"Math is Cool" Masters -- 2023-24

6th Grade

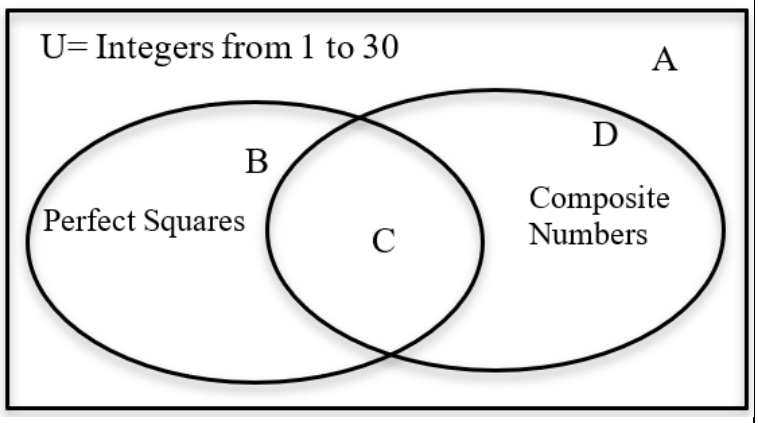
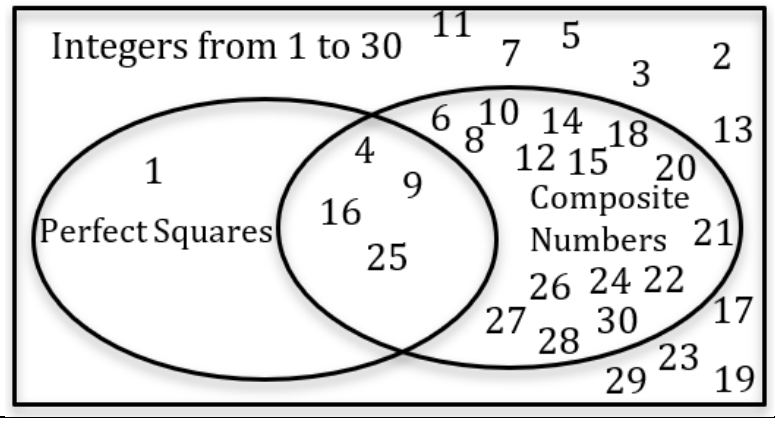
Mental Math Solutions

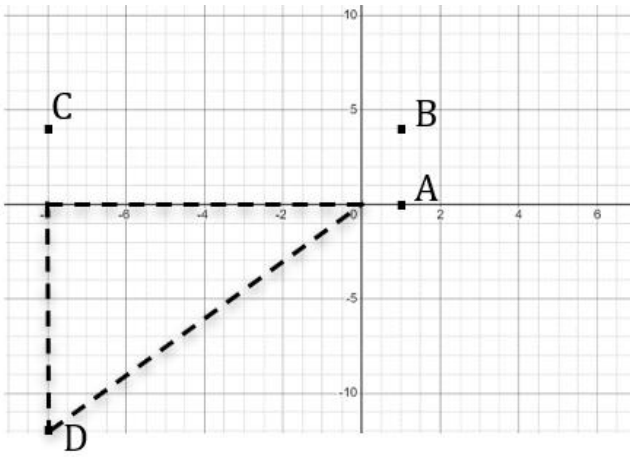
6th	Answer	Solution
1	70	What is the sum of the largest prime number less than thirty and the smallest prime number greater than forty? **$29 + 41 = 70$**
2	39 [integers]	How many integers are there from twenty-seven to sixty-five inclusive? **$65 - 26 = 39$**
3	[5x =] 60	If three X equals thirty-six, then what does five X equal? **$3x = 36 \rightarrow x = 12 \rightarrow 5x = 60$**
4	[A + B =] 51	The probability of drawing a red ten from a standard deck of cards is one over twenty-six. As a reduced common fraction, the probability of not drawing a red ten is A over B. What is A plus B? **P(not red 10) = 50/52 = 25/26, and 25 + 26 = 51**
5	72 [pints]	An amphora was a unit of measurement of volume in Greco-Roman times and is equal to nine U.S. customary gallons. How many pints are in an amphora? **There are 8 pints to a gallon, so $9 \cdot 8 = 72$**
6	64 [jelly beans]	A bowl has thirty red jelly beans, twenty green jelly beans, fifteen blue jelly beans, six white jelly beans, and eleven black jelly beans. If jelly beans are taken from the bowl at random, how many must be taken to guarantee two green jelly beans have been taken? **$30 + 15 + 6 + 11 + 2 = 64$**
7	144 [seconds]	Jacob takes three minutes to run a lap on a certain track. Jackson runs at an average rate that is one-and-a-quarter times as fast as Jacob. In seconds, how long does it take Jackson to run a lap on the same track? **1.25 as fast means 4/5 the time, so $4/5 \cdot 180 = 144$**
8	[P + Q =] 23	If A is seventy percent of B, and B is five-eighths of C, then as a reduced common fraction A over C equals P over Q. What is P plus Q? **Let A = 7, then B = 10. If B = 10, then C = 16. Then A/C = 7/16, and 7 + 16 = 23**

"Math is Cool" Masters -- 2023-24
6th Grade
Individual Test Solutions

6th	Answer	Solution
1	30 [%]	The probability of rain on a certain Monday is 70%. As a percent, what is the probability that it won't rain on that same Monday? **100 – 70 = 30**
2	4 [cards]	In a standard 52-card deck of playing cards, how many cards are red multiples of 5? **Two red 5s and two red 10s make 4 cards**
3	6 [orders]	In how many orders can three friends stand in line at a concession stand? **3! = 6**
4	20 [cats]	The ratio of cats to dogs in a shelter is 5:4. How many cats are there if there are 16 dogs in the shelter? **5/4 = x/16 → x = 20**
5	85	What is the largest 2-digit multiple of 17? **17 · 5 = 85 and 17 · 6 = 102, so the answer is 85**
6	47 [nickels]	Kyla has $2.35 worth of nickels. How many nickels does she have? **235/5 = 47**
7	0	Evaluate 3x – 15y if x = 45 and y = 9. **3·45 – 15·9 = 0**
8	50 [cm]	How many centimeters are in one-half of a meter? **There are 100 cm in 1 m, so there are 50 cm in ½ m**
9	200 [words per minute]	Cici reads a 1000-word article in five minutes. How many words per minute is Cici's reading rate? **1000/5 = 200**
10	64 [inches]	Tim is 5 feet 8 inches tall, Tara is 5 feet 3 inches tall, and Tanya is 5 feet 1 inch tall. In inches, what is the mean of the three heights? **5 ft 8 in = 68 in, 5 ft 3 in = 63 in, and 5 ft 1 in = 61 in, so (68 + 63 + 61)/3 = 192/3 = 64**
11	49	Evaluate: $3 \cdot 5^2 - 104/4$ **$3 \cdot 5^2 - 104/4 = 75 - 26 = 49$**
12	25 [%]	What percent of 56 is 14? **14/56 = x/100 → x = 25**

13	73	The first three terms of an arithmetic sequence are 3, 13, 23. What is the eighth term in the sequence? **To get the 8th terms add 10 to 3 seven times, or $3 + 10 \cdot 7 = 73$.**
14	[a + b =] 15	My pin consists of three digits a, b, and c, where a < b < c. If b – a = c – b, and c = 9, then what is the largest possible value of a + b? **The pin could 159, 369, 579, or 789, so the largest possible value of a + b = 7 + 8 = 15**
15	10 [cm]	A 10 cm by 15 cm rectangle is divided into 25 smaller congruent rectangles. What is the perimeter in cm of one of the smaller rectangles. **$10 \cdot 15 = 150$ and $150/25 = 6$, so the area of each of the 25 smaller rectangles is 6 and the dimensions must be 2 x 3, since $10/5 = 2$ and $15/5 = 3$. The perimeter is $2 \cdot 2 + 2 \cdot 3 = 10$.**
16	57600 [ft^2]	An American football field is 120 yards by $53\frac{1}{3}$ yards. In square feet, what is the area of an American football field? **$120 \cdot 3 = 360$ and $53\frac{1}{3} \cdot 3 = 160$ and $360 \cdot 160 = 36000 + 21600 = 57600$**
17	66 [minutes]	Four rogues can disarm 12 traps in 44 minutes. In minutes, how long will it take 2 rogues to disarm 9 traps? **4 rogues : 12 traps : 44 min** **1 rogue : 3 traps : 44 min** **2 rogues : 3 traps : 22 min** **2 rogues : 9 traps : 66 min**
18	6	Evaluate: $1 + \frac{1}{2} + \frac{1}{\frac{1}{2}} + \frac{1}{\frac{1}{\frac{1}{2}}} + \frac{1}{\frac{1}{\frac{1}{\frac{1}{2}}}}$ **$1 + \frac{1}{2} + \frac{1}{\frac{1}{2}} + \frac{1}{\frac{1}{\frac{1}{2}}} + \frac{1}{\frac{1}{\frac{1}{\frac{1}{2}}}} = 1 + 1/2 + 2 + 1/2 + 2 = 6$**
19	[A + B =] 31	A rectangular prism has dimensions 2 cm by 3 cm by 5 cm. A second rectangular prism has dimensions 4 cm by 9 cm by 25 cm. As a reduced common fraction, the ratio of the volume of the first prism to the volume of the second prism is A/B. What is A + B? **$V_{small} = 2 \cdot 3 \cdot 5 = 30$** **$V_{large} = 4 \cdot 9 \cdot 25 = 900$** **$V_{small}/V_{large} = 30/900 = 1/30$ and $1 + 30 = 31$**

20	[D – A =] 5	 In the Venn Diagram shown here, the Universal set is all integers from 1 to 30, inclusive. Let A = the number of integers inside the rectangle, but outside the two overlapping ovals. Let B = the number of integers inside the Perfect Squares oval not including the part where it overlaps with the Composite Numbers oval. Let C = the number of integers in the overlap of the two ovals. Let D = the number of integers in the Composite Numbers oval not including the overlap with the Perfect Squares oval. What is D – A? **In the completed Venn Diagram below there are 15 integers in section D and 10 integers in section A, so D – A = 15 – 10 = 5** 
21	[x =] 5	What is the sum of all possible solutions for x in the following equation? $\sqrt{21-x} = x - 1$ **$\sqrt{21-x} = x - 1 \rightarrow 21 - x = x^2 - 2x + 1 \rightarrow 0 = x^2 - x - 20 \rightarrow 0 = (x - 5)(x + 4) \rightarrow x = 5$ and $x = -4$, but $\sqrt{x}$ is defined as just the positive solutions, so $x = -4$ is not a solution.**
22	5 [%]	A standard 8-sided die (numbered 1 – 8) is rolled with a standard 20-sided die (numbered 1 – 20). As a percentage, what is the probability that doubles are rolled? **There are $8 \cdot 20$ possible pairs, and 8 of the possible pairs are doubles, so P(doubles) = 8/160 = 1/20= 5/100 = 5%**

23	[X + Y =] 17	On a coordinate plane, point A is 1 unit to the right of the origin. Point B is 4 units directly above point A, point C is 9 units directly to the left of point B and point D is 16 units directly below point C. In simplest radical form, point D is $X\sqrt{Y}$ units from the origin. What is X + Y? **The distance from D to the origin is the length of the hypotenuse of a right triangle shown in the diagram below with legs of 8 and 12, so the distance is $\sqrt{8^2 + 12^2} = \sqrt{208} = 4\sqrt{13}$ and 4 + 13 = 17.** 
24	[C – A =] 8	Four positive integers are represented by A, B, C, and D. The positive difference between A and D is 10. The positive difference between B and C is 9. The positive difference between the two smallest integers is 1. The largest of the four integers is represented by D, and the smallest of the four integers is represented by B. What is the value of C – A? **The order of the four letters from largest to smallest is DCAB. Let D = any integer greater than 11, such as 12. If D = 12 and A = 2, then B = 1 or 3, but B is the smallest integer, so it must equal 1, and so C = 10. Therefore, C – A = 10 – 2 = 8**
25	[A + B =] 115	The solution to the following inequality is x > N, where N is a reduced common fraction in the form A/B. What is A + B? $$\frac{5}{3}x - \frac{7}{9} > \frac{1}{6}x + \frac{11}{12}$$ **$\frac{5}{3}x - \frac{7}{9} > \frac{1}{6}x + \frac{11}{12} \rightarrow 60x - 28 > 6x + 33 \rightarrow 54x > 61 \rightarrow x > 61/54$ and 61 + 54 = 115**

26	78	What is the largest 2-digit integer that is the product of three distinct prime numbers? **$2 \cdot 3 \cdot 5 = 30$** **$2 \cdot 3 \cdot 7 = 42$** **$2 \cdot 3 \cdot 11 = 66$** **$2 \cdot 3 \cdot 13 = 78$** **$2 \cdot 3 \cdot 17 = 102$** **$2 \cdot 5 \cdot 7 = 70$** **$2 \cdot 5 \cdot 11 = 110$** **$3 \cdot 5 \cdot 7 = 105$** **Going systematically through the list of numbers that are the product of three primes shows that 78 is the largest 2-digit integer that is the product of 3 distinct primes.**
27	36	A data set of seven positive integers has a distinct mode equal to 10. The mean and median of the set are also 10. What is the largest possible number in the set? **Since the median is 10, the fourth number needs to be 10. To maximize the largest number, make the 4^{th}, 5^{th}, and 6^{th} numbers all 10, and the first three numbers can be 1, 1, and 2, as opposed to 1, 1, and 1 where 10 would no longer be the unique mode. So, the largest number would be 70 – 10 – 10 – 10 – 1 – 1 – 2 = 36**
28	15 [integers]	How many 3-digit integers exist such that the product of the three digits is 18? **129 → 3! or 6 arrangements** **136 → 3! or 6 arrangements** **233 → 3!/2! or 3 arrangements** **6 + 6 + 3 = 15**
29	[P + Q =] 23	Let A equal 200% of B, 400% of C, and 600% of D. As a reduced common fraction (B + C + D)/A = P/Q. What is P + Q? **Since A is 2 times B, 4 times C, and 6 times D, let A be an integer that is divisible by 2, 4, and 6, such as 12. If A = 12, then B = 6, C = 3, and D = 2, and (6 + 3 + 2)/12 = 11/12 and 11 + 12 = 23.**
30	[N =] 6	Let 405/144 be the first term of a geometric sequence with a common ratio of 2/3. Let the N^{th} term of the sequence be the first term with a value less than 1/2. What is N? **405/144, 135/72, 45/36, 15/18, 5/9, 10/27, . . . The first term that is less than 1/2 is 10/27 which is the 6^{th} term, so N = 6.**

<table>
<tr>
<td>31</td>
<td>[A + B =] 811</td>
<td>As a reduced common fraction, the median of the following data set is A/B. What is A + B?
$\frac{8}{11}, \frac{24}{27}, \frac{20}{23}, \frac{16}{19}, \frac{28}{31}, \frac{12}{15}$
Since the difference between the denominator and the numerator in every fraction is 3, the larger the numerator and denominator are, the closer each fraction is to one, so ascending order would be $\frac{8}{11}, \frac{12}{15}, \frac{16}{19}, \frac{20}{23}, \frac{24}{27}, \frac{28}{31}$. The median is the average of $\frac{16}{19}$ and $\frac{20}{23}$ or $\frac{\frac{16}{19}+\frac{20}{23}}{2} = \frac{\frac{368+380}{437}}{2} = \frac{748}{874} = \frac{374}{437}$ and 374 + 437 = 811</td>
</tr>
<tr>
<td>32</td>
<td>100021 [base 3] [100021_3]</td>
<td>What is the product of $101_2 \cdot 302_4$ in base 3?
$101_2 = 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 5$
$302_4 = 3 \cdot 4^2 + 0 \cdot 4^1 + 2 \cdot 4^0 = 50$
$5 \cdot 50 = 250 = 1 \cdot 243 + 0 \cdot 81 + 0 \cdot 27 + 0 \cdot 9 + 2 \cdot 3 + 1 \cdot 3^0 = 1 \cdot 3^5 + 0 \cdot 3^4 + 0 \cdot 3^3 + 0 \cdot 3^2 + 2 \cdot 3^1 + 1 \cdot 3^0 = 100021_3$.</td>
</tr>
<tr>
<td>33</td>
<td>3061</td>
<td>A sequence of integers is generated according to the following rules. If any term is a perfect square, the next term is its square root. If a term is an even number and not a perfect square, it has an odd number added to it. The first non-perfect square even number in the sequence has 1 added to it, the second non-perfect square even number has 3 added to it, and so on, adding a successive odd number to each successive non-perfect square even number. Successive non-perfect square odd numbers in the sequence are multiplied by successive even numbers, beginning with the first non-perfect square odd number being multiplied by 2, the next non-perfect square odd number being multiplied by 4, and so on. What is the 14th term in the sequence if the first term is 10?
The series looks like this:
<table>
<tr><td>1</td><td>10</td></tr>
<tr><td>2</td><td>10+1=11</td></tr>
<tr><td>3</td><td>11·2=22</td></tr>
<tr><td>4</td><td>22+3=25</td></tr>
<tr><td>5</td><td>$\sqrt{25}$=5</td></tr>
<tr><td>6</td><td>5·4=20</td></tr>
<tr><td>7</td><td>20+5=25</td></tr>
<tr><td>8</td><td>$\sqrt{25}$=5</td></tr>
<tr><td>9</td><td>5·6=30</td></tr>
<tr><td>10</td><td>30+7=37</td></tr>
<tr><td>11</td><td>37·8=296</td></tr>
<tr><td>12</td><td>296+9=305</td></tr>
<tr><td>13</td><td>305·10=3050</td></tr>
<tr><td>14</td><td>3050+11=3061</td></tr>
</table>
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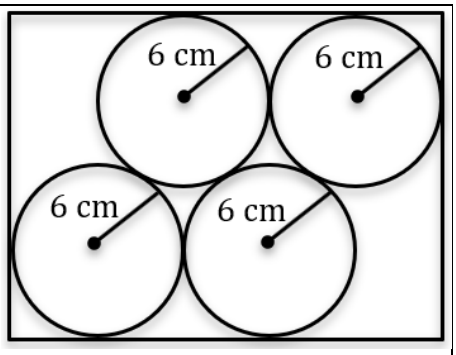
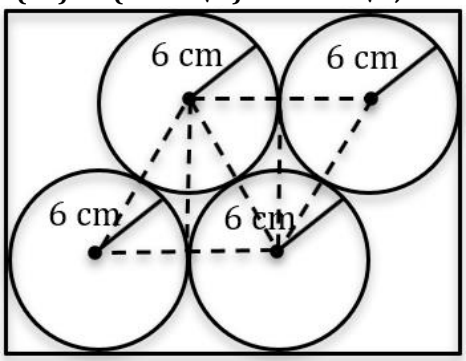
</tr>
</table>

34	[A + B =] 1114	An unfair penny has a probability of flipping heads of 3/5, an unfair nickel has a probability of flipping heads of 5/9, and an unfair dime has a probability of flipping heads of 6/13. When the three coins are flipped together, as a reduced common fraction the probability of at least one of the coins showing heads is A/B. What is A + B? **P(at least one heads) = 1 – P(TTT)** **$1 - 2/5 \cdot 4/9 \cdot 7/13 = 585/585 - 56/585 = 529/585$ and 529 + 585 = 1114**
35	72 [seconds]	Biff and Eho are running at steady rates in opposite directions on a circular track. Biff takes 120 seconds to complete a lap while it takes Eho 180 seconds to complete a lap. Once they pass each other, how many seconds will it take them to pass each other again? **Eho takes 3 seconds to go the same distance as Biff in 2 seconds. So, Biff can go 1.5 the distance in the same amount of time as Eho. If Eho's distance before they pass each other again is d, then Biff's distance before they pass is 1.5d and the ratio of Biff's distance to Eho's distance is $1.5d/d = 3/2$. In other words, they will pass each other when Biff has traveled 3/5 of the circle and Eho has traveled 2/5 of the circle. The number of seconds will be $3/5 \cdot 120 = 2/5 \cdot 180 = 72$.**
36	720	The vertices of a regular octahedron are labeled with the integers 1 through 6, in some order, with each integer used exactly once. Each edge of the octahedron is labeled with the sum of the integers at its endpoints. Each face is labeled with the product of the labels of its three edges. What is the largest possible difference between the labels of any two adjacent faces? **The face with the largest label will have vertices labeled with 4, 5, and 6, edges labeled with 4 + 5 = 9, 4 + 6 = 10, and 5 + 6 = 11. The label of this face will be $9 \cdot 10 \cdot 11 = 990$. An adjacent face must share exactly one edge with this first face, so the smallest possible label of an adjacent face will be the one that shares the two smallest vertices, 4 and 5, and has the third vertex labeled 1. The edges of this adjacent face will be labeled with 5 = 1 + 4, 6 = 1 + 5, and 9 = 4 + 5, so the label for the adjacent face would be $5 \cdot 6 \cdot 9 = 270$. Then 990 – 270 = 720.**

37	[A + B + C =] 99	 In the figure shown, there are four congruent circles that are externally tangent to each other and that are internally tangent to the sides of the surrounding rectangle. The radius of each circle is 6 cm. In centimeters, the perimeter of the rectangle is A + B$\sqrt{C}$. What is A + B + C? Note: the value of C does not have a factor that is a perfect square other than 1. **The width of the rectangle is 5 · 6 = 30. The height of the rectangle is 2 · 6 plus the height of the dashed equilateral triangles shown below. The height of the dashed equilateral triangles is also the length of the long leg of the 30-60-90 triangles that are created by drawing in the heights. The short leg of these 30-60-90 triangles is 6, so the long leg and the height of the equilateral triangles is $6\sqrt{3}$. Therefore, the height of the rectangle is $12 + 6\sqrt{3}$. The perimeter of the rectangle is $2(30) + 2(12 + 6\sqrt{3}) = 84 + 12\sqrt{3}$, and 84 + 12 + 3 = 99.** 

38	[A + B =] 4	A box contains only dimes and nickels. If there were 10% more dimes, there would be 4% more money in the box. As a reduced common fraction, the ratio of the original number of dimes to the original number of nickels in the box is A/B. What is A + B? **Let D = the original number of dimes in the box** **Let N = the original number of nickels in the box** **Let M = the combined value in cents of the original number of dimes and nickels in the box** **10D + 5N = M - (1)** **11D + 5N = 1.04M - (2)** **Subtracting (2) – (1) above yields:** **D = 0.04M - (3)** **Multiply both sides by 10** **10D = 0.4M - (4)** **Substitute 0.4M for 10D in (1) to get:** **5N = 0.6M - (5)** **Divide both sides by 5 to get:** **N = 0.12M - (6)** **Divide (3) by (6) to get:** **D/N = 0.04M/0.12M = 1/3 and 1 + 3 = 4**
39	[A + B + C =] 15	Consider the equation: $\sqrt{88 + 30\sqrt{7}}$ = A + B$\sqrt{C}$ where A, B, and C are positive integers and C does not have a factor that is a perfect square other than 1. What is A + B + C? **If $\sqrt{88 + 30\sqrt{7}}$ = A + B$\sqrt{C}$, then 88 + 30$\sqrt{7}$ = (A + B$\sqrt{C}$)^2 = A^2 + 2AB$\sqrt{C}$ + B^{2}C, which means A^2 + B^{2}C = 88 and 2AB$\sqrt{C}$ = 30$\sqrt{7}$. So, C = 7 and A and B are a factor pair of 15, such as 1 and 15, or 3 and 5. Substituting 7 for C and guessing and checking with A and B in the equation A^2 + B^{2}C = 88 yields A = 5 and B = 3. So, 5 + 3 + 7 = 15.**

40	660	In a hand of 7 cards drawn from multiple standard decks there are two identical Ace of Spades, two identical Jack of Hearts, and three cards that are distinct from the Aces and Jacks and from each other. In how many distinct ways can the cards be put in order if the Aces cannot be next to each other and the Jacks cannot be next to each other? **If there were no restrictions, the number of arrangements would be 7!/(2!2!) = 1260. Count the number of orders in which the Aces are together plus the number of orders in which the Jacks are together when the Aces are not together and subtract from 1260. Whenever the Aces are together the remaining 5 cards can be arranged 5!/2! = 60 ways. If you ignore the other cards there are 6 ways the Aces can be together (see cases 1 – 6 below), so 6(60) = 360 would be the total number of orders involving two adjacent Aces. There would then also be 360 orders involving two adjacent Jacks, but some of them have already been counted as part of the orders involving two adjacent Aces. To count the orders that simultaneously involve two adjacent Aces and two adjacent Jacks, consider the following cases and the number of orders involving adjacent Jacks for each one:** **A A _ _ _ _ _ - 4(3!) = 24 - case 1** **(For this first case there are 4 possible locations for adjacent Jacks** **A, A, J, J, _, _ , _** **A, A, _, J, J, _, _** **A, A, _ _, J, J, _** **A, A, _, _, _ J, J** **And for each of these four locations, the remaining three cards can be arranged in 3! orders, which is where the 4(3!) calculation comes from.)** **_ A A _ _ _ _ - 3(3!) = 18 - case 2** **_ _ A A _ _ _ - 3(3!) = 18 - case 3** **_ _ _ A A _ _ - 3(3!) = 18 - case 4** **_ _ _ _ A A _ - 3(3!) = 18 - case 5** **_ _ _ _ _ A A - 4(3!) = 24 - case 6** **Therefore, the total number of arrangements of adjacent Jacks when Aces are not adjacent would be 360 – (24(2) + 18(4)) = 360 – 120 = 240.** **So, the final answer would be 1260 – (360 + 240) = 660**

"Math is Cool" Masters -- 2023-24

6th Grade

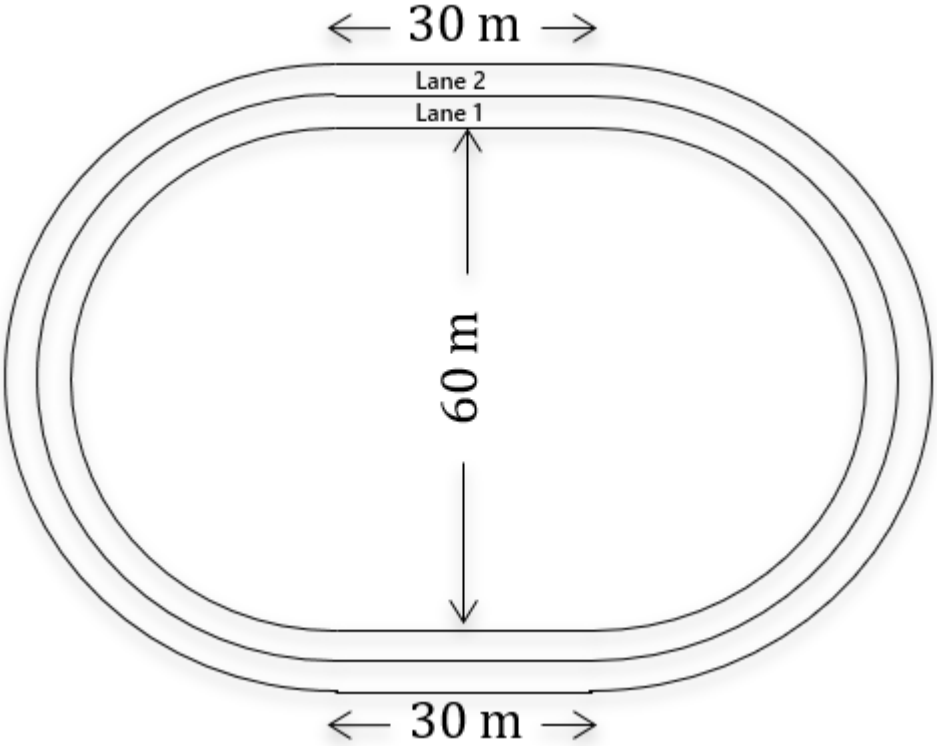
Multiple Choice Solutions

6th	Answer	Solution
REFER TO THE FOLLOWING INFORMATION FOR PROBLEMS #1 THROUGH #3. A semiprime is a natural number that is the product of exactly two prime numbers (not necessarily distinct). For example, 22 is a semiprime, because 22 = 2 · 11 and both 2 and 11 are prime numbers, whereas 12 is not a semiprime, because it has at least one factor that is a composite number.		
1	D	What is the largest 2-digit semiprime? A) 91 B) 93 C) 94 D) 95 E) 97 **90 is divisible by 9, 91 = 7 · 13, 92 is divisible by 4, 93 = 3 · 31, 94 = 2 · 47, 95 = 5 · 19, 96 is divisible by 4, 97 is prime, 98 is divisible by 49, 99 is divisible by 9, so the answer is 95**
2	C	Some semiprimes are perfect squares. What is the sum of all 2-digit perfect-square semiprimes? A) 34 B) 58 C) 74 D) 87 E) 155 **$5^2 = 25$ and $7^2 = 49$, and 25 + 49 = 74**
3	E	What is the positive difference between the largest 3-digit perfect-square semiprime and the smallest 2-digit non-perfect-square semiprime? A) 936 B) 937 C) 945 D) 947 E) 951 **2 · 5 = 10 is the smallest 2-digit non-perfect-square semiprime.** **$31^2 = 961$ and is the largest 3-digit perfect-square semiprime.** **961 – 10 = 951**
REFER TO THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #4 THROUGH #7. Consider the following data set: {10, 12, 14, 16, 18, 20}		
4	A	What is the mean of the data set? A) 15 B) 18 C) 19 D) 20 E) 21 **(10 + 12 + 14 + 16 + 18 + 20)/6 = 15**

5	B	How many pairs of integers between 10 and 20 inclusive can be added to the data set, such that the mean does not change? Note: the order of the two integers does not matter and they may be the same as each other. A) 5 B) 6 C) 10 D) 11 E) 12 **Since the mean is 15, any pair of numbers that add up to $2 \cdot 15 = 30$ would keep the mean the same.** **(10, 20), (11, 19), (12, 18), (13, 17), (14, 16), and (15, 15) make 6 pairs.**
6	A	A random number in the set is replaced with a different integer between 10 and 20 inclusive. In other words, a number cannot be replaced by itself. What is the probability that the new median is greater than 16? A) 1/5 B) 7/30 C) 4/15 D) 3/10 E) 1/3 **10, 12, or 14 would need to be replaced by 17, 18, 19, or 20, so P(median > 16) = 4/10 = 2/5.** **16, 18, or 20 can be replaced by any number in the set and the median will never be more than 16, so P(median > 16) = 0.** **P(median > 16) = $\frac{1}{2} \cdot \frac{2}{5} + \frac{1}{2} \cdot 0 = 1/5$.**

7	B	How many data sets consisting of 6 distinct two-digit positive integers have a mean of 14? A) 23 B) 26 C) 27 D) 28 E) 30 **$6 \cdot 14 = 84$** **There is 1 way for the smallest 5 numbers to be 10, 11, 12, 13, and 14: {10, 11, 12, 13, 14, 24}. Not counting the first set mentioned above, there are 4 ways for the first four numbers to be 10, 11, 12, and 13: {10, 11, 12, 13, 15, 23}, {10, 11, 12, 13, 16, 22}, {10, 11, 12, 13, 17, 21}, and {10, 11, 12, 13, 18, 20}. Not counting the first one in the group listed above there are 7 ways for the first three numbers to be 10, 11, and 12: {10, 11, 12, 14, 15, 22}, {10, 11, 12, 14, 16, 21}, {10, 11, 12, 14, 17, 20}, {10, 11, 12, 14, 18, 19}, {10, 11, 12, 15, 16, 20}, {10, 11, 12, 15, 17, 19}, and {10, 11, 12, 16, 17, 18}. Not counting the first one in the group listed above there are 6 ways for the first two numbers to be 10 and 11: {10, 11, 13, 14, 15, 21}. {10, 11, 13, 14, 16, 20}, {10, 11, 13, 14, 17, 19}, {10, 11, 13, 15, 16, 19}, {10, 11, 13, 15, 17, 18}, and {10, 11, 14, 15, 16, 18}. Not counting the first on in the group listed above there are 5 ways for the first number to be 10: {10, 12, 13, 14, 15, 20}, {10, 12, 13, 14, 16, 19}, {10, 12, 13, 14, 17, 18}, {10, 12, 13, 15, 16, 18}, and {10, 12, 14, 15, 16, 17}. There are 3 ways for the smallest number to be 11: {11, 12, 13, 14, 15, 19}, {11, 12, 13, 14, 16, 18}, and {11, 12, 13, 15, 16, 17}. There are 0 ways for the smallest number to be anything other than 10 or 11.** **$1 + 4 + 7 + 6 + 5 + 3 = 26$**

REFER TO THE FOLLOWING INFORMATION FOR PROBLEMS #8 THROUGH #10.
Each lane in the track below consists of two straightaways and two semicircles. The length of the straightaways in both lanes is 30 meters. The width of each lane is 2 meters. The distance between the inner edge of the two straightaways on the inside lane is 60 meters. The inside lane is lane 1 and the outside lane is lane 2.



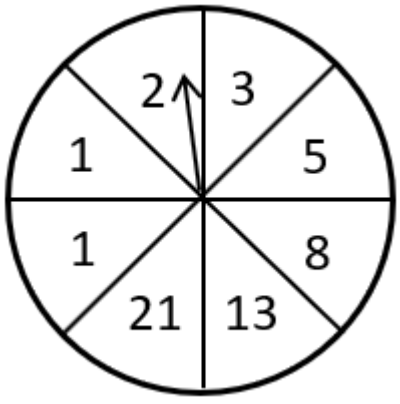
8	*C*	What is the shortest distance a runner can take in completing one lap in lane 1 of the track? Note: the shortest distance would be achieved by running directly on the inner edge of lane 1. A) 60 + 30π meters B) 30 + 60π meters C) 60 + 60π meters D) 60 + 900π meters E) 60 + 1024π meters **$30 \cdot 2 + 60\pi/2 \cdot 2 = 60 + 60\pi$**
9	*E*	What is the area of lane 1? A) 120 + 4π m^2 B) 120 + 62π m^2 C) 124π m^2 D) 60 + 124π m^2 E) 120 + 124π m^2 **The area of the 2 straightaways is $2 \cdot 30 \cdot 2 = 120$. The area of the two curved sections is $2((32^2\pi - 30^2\pi)/2) = 1024\pi - 900\pi = 124\pi$. The total area of lane 1 is $120 + 124\pi$.**

10	B	One runner consistently runs the shortest distance within lane 1 and a second runner consistently runs the shortest distance within lane 2. Both runners start at the same time next to each other at one end of one of the straightaway sections and run at the same average rate and in the same direction. At some point the runner in lane 1 will pass the runner in lane 2. During which lap for the runner in lane 1 will this happen? A) 19^{th} lap B) 20^{th} lap C) 21^{st} lap D) 22^{nd} lap E) 25^{th} lap **The runner in lane 2 falls behind the runner in lane 1 by 4π meters each lap, so the number of laps will be $(60 + 60\pi)/4\pi \approx 60/12.56 + 15$. Since $60/12.56$ is between 4 and 5, the runner in lane 1 will pass the runner in lane 2 between completing lap 19 and completing lap 20, so the passing will happen during lap 20 for the runner in lane 1.**

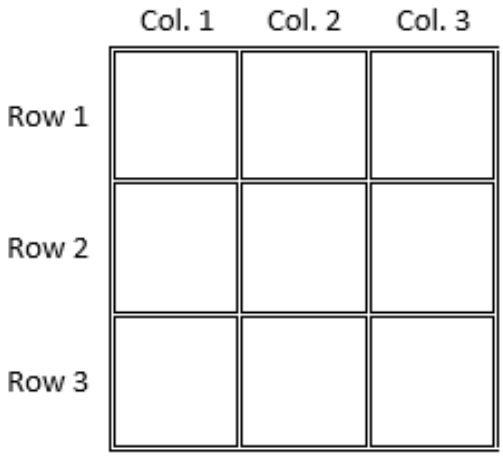
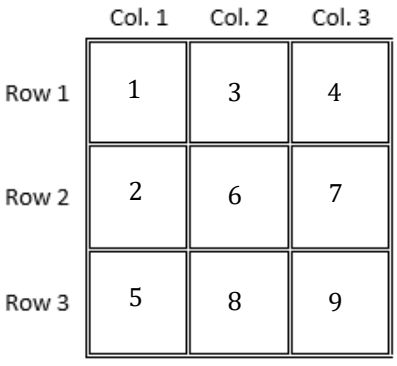
"Math is Cool" Masters -- 2023-24

6th Grade

Team Test Solutions

6th	Answer	Solution
1	[A =] 64	The area of a circle whose circumference is 16π centimeters is Aπ cm^2. What is A? **If C = 16π, then d = 16, and r = 8, so the area is $8^2\pi = 64\pi$, and A = 64.**
2	[A + B + C + D =] 54	A and B are two different integers greater than 10, but less than 20. C and D are two different integers greater than 0, but less than 10. What is the greatest possible sum of A + B + C + D? **A = 19, B = 18, C = 9, and D = 8, and 8 + 9 + 18 + 19 = 54**
3	99	What is the median of the data set? {11, 200, 53, 162, 125, 71, 99} **When the numbers are put in ascending order, 11, 53, 71, 99, 125, 162, 200, the middle number, 99, is the median.**
4	20 [card houses]	Brianna takes 27 minutes to build 3 card houses. At this rate, how many card houses can Brianna build in 3 hours? **3 houses in 27 min = 1 house in 9 min = 20 houses in 180 min = 20 houses in 3 hours**
5	[A + B =] 17	The 8 sectors in the spinner shown below are congruent and the spinner is spun twice. The probability as a reduced common fraction that the sum of the two resulting numbers is 2 is A/B. What is A + B?  **P(sum of 2) = P(1, 1) = 2/8 · 2/8 = 1/16 and 1 + 16 = 17**
6	60 [%]	Let A = 30% of B and 50% of C. As a percent, what is C/B? **A = 0.3B and A = 0.5C, so 0.3B = 0.5C → C/B = 0.3/0.5 = 60%**

7	27 [ways]	In how many different ways can you add together one or more perfect squares to make a sum of 30? The order of the numbers does not matter. For example, two different ways to get a sum of 40 would be (1) four 1s + nine 4s and (2) four 1s + one 36. **(Thirty 1s), (twenty-six 1s + one 4), (twenty-two 1s + two 4s), (twenty-one 1s + one 9), (eighteen 1s + three 4s), (seventeen 1s + one 4 + one 9), (fourteen 1s + four 4s), (fourteen 1s + one 16), (thirteen 1s, two 4s, and one 9), (twelve 1s + two 9s), (ten 1s + five 4s), (ten 1s + one 4 + one 16), (nine 1s + three 4s + one 9), (eight 1s + one 4 + two 9s), (six 1s + six 4s), (six 1s + two 4s + one 16), (five 1s + one 25), (five 1s + one 9 + one 16), (four 1s + two 4s + two 9s), (three 1s + three 9s), (two 1s + seven 4s), (two 1s + three 4s + one 16), (one 1 + one 4 + one 9 + one 16), (one 1 + one 4 + one 25), and (three 4s + two 9s) make a total of 25 ways.**
8	1800 [ft^2]	Marion has 120 ft of fencing material to use to enclose a rectangular part of her back yard. Her house is long enough to be used as one of the long sides of the enclosed rectangular area, so the fencing only needs to be used on the two short sides and the other long side of the rectangle. What is the number of square feet in the largest area that she can enclose with this amount of fencing? **The largest possible area corresponds to the y-value of the vertex of the quadratic equation: $x(120 - 2x) = \text{Area}$. The x-intercepts are 0 and 60, so the x-value of the vertex is 30. Plug in 30 to get the y-value, $30(120 - 60) = 1800$.**

9	589	In the following grid, place each of the digits 1 through 9 into a box, using each digit exactly once, according to the following rules. What 3-digit integer is in Row 3, reading from left to right? 1. Every digit is smaller than the digit immediately to its right and smaller than the digit immediately below it. 2. The three digits in the second row sum to 15. 3. The three digits in the second column sum to 17. 4. The digit 5 is not in the first row.  

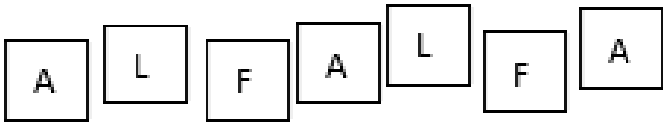
10	[B – A =] 16	The integer 12 has exactly 6 factors, 1, 2, 3, 4, 6, and 12. All integers from 1 to 100, including 12 and having exactly 6 factors can be written in a list in ascending order. Let A and B be the two adjacent integers in the list that have the greatest difference between each other, where B > A. What is B – A? **The number must have a prime factorization in the form $a^2 \cdot b$ or a^5, in order to have 6 factors. So, the list is:** **$2^2 \cdot 3 = 12$** **$3^2 \cdot 2 = 18$** **$2^2 \cdot 5 = 20$** **$2^2 \cdot 7 = 28$** **$2^5 = 32$** **$2^2 \cdot 11 = 44$** **$3^2 \cdot 5 = 45$** **$5^2 \cdot 2 = 50$** **$2^2 \cdot 13 = 52$** **$3^2 \cdot 7 = 63$** **$2^2 \cdot 17 = 68$** **$5^2 \cdot 3 = 75$** **$2^2 \cdot 19 = 76$** **$2^2 \cdot 23 = 92$** **$7^2 \cdot 2 = 98$** **$3^2 \cdot 11 = 99$** **The greatest difference is between 76 and 92, and 92 – 76 = 16.**

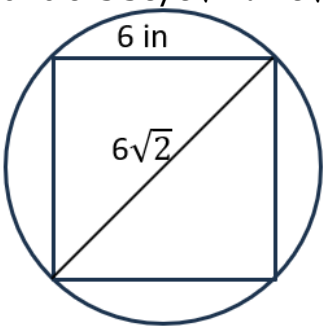
"Math is Cool" Masters -- 2023-24

6th Grade

Triple Jump Solutions

6th	Answer	Solution
1	5	Evaluate: $(29 - 11) - 6 \cdot 2^2 + (165 \div 15)$ **$(29 - 11) - 6 \cdot 2^2 + (165 \div 15) = 18 - 24 + 11 = 5$**
2	240 [cubits]	A cubit is an ancient unit of length, approximately equal to the length of a forearm. It was typically about 18 inches. Assuming a cubit equals 18 inches, how many cubits are in the length of a field that is 120 yards long? **1 yard = 36 inches = 2 cubits, so 120 yards = 240 cubits.**
3	150 [pennies]	Donald wants to give his penny collection to his three nephews, Huey, Dewey, and Louie. He gives Huey 30 percent of his pennies, he gives Dewey 1/3 of his pennies, and he gives Louie the remaining 55 pennies. In total, how many pennies were in his collection? **$1 - 30/100 - 1/3 = 1 - 9/30 - 10/30 = 11/30$** **$11/30 = 55/x \rightarrow x = 150$**
4	41	The first three numbers in a sequence are 1, 4, and 8, where each successive term is generated by adding the number that corresponds to the number of characters used in the English spelling of the previous term. For example, 1 is spelled 'one', and 1 + 3 = 4. What is the 7th term in the sequence? Note: the spelling of a number like twenty-five includes a hyphen, which makes the character count in this case 11 characters. **1, 4, 8, 13, 21, 31, 41**
5	[N =] 41	The set of integers shown have the same median and mode. What is the value of N? {78, 15, 26, 60, 41, 37, 52, N} **If you put the numbers in order, the N can be before or after the 41, but for the mode to equal the median, N must be 41.** **{15, 26, 37, 41, N, 52, 60, 78} or {15, 26, 37, N, 41, 52, 60, 78}**

6	[A + B =] 16	The following expression can be simplified to a reduced common fraction A/B. What is A + B? $\dfrac{\frac{3}{8}-\frac{5}{4}+\frac{7}{2}}{\frac{5}{8}-\frac{7}{4}+\frac{9}{2}}$ $\frac{\frac{3}{8}-\frac{5}{4}+\frac{7}{2}}{\frac{5}{8}-\frac{7}{4}+\frac{9}{2}} = \frac{\frac{3}{8}-\frac{10}{8}+\frac{28}{8}}{\frac{5}{8}+\frac{14}{8}-\frac{36}{8}} = \frac{3-10+28}{5-14+36} = \frac{21}{27}$ **= 7/9 and 7 + 9 = 16**
7	48	In the expression AB – CD, AB and CD are 2-digit positive integers, and A, B, C, and D can each be replaced by a different one of the four digits, 1, 3, 5, and 7. What is the positive difference between the largest possible positive value and the smallest possible positive value of AB – CD? **Largest possible value: 75 – 13 = 62** **Smallest possible value: 51 – 37 = 14** **62 – 14 = 48**
8	[P + Q =] 6	Biff has a set of tiles spelling ALFALFA. Eho randomly chooses a set of 4 tiles from Biff's set without  replacement. As a reduced common fraction, the probability that Eho can create a palindrome using all 4 tiles is P/Q. What is P + Q? **Tile sets that he can create a palindrome with include: {AAFF}, {AALL}, and {FFLL}. The number of ways sets of four tiles can be chosen is $_7C_4$ = 7!/[(7 – 4)!4!] = 35. The number of ways to choose {AAFF} is $_3C_2 \cdot {}_2C_2$ = 3. The number of ways to choose {AALL} is $_3C_2 \cdot {}_2C_2$ = 3. The number of ways to choose {FFLL} is $_2C_2 \cdot {}_2C_2$ = 1. So, P(able to create palindrome) = (3 + 3 + 1)/35 = 1/5 and 1 + 5 = 6.**

9	[A + B =] 5	A square with perimeter twenty-four inches is inscribed inside a circle. As a simplified fraction the ratio of the area of the square to the circumference of the circle is $\frac{A\sqrt{B}}{\pi}$, where A and B are single-digit integers. What is A plus B? **The area of the square is 36 in^2 and the circumference of the circle is $6\sqrt{2}\pi$ in. The ratio of the area of the square to the circumference of the circle is $36/6\sqrt{2}\pi = 6\sqrt{2}/2\pi = 3\sqrt{2}/\pi$, and 3 + 2 = 5.** 

10	[r - c =] 49	The positive integers are arranged in the following infinite pattern. Each integer has a certain row and column position, indicated by (r, c). For example, the number 9 is in position (2, 3). Find the location (r, c) that the number 2024 is in. What is the value of r – c? **The first row consists of triangular numbers, which has the formula n(n + 1)/2, where n = the position number of each triangular number. For example, the 2nd triangular number is 2(2 + 1)/2 = 3. Determine the closest triangular number to 2024 using the formula, n(n + 1)/2 = 2024 → 63(64)/2 = (3600 + 180 + 252)/2 = 2016, which would be at the top of the 63rd column and have the location (1, 63). The location of 2017 is (64, 1), 2018 is (63, 2), 2019 is (62, 3), . . . 2022 is (59, 6), 2023 is (58, 7), and 2024 is (57, 8), so r - c = 49.**

		Column Number:					
		1	**2**	**3**	**4**	**5**	**6**
Row Number:	**1**	1	3	6	10	15	…
	2	2	5	9	14	20	…
	3	4	8	13	19	26	…
	4	7	12	18	25	33	…
	5	11	17	24	32	41	…
	6	…	…	…	…	…	…

"Math is Cool" Masters -- 2023-24

6th Grade

College Bowl Round #1 Solutions

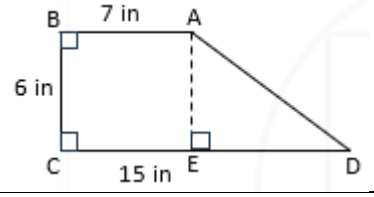
6th	Answer	Solution
1	[x =] -14	Solve for X: two hundred and forty minus fifteen X equals four hundred and fifty $240 - 15x = 450 \rightarrow -15x = 210 \rightarrow x = -14$
2	33	What is fifty-one minus forty-two plus thirty-three minus twenty-four plus fifteen? $51 - 42 + 33 - 24 + 15 = 33$
3	80	The first three terms of an arithmetic sequence are three, fourteen, and twenty-five. What is the eighth term in the sequence? $3, 14, 25, 36, 47, 58, 69, \underline{80}$ **or** $\underline{3 + 7 \cdot 11 = 80}$
4	97	Axel's first three test scores are eighty-nine, seventy-six, and ninety-one. What average score does he need to get on the next two tests to bring his average to ninety? $90 \cdot 5 = 450$ and $450 - 89 - 76 - 91 = 194$ and $194/2 = 97$.
5	[A = B =] 29	If X over Y equals eight over nine and Y over Z equals three over seven, then as a reduced common fraction X over Z equals A over B. What is A plus B? $x/y = 8/9$ and $y/z = 3/7 = 9/21$, so $x/z = 8/21$ and $8 + 21 = 29$ **or** $y = 9x/8$ and $y = 3z/7$, so $9x/8 = 3z/7 \rightarrow 63x = 24z \rightarrow x/z = 24/63 = 8/21$
6	[A =] 12	A cylinder has a volume of one hundred and forty-four pi cubic inches. If the height of the cylinder is four inches, then the circumference of its base is A pi inches. What is A? $144\pi/4 = 36\pi$, so the radius is 6 and the circumference is 12π.
7	4 [perfect cubes]	How many perfect cubes exist between three hundred and one thousand three hundred? $6^3 = 216$, $7^3 = 343$, $8^3 = 512$, $9^3 = 729$, $10^3 = 1000$, $11^3 = 1331$

8	504 [in^2]	How many square inches are in three-point-five square feet? **$3.5 \cdot 144 = 504$**
9	18 [miles]	In miles, how far does a car traveling at an average rate of forty-five miles per hour go in twenty-four minutes? **45 miles in 60 min = 3/4 mile in 1 min = 18 miles in 24 min**
10	83 [%]	Stef Curry has made ninety-one percent of his free throws in his career. Rounded to the nearest whole percentage and based on his career free throw percentage, what is the probability that he will make two free throws in a row? **$0.91 \cdot 0.91 = 0.8281 \approx 83\%$**

"Math is Cool" Masters -- 2023-24

6th Grade

College Bowl Round #2 Solutions

6th	Answer	Solution
1	215 [cents]	A five-gallon bucket of paint costs forty-three dollars. In cents, what is the cost of the paint per quart? **There are 20 quarts in 5 gallons, so 4300/20 = 215.**
2	8 [threes]	A short story is printed on thirty-three pages and every page is numbered, starting with page one. How many threes are printed in the page-numbering of the short story? **3, 13, 23, 30, 31, 32, 33 makes a total of 8 threes**
3	[A + B =] 12	The solution to the equation five X over nine equals twenty-five over six is X equals A point B, where A and B are single digits. What is A plus B? $\mathbf{5x/9 = 25/6 \rightarrow 30x = 225 \rightarrow x = 7.5}$**, and 7 + 5 = 12**
4	124	What is the sum of the positive factors of forty-eight? **1 + 2 + 3 + 4 + 6 + 8 + 12 + 16 + 24 + 48 = 124**
5	38 [inches]	Trapezoid ABCD has side AB parallel to side CD and side BC perpendicular to sides AB and CD. If AB equals seven inches, BC equals six inches, and CD equals fifteen inches, in inches what is the perimeter of ABCD? **Adding altitude $\overline{AE}$ to the figure below reveals that ΔAED is a right triangle with legs 6 and 8, so AD must be 10 and the perimeter is 6 + 7 + 15 + 10 = 38.** 
6	[P + Q =] 121	Six scrabble tiles have the letters A, B, C, D, E, and F on them. Three tiles are chosen at random without replacement. As a reduced common fraction, the probability that the first tile is A, the second tile is B, and the third tile is C is P over Q. What is P plus Q? $\mathbf{1/6 \cdot 1/5 \cdot 1/4 = 1/120}$**and 1 + 120 = 121**
7	488	What is the sum of the first five terms in the geometric sequence whose first term is eight and whose common ratio is negative three? **8 + -24 + 72 + -216 + 648 = 488**

8	55	A palindrome is an integer that reads the same forwards and backwards, such as one hundred fifty-one. What is the mean of the nine smallest palindromes that are greater than ten? **$(11 + 22 + 33 + 44 + 55 + 66 + 77 + 88 + 99)/9 = 55.$**
9	[D =] 15	Let A, B, C, and D be distinct positive integers. If A equals seventeen and is greater than B and C and B is greater than C and D, then what is the largest possible value of D? **Since A > B > C or D, we don't know whether C > D or not. So, the largest possible value of D is 15, if D > C.**
10	896	What is thirty-two times twenty-eight? **$(30 + 2)(30 - 2) = 900 - 4 = 896$**

"Math is Cool" Masters -- 2023-24

6th Grade

College Bowl Round #3 Solutions

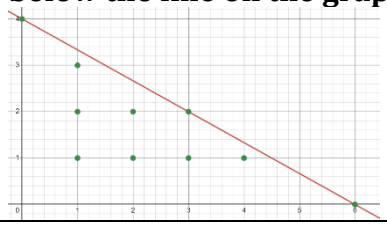
6th	Answer	Solution
1	11	Evaluate: the quantity four divided by one-fourth minus the quantity three divided by one-third plus the quantity two divided by one-half. $\frac{4}{\frac{1}{4}} - \frac{3}{\frac{1}{3}} + \frac{2}{\frac{1}{2}} = 16 - 9 + 4 = 11$
2	56	What is the median of the prime numbers between forty-nine and sixty? $(53 + 59)/2 = 56$
3	3	What is twelve percent of twenty-five? $12/100 = x/25 \rightarrow x = 3$
4	19 [sandwiches]	Natalie takes five minutes to make three sandwiches. Penn takes six minutes to make four sandwiches. How many sandwiches can Natalie and Penn make working together in fifteen minutes? **Natalie: 3 sandwiches in 5 min = 9 sandwiches in 15 min** **Penn: 4 sandwiches in 6 min = 10 sandwiches in 15 min** $10 + 9 = 19$
5	6	What is the sum of the solutions to the equation: X squared minus six X minus seventy-two equals zero? $x^2 - 6x - 72 = 0 \rightarrow (x - 12)(x + 6) = 0 \rightarrow x = 12$ **and** $x = -6$ **and** $12 + -6 = 6$
6	507 [cubic inches]	Find the volume in cubic inches of a pyramid with a square base, where the side length of the base is thirteen inches and the height of the pyramid is nine inches. $V = 1/3\ Bh = (1/3)(13)^2(9) = 507$ **cu inches**
7	[A + B =] 89	The first five terms of a sequence are one, one, two, three, and five. As a reduced common fraction, the ratio of the tenth term over the ninth term is A over B. What is A plus B? **The first 10 terms are 1, 1, 2, 3, 5, 8, 13, 21, 34, 55. The ratio is 55/34 and 55 + 34 = 89.**
8	78	What is the sum of the smallest two prime numbers that are greater than twenty and differ by four? $37 + 41 = 78$

9	14	Aaron's number is ten more than Brian's number. Clarissa's number is six less than Brian's number. Deepta's number is two more than Clarissa's number, which is thirteen. What is the positive difference between Aaron's number and Deepta's number? **C = 13, D = 15, B = 19, A = 29** **29 – 15 = 14**
10	16 [even numbers]	How many even numbers are between fifty-one and eighty-three? **52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82**

"Math is Cool" Masters -- 2023-24

6th Grade

College Bowl Round #4 Solutions

6th	Answer	Solution
1	[C =] 10	Four hundred times five hundred times six hundred times seven hundred equals A point B times ten to the C in scientific notation, where A and B are positive single-digit integers, and C is a positive integer. What is C? **$400 \cdot 500 \cdot 600 \cdot 700 = 840 \cdot 100{,}000{,}000 = 8.4 \cdot 10^{10}$, and C = 10.**
2	[x =] 3	Solve the following equation for X: Seven X plus nineteen equals three X plus thirty-one **$7x + 19 = 3x + 31 \rightarrow 4x = 12 \rightarrow x = 3$**
3	63	What is the sum of the first six terms in the series that begins with one and in which each successive term is two times the previous term? **$1 + 2 + 4 + 8 + 16 + 32 = 63$**
4	[A + B =] 13	Nine cards are numbered one through nine. Two cards are selected at random without replacement. As a reduced common fraction, the probability that the numbers on the cards add up to seven is A over B. What is A plus B? **There are $9 \cdot 8 = 72$ ways to select two cards without replacement. Card selections that add to 7 include (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), and (6, 1), so $P(7) = 6/72 = 1/12$ and $1 + 12 = 13$.**
5	7 [points]	A line with equation six X plus nine Y equals thirty-six is drawn on a coordinate plane. How many points, X comma Y, are there below the line in which both X and Y are positive integers? **There are 7 points with positive integer coordinated below the line on the graph below.** 

6	37	My number is an integer between zero and fifty. It is not a multiple of two, three, or five. My number is between thirty-two and forty. What is my number? **All numbers between 32 and forty are divisible by 2, 3, or 5, except for 37.**
7	8 [km^2]	One mile equals about one-point-six-one kilometers. Rounded to the nearest whole number, what is the number of square kilometers in three square miles? $\mathbf{1.61 \cdot 1.61 \approx 2.56}$ **and** $\mathbf{2.56 \cdot 3 \approx 7.68 \approx 8}$
8	[*A* =] 32	Eighty is two hundred and fifty percent of *A*. What is *A*? $\mathbf{80 = 2.5A \rightarrow A = 80/2.5 = 32}$
9	375	What is the median of the first six powers of five? In other words, five to the first through five to the sixth. **The median will be the average of** $\mathbf{5^3}$ **and** $\mathbf{5^4}$**, or** $\mathbf{(125 + 625)/2 = 750/2 = 375}$
10	21 [minutes]	In minutes, how long does it take a train traveling at an average speed of twenty miles per hour to travel seven miles? **20 miles in 60 min = 1 mile in 3 min = 7 miles in 21 min**

"Math is Cool" Masters -- 2023-24

6th Grade

College Bowl Round #5 Solutions

6th	Answer	Solution
1	5	What is the sixth term in the arithmetic sequence whose first three terms are two hundred, one hundred sixty-one, and one hundred twenty-two? **200, 161, 122, 83, 44, 5**
2	56 [inches]	A rectangle has a length of seven inches and width of twenty-eight inches. In inches, what is the perimeter of a square whose area is the same as the area of the rectangle? **$7 \cdot 28 = 196$ and $\sqrt{196} = 14$, so the perimeter of the square is $4 \cdot 14 = 56$**
3	177	Evaluate: two to the fifth power plus three to the fourth power plus four to the third power **$2^5 + 3^4 + 4^3 = 32 + 81 + 64 = 177$**
4	[A + B =] 49	As a reduced common fraction, the median of the following data set is A over B. What is A plus B? {eleven tenths, eleven halves, eleven eighths, eleven fourths} **The order is 11/10, 11/8, 11/4, 11/2 and the median is (11/8 + 11/4)/2 = (44/32 + 88/32)/2 = (132/32)/2 = (33/8)/2 = 33/16 and 33 + 16 = 49**
5	[A + B =] 7	Jorge takes ten minutes to dig one hole. Laurie digs eight holes in one hour. The ratio of Jorge's hole-digging rate to Laurie's hole-digging rate in holes per second is A over B. What is A plus B? **George: 1 hole in 10 min = 1/10 hole in 1 min = 1/600 hose in 1 sec** **Laurie: 8 holes in 1 hr = 8/60 or 2/15 holes in 1 min = 2/900 or 1/450 holes in 1 sec** **The ratio is (1/600)/(1/450) = 450/600 = 3/4 and 3 + 4 = 7** **or** **The ratio will be the same whether it's holes per hr, holes per min, or holes per sec and George can dig 6 holes per hour while Laurie can dig 8 holes per hour and 6/8 = 3/4**

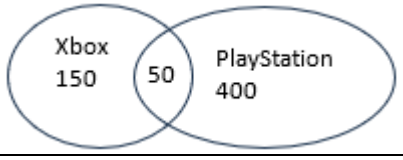
6	[A + B + C =] -8	When the two binomials five X plus three and seven X minus eight are multiplied the result is A X squared plus B X plus C, where B and C are negative integers. What is A plus B plus C? **$(5x + 3)(7x - 8) = 35x - 19x - 24$ and $35 + -19 + -24 = -8$**
7	[A + B =] 55	A fun-size bag of M&Ms has three brown, four orange, two yellow, six green, and five blue M&Ms in it. If two M&Ms are randomly taken from the bag, the probability as a reduced common fraction that at least one of them is blue is A over B. What is A plus B? **P(at least 1 blue) = 1 – P(no blue) = $1 - 15/20 \cdot 14/19 = 380/380 - 210/380 = 170/380 = 17/38$ and $17 + 38 = 55$**
8	2	Abe arrived first at the party. Bonita didn't arrive last. Carter arrived after Abe. Doris arrived after Bonita but before Carter. Who was the second person to arrive at the party. Answer as an integer: one for Abe, two for Bonita, three for Carter, or four for Doris. **The order is Abe, Bonita, Doris, Carter**
9	[x + y =] 23	Let two over three equal X over thirty-six and let four over five equal Y over eighty-five. If X over Y is a reduced common fraction, what is X plus Y? **$2/3 = x/36 \rightarrow x = 24$** **$4/5 = y/85 \rightarrow y = 68$** **$x/y = 24/68 = 6/17$ and $6 + 17 = 23$**
10	484 [ft^2]	One acre is equivalent to forty-three thousand five hundred and sixty square feet. How many square feet are in one ninetieth of an acre? **$43560 \cdot 1/90 = 4356/9 = 1452/3 = 484$**

"Math is Cool" Masters -- 2023-24

6th Grade

College Bowl Round #6 Solutions

6th	Answer	Solution
1	18	What is the sum of all integers that are greater than the square root of twenty and less than the square root of sixty? $\sqrt{20} < 5, 6, \text{ and } 7 < \sqrt{60}, \text{ and } 5 + 6 + 7 = 18$
2	[A + B =] 46	A bowl has red, green, and blue marbles in it, and one marble will be randomly selected. The probability of selecting a red marble is two over five and the probability of selecting a green marble is two over seven. As a reduced common fraction, the probability of selecting a blue marble is A over B. What is A plus B? $P(\text{blue}) = 1 - 2/5 - 2/7 = 1 - 24/35 = 11/35 \text{ and } 11 + 35 = 46$
3	11	What is the mean of the odd numbers between six and sixteen? **The odd numbers between 6 and 16 are 7, 9, 11,13, 15, and the middle number is both the median and the mean**
4	14 [jiggles]	Seven jiggles make four wiggles. Six wiggles make fifteen bobbles. How many jiggles do twenty bobbles make? $7J = 4W \rightarrow 42J = 24W, 6W = 15B \rightarrow 24W = 60B, \text{ so } 42J = 60B \rightarrow 14J = 20B$
5	12 [ways]	How many distinct ways are there to arrange the letters in the word SENSES, spelled S-E-N-S-E-S, if none of the Ss can be next to each other? **For each arrangement of the three Ss, the remaining three letters (2 Es and 1 N) can be arranged 3!/2! = 3 ways** **There are 4 ways for the Ss to not be next to each other: S _ S _ S _, _ S _ S _ S, S _ _ S _ S, S _ S _ _ S** $\text{So, the answer is } 4 \cdot 3 = 12$
6	[P + Q =] 6	If four X minus A equals six X minus B and B minus A equals three, then X equals P point Q, where P and Q are single-digit integers. What is P plus Q? $4x - A = 6x - B \rightarrow 2x = B - A \rightarrow 2x = 3 \rightarrow x = 1.5 \text{ and } 1 + 5 = 6$

7	4 [mph]	Jen takes five minutes to ride her bike to school and seven minutes to ride back home. If the distance between home and school is four-tenths of a mile, in miles per hour, what is her average rate for the two combined bike rides? **The two trips combined cover 0.8 miles in 12 minutes = 1 mile in 15 min = 4 miles per hour**
8	450	Of six hundred people surveyed, one-fourth of the two hundred people who reported owning an Xbox also own a PlayStation. If each person surveyed owns an Xbox, a PlayStation, or both, how many own a PlayStation? **If there are 50 who own both and the total must add up to 600, there would be 450 PlayStation owners.** 
9	1980 [feet]	There are five thousand two hundred and eighty feet in a mile. A furlong is one-eighth of a mile. How many feet are in three furlongs? $\mathbf{5280/8 = 660 \text{ and } 660 \cdot 3 = 1980}$
10	60 [inches]	In inches, what is the perimeter of a square whose area is two hundred and twenty-five square inches? $\mathbf{\sqrt{225} = 15 \text{ and } 15 \cdot 4 = 60}$

"Math is Cool" Masters -- 2023-24

6th Grade

College Bowl Round EXTRA Solutions

6th	Answer	Solution
1	36	What integer between thirty and forty-five has exactly nine positive integer factors? **An odd number of factors indicates that the number is a perfect square, so it must be 36, whose 9 factors are 1, 2, 3, 4, 6, 9, 12, 18, and 36**
2	30 [integers]	How many integers between one hundred and two hundred have a ones digit that is a non-zero multiple of three? **There are 3 in each of the 10 groups of 10, and $3 \cdot 10 = 30$**
3	202202	What is one thousand and one times two hundred and two? $\mathbf{1001 \cdot 202 = 202202}$
4	[A + B =] 37	A fair coin is flipped and then a pair of standard six-sided dice are rolled. As a reduced common fraction, the probability that the result is heads followed by a sum of three is A over B. What is A plus B? **P(H) = 1/2, P(sum of 3) = 1/18, $1/2 \cdot 1/18 = 1/36$, and $1 + 36 = 37$.**
5	[A =] 48	In square centimeters, the surface area of a right circular cylinder with radius two centimeters and height ten centimeters is A pi. What is A? $\mathbf{SA_{cone} = 2\pi r^2 + 2\pi rh}$ $\mathbf{2\pi \cdot 2^2 + 2\pi \cdot 2 \cdot 10 = 48\pi}$
6	[x =] 13	Solve the equation for X: five X minus thirteen equals two X plus twenty-six $\mathbf{5x - 13 = 2x + 26 \rightarrow 3x = 39 \rightarrow x = 13}$