible by 3 = $(2^0 + 2^1 + 2^2 + 2^3)(5^0 + 5^1)$ = 15 x 6 = 90
	4	5 [= x]	Let x be a number such that the following matrix does not have an inverse. What is x? $\begin{bmatrix} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & x \end{bmatrix}$ X = 5 will make the determinant = 0, therefore it will not have an inverse.
5		2,500,000 [cubic cm]	How many cubic centimeters are in 2.5 cubic meters? 1 cubic meter = 100x100x100 = 1000000 cubic cm. 2.5(1,000,000) = 2,500,000
	5	-2	Find the value of: $\log_2(\log_{16}(\log_5 25))$ $\text{Log}_5 25 = 2$ $\text{Log}_{16} 2 = \frac{1}{4}$ $\text{Log}_2(1/4) = -2$

"Math is Cool" Masters -- 2023-24

High School

College Bowl Round #1 Solutions

	Answer	Solution
1	8 [points]	What is the maximum number of distinct points of intersection that a circle can have with a square? 2 on each side of the square.
2	9 [integers]	How many composite integers are between thirty and forty, inclusive? 11 integers total, inclusive. 31 and 37 are prime, so 9 compositive integers.
3	0	What is the remainder when eight factorial is divided by six? 8! Includes factors of 6, so it divides out evenly.
4	120 [Cheez-its]	Biff and Eho each have the same number of Cheez-its. Biff eats two hundred thirty seven Cheez-its and Eho eats one hundred fifty three Cheez-its and still has four sevenths of his Cheez-its left. How many Cheez-its does Biff have left? x = starting no. for each x – 153 = (4/7)x (3/7)x = 153 x = 357 Biff has 357 – 237 = 120 left
5	22 [= A + B]	Yessica randomly picks two cards without replacement from a group of six cards numbered two, three, five, six, seven and ten. The probability that the product of the two numbers on the cards is a multiple of ten is a reduced common fraction A over B. What is A + B? There are 6C2 = 15 ways to select 2 cards. A 10 + any card will give a multiple of 10 (5 ways). A 5 + 2 or 5 + 6 will give a multiple of 10 (2 ways). Thus, P = 7/15. 7+15 = 22
6	30 [square units]	Find the area in square units of the triangle with vertices at the following points: six comma eight (pause) nine comma two (pause) and seventeen comma six. Shoelace method: Line the points up in columns: (6, 8) (9, 2) (17, 6) (6, 8) first point is repeated A = ½ times |(12 + 54 + 136) – (72 + 34 + 36)| = ½ times |60| = 30

7	190 [ways]	How many ways are there to choose exactly two pets from a pet rescue facility that currently has five hamsters, seven hedgehogs and eight pygmy goats? 20 total animals 20C2 = 190
8	-6 [=x]	Solve for x: Negative 4x plus fifteen equals negative 9x minus fifteen -4x + 15 = -9x – 15 5x = -30 X = -6
9	42	What is the next number in the sequence that begins as follows: Two, two, four, six, ten, sixteen, twenty-six, and so on. Fibonacci-like sequence, add the previous two terms.
10	50	What is the quotient when one billion is divided by the product of two to the eighth and five to the seventh? $2^8 5^7 = 2(10^7)$ $10^9/10^7 = 100$ 100/2 = 50

"Math is Cool" Masters -- 2023-24

High School

College Bowl Round #2 Solutions

	Answer	Solution
1	1 [kg]	A full can of kerosene weighs 8 kg. Half the kerosene is poured out of the can, after which the can weighs 4.5 kg. What is the weight of the empty can, in kilograms? Therefore, half the kerosene weighs 8 – 4.5 = 3.5 kg, so all of the kerosene weighs 7 kg, and the can weighs 1 kg.
2	36	What is the number of lines of symmetry for an equilateral triangle times the number of edges in a rectangular prism? 3x12 = 36
3	16 [= sum]	Three consecutive integers sum to eight hundred and thirty-four. What is the sum of the digits of the smallest number? x + (x + 1) + (x + 2) = 834 x = 277 2+7+7 = 16
4	30 [integers]	How many positive integers less than fifty are not divisible by either three or eleven? 16 are divisible by 3. 4 are divisible by 11. One (33) is an overlap. 16 + 4 – 1 = 19 are divisible by one or the other or both. Therefore, 49 – 19 = 30 are NOT divisible by either three or eleven.
5	90 [paths]	Point A and Point B are points in coordinate space. Point A has coordinates zero, zero, zero and Point B has coordinates two, two, two. How many unique paths are there from Point A to Point B that move from one lattice point to another along the straight lines connecting them in the positive x, y or z direction? A total of six straight moves need, two in the x direction, two in the y direction, and two in the z direction, or: xxyyzz The number of unique arrangements is: 6!/(2!2!2!) = 90
6	60 [nickels]	Mario has fifteen dollars in nickels, dimes and quarters. He has twice as many nickels as quarters. He has fifteen more dimes than quarters. How many nickels does Mario have? N = 2Q D = Q + 15 5N + 10D + 25Q = 1500 5(2Q) + 10(Q + 15) + 25Q = 1500 45Q = 1350 Q = 30, N = 60

7	8 [= A]	The six digit integer three A six A nine two is divisible by eleven. What is the value of A? If the alternating sum of digits is divisible by 11, then the number is divisible by 11. 3 – A + 6 – A + 9 – 2 16 – 2A Is only divisible by 11 if A = 8
8	50 [= mean]	What is the mean of the first ninety-nine counting numbers? Sum of first 99 = (99)(100)/2 Divide that by 99 to get the mean = 100/2 = 50
9	27 [%]	What is fifty-four divided by two hundred, expressed as a percentage? 54/200 = 27/100 = 27%
10	-36 [= product]	What is the product of the next two numbers in the arithmetic (proctor – pronounced air-ith-MET-ic) sequence that begins as follows: Forty-eight, thirty-five, twenty-two, and so on. Subtract 13 each time: 45, 35, 22, 9, -4

"Math is Cool" Masters -- 2023-24

High School

College Bowl Round #3 Solutions

	Answer	Solution
1	6561	What is nine to the fourth power? 9x9x9x9 = 6561
2	13 [cm]	If a rhombus with area twenty-six square centimeters has one diagonal of length four centimeters, what is the length in centimeters of the other diagonal? A = pq/2 26 = 4q/2 Q = 26/2 = 13 cm
3	60 [%]	A standard ten-sided die is numbered on its faces with the integers zero through nine. When the die is rolled, what is the probability as a percent that the number showing is five or less? There are 6 outcomes that are 5 or less. 6/10 = 60%
4	8 [hours]	If I can eat one bagel in four hours and Emily can eat one bagel in eight hours, how many hours would it take for us to eat three bagels together? a = 4 b = 8 time together for one bagel = (4)(8)/(4+8) = 32/12 time together for 3 bagels = 3(32)/12 = 8
5	24 [ounces]	Vivek has four pet rats, each of which weighs a whole number of ounces. The median weight of the rats is eleven ounces, and the mean weight of the rats is twelve ounces. What is the greatest possible difference between the weight of the heaviest and lightest rat, in ounces? The sum of the middle two must be 22, to give a median of 11. The total sum equals 48. 48 – 22 = 26. The lightest rat = 1 oz and the heaviest rat = 25 oz to give a difference of 25 – 1 = 24.
6	101 [= sum]	When the positive integer divisors of three hundred eighty-five are arranged from least to greatest, what is the sum of the fourth, fifth and sixth divisors? $385 = 5^1 7^1 11^1$ Divisors: 1, 5, 7, 11, 35, 55, … 11+35+55 = 101

7	12 [ways]	How many ways can the digits one, two, three and four be arranged in a line, such that no adjacent digits differ by more than two? Total ways = 4! = 24 However, 1 and 4 cannot be next to each other. Have to exclude: 1 4 2 3 1 4 3 2 4 1 2 3 4 1 3 2 2 1 4 3 3 1 4 2 2 4 1 3 3 4 1 2 2 3 1 4 3 2 1 4 2 3 4 1 3 2 4 1 24 – 12 = 12 ways.
8	12	What is thirty-nine sevenths divided by thirteen twenty-eighths? (39/7)/(13/28) = (39/7)*(28/13) = 12
9	60 [sq. units]	What is the area in square units of triangle ABC with vertices at point A two comma three, point B seventeen comma eleven and point C seventeen comma three? It is a right triangle with length 15 and height 8. A = 1/2(15)(8) = 60
10	128	Simply the expression: thirty-two raised to the seven-fifths $32^{7/5} = (32^{1/5})^7 = 2^7 = 128$

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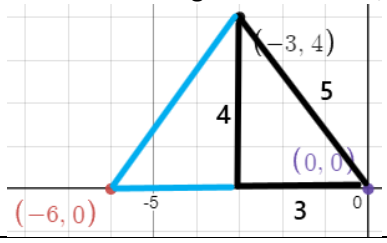
High School

College Bowl Round #4 Solutions

	Answer	Solution
1	12060 [seconds]	How many seconds are in three hours and twenty-one minutes? 3x60 + 21 = 201 minutes 201x60 = 12060
2	17	What is the largest prime factor of two thousand forty? $2040 = 2^3x3x5x17$
3	78 [sq units]	Twenty-seven unit cubes are arranged to form a larger three by three by three cube. The center unit cube from each face is then removed. In square units, what is the surface area of the resulting solid? Of the original surface area, which is 9x6 = 54, 6 square units have been removed, leaving 48. However, each "hole" in the center contains 5 square units. 48 + 6x5 = 78.
4	35	The sum of four consecutive odd integers is 128. What is the largest of the four integers? x+(x+2)+(x+4)+(x+6) = 128 4x = 116 X = 29, x + 6 = 35
5	16 [units]	A line segment has endpoints at A with coordinates zero comma zero and B with coordinates negative three comma four. Point C is the image of point B translated down four units and left three units. What is the perimeter of triangle ABC, in units? Two 3-4-5 triangles are formed, and the base = 6. 5+5+6 = 16 
6	2 [= median]	If all test scores are integers from zero to one hundred inclusive, what is the least possible median of five test scores that add up to two hundred four? Put the top two scores both at 100. The sum of the bottom three needs to equal 4. The minimum median is 2 to give the required total: 1 1 2 100 100

7	9 [= 5th term]	In order, the first three terms of an arithmetic sequence are x, six, and 2x minus three. What is the fifth term of the sequence? d = common difference $x + d = 6$ $6 + d = 2x - 3$ Solve system to find that x = 5, d = 1. Therefore the first 5 terms are: 5, 6, 7, 8, 9.
8	32	What is the sum of the entries in the sixth row of Pascal's triangle, where the first row consists of a single one? The sum of the elements of row 'n' = 2^n. n = 0 for the first row, so n = 5 for the sixth row. $2^5 = 32$. Alternatively, add: 1 + 5 + 10 + 10 + 5 + 1 = 32
9	1 [=x]	What is the greater of the two solutions to the following equation: negative five x-squared plus two x plus three equals zero. $-5x^2 + 2x + 3 = 0$ $5x^2 - 2x - 3 = 0$ $(5x + 3)(x - 1) = 0$ $x = -3/5, x = 1$
10	2 [whole numbers]	How many of the following represent whole numbers? Twelve, four-tenths, negative sixty-four, pi, zero divided by ten, five minus eight 12, 0.4, -64, π, 0/10 = 0, 5 – 8 = -3 Whole numbers include the counting numbers and 0. They do not include fractions, decimals, or negative numbers. 12 and 0 are the only whole numbers.

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College Bowl Round #5 Solutions

	Answer	Solution
1	11114	What is the sum of eleven, one hundred one, one thousand one, and ten thousand one? 11 + 101 + 1001 + 10001 = 11,114
2	60 [°]	What is the measure in degrees of one exterior angle of a regular hexagon? The sum of the exterior angles for any polygon = 360°. 360°/6 angles = 60° per angle.
3	42 [Zoops]	If two Bleps equals fifteen Drips and five Drips equals twenty-eight Zoops, how many Zoops equal one Blep? 2B = 15D 5D = 28Z ➔ 15D = 84Z = 2B 1B = 42Z
4	27 [=b]	What is the value of 'b' in the following geometric sequence: nine, a, b, c, eighty-one r = $\sqrt{3}$ 9 * $\sqrt{3}$ * $\sqrt{3}$ = 27
5	6 [hours]	Josie can solve 30 problems in 4 hours when she works alone. Tessa can solve 33 problems in 6 hours when she works alone. If they both work together the entire time, how many hours will it take them to solve 78 problems? J: 30/4 = 7.5 p/hour T: 33/6 = 5.5 p/hour J+T = 13 p/hour 78/13 = 6 hours
6	10000 [= x]	Find the value of x if log base ten of x equals four. $x = 10^4$
7	64	A yogurt stand offers 6 different kinds of toppings: sprinkles, m&ms, cookie crumbs, whipped cream, licorice bites and sweet pickles. You can put as many different toppings on as you want, from 0 to 6. How many different ways can a frozen yogurt be made by picking from these toppings? For each topping there are two choices – either add it or not. Therefore, the number of ways = 2^6 = 64
8	4 [divisors]	How many divisors of sixty-four are perfect squares? 1, 4, 16, 64

9	44	Seventy-five percent of a number is eighty-eight. What is three-eighths of the number? ¾ = 6/8, therefore if 6/8 of the number = 88, then 3/8 is half of that, or 44.
10	5 [times]	If one order of fries and five burgers costs twice as much as three orders of fries and two burgers, how many times as much does a burger cost compared to one order of fries? F + 5B = 2(3F + 2B) = 6F + 4B 1B = 5F

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College Bowl Round #6 Solutions

	Answer	Solution
1	91	What is the sum of the first six positive perfect squares? 1+4+9+16+25+36 = 91
2	-25 [= y coordinate]	What is the y-coordinate of the vertex of the parabola given by f of x equals quantity x minus four times quantity x plus six? $f(x) = (x – 4)(x + 6) = x^2 + 2x – 24$ x-coor = -b/2a = -2/2 = -1 y-coor = (-1 – 4)(-1 + 6) = -25
3	61 [%]	There are two hundred students at this competition. Twenty-two are in Geometry class, fifty-six are in Algebra two, sixty are in Pre-Calc and sixty-two are in Calculus. If one student is randomly selected for the grand prize, what is the probability in percent that they are either in Pre-Calc or Calculus? (60 + 62)/200 = 0.61 = 61%
4	81 [factors]	How many positive factors does one hundred million have? $100,000,000 = 10^8 = 2^8 5^8$ Therefore, it has (8 + 1)(8 + 1) = 81 factors
5	55 [product]	Point P with coordinates four comma eight is dilated by a scale factor of three halves around the point two comma two. What is the product of the x and y coordinates of the new dilated point? (4, 8) is 2 points to the left of (2, 2) and 6 points above. Therefore move an additional 1 point to the left and 3 points up, ending at the point (5, 11). 5x11 = 55
6	8 [= sum]	What is the sum of the terms in the infinite sequence that begins: four plus two plus one plus one-half and so on. S = a1/(1 – r) = 4/(1 – ½) = 4/(1/2) = 8
7	2 [people]	At the end of a party, everyone present shakes hands one time with every other person. A latecomer arrives and shakes hands only with the people that he knows. Altogether, sixty-eight handshakes occurred. How many people did the latecomer know? Handshake formula is: n(n – 1)/2 If n = 12 people, there are 66 handshakes. Therefore the latecomer must have done two handshakes.
8	26 [= mean]	What is the mean of the prime numbers between twenty and thirty? (23+29)/2 = 26

9	10	If x equals negative three, evaluate the following expression: six minus two minus 2x 4 – 2x 4 – 2(-3) 4 – (-6) = 4 + 6 = 10
10	15 [= x]	The product one hundred twenty eight times two hundred fifty six can be written as two raised to the power of x. What is x? $128 \cdot 256 = 2^7 \cdot 2^8 = 2^{15}$

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High School

College Bowl Extra Questions Solutions

	Answer	Solution
1	-2 [= slope]	What is the slope of the line that goes through the points seven-halves comma negative one and negative fifteen-halves comma twenty-one? (7/2, -1) and (-15/2, 21) Slope = (21 - -1)/(-15/2 – 7/2) = 22/(-22/2) = 22/-11 = -2
2	85 [cents]	A raffle has two hundred free tickets. One ticket will win a one-hundred seventy dollar prize. The remaining tickets will win nothing. If you have one ticket, what is your expected winnings in cents? The probability of winning is 1/200. Since there is no cost or penalty to not winning, the expected value is (1/200)(170) = 0.85 = 85 cents.
3	-12 [= cube root]	What is the cube root of negative one thousand seven hundred twenty-eight? (-12)(-12)(-12) = -1728
4	363 [cm^2]	Using a value of three to approximate pi, what is the surface area in square centimeters of a sphere with a diameter of eleven centimeters? SA = $4\pi r^2$ = 4(3)(11/2)^2 = 3*121 = 363
5	6 [= x]	What is the smallest positive integer value of x for which fifty-four times x is a perfect square? $54 = 2x3x3^2$ Therefore, need additional factors of 2 and 3 to get a perfect square.
6	135 [= the sum]	If the mean of five numbers is twenty-seven, what is the sum of the five numbers? 27*5 = 135