"Math is Cool" Championships -- 2024-25

High School

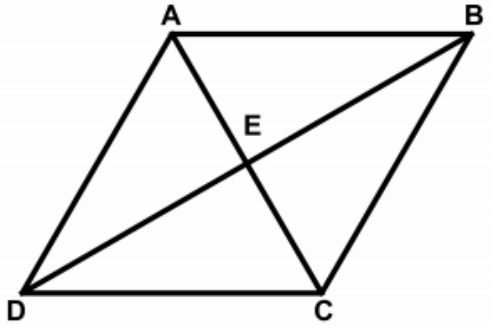
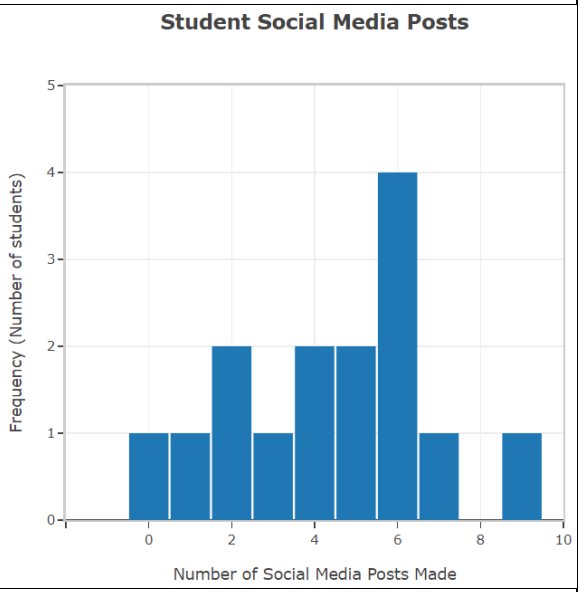
Mental Math Solutions

	Answer	Solution
1	23300	What is forty-one thousand seven hundred twenty minus eighteen thousand four hundred twenty? 41720 – 18420 = 23300
2	7 [integers]	How many integers 'x' satisfy the following equation: The absolute value of 'x' is less than pi $\|x\| < 3.14$ $-3.14 < x < 3.14$ Integers that satisfy are: -3, -2, -1, 0, 1, 2, 3
3	4 [= $b^2 - ac$]	If a, b and c are positive consecutive even integers, with c greater than b greater than a, what is the value of b-squared minus 'ac'? Example: 2, 4, 6 = a, b, c $4^2 – (2)(6) = 16 – 12 = 4$
4	330[square inches]	The sides of a triangle are eleven, sixty and sixty-one inches. In square inches, what is the area of the triangle? A = ½ bh = ½(11)(60) = 330
5	21	What number is one-third of the way from negative four to seventy-one, and closer to negative four? The distance from -4 to 71 = 75, so 25 units is 1/3 of the distance. -4 + 25 = 21.
6	400 [minutes]	Two students working at a constant rate can paint one classroom in five hours. How many minutes would it take three students working at the same constant rate to paint two classrooms? 2 students: 1 job: 5 hours 10 student-hours/1 job 3 students: 2 jobs: ? hours Need 20 student-hours/2 jobs 20/3 students = 6 2/3 hours = 400 minutes
7	25 [points]	Amy's average on her six math tests this semester is seventy-five points. Fortunately, her teacher drops each student's lowest grade, which raises Amy's average to eight-five. What was her lowest test grade, in points? 75x6 = 450 point sum for 6 tests 85*5 = 425 point sum for 5 tests 450 – 425 = 25 points

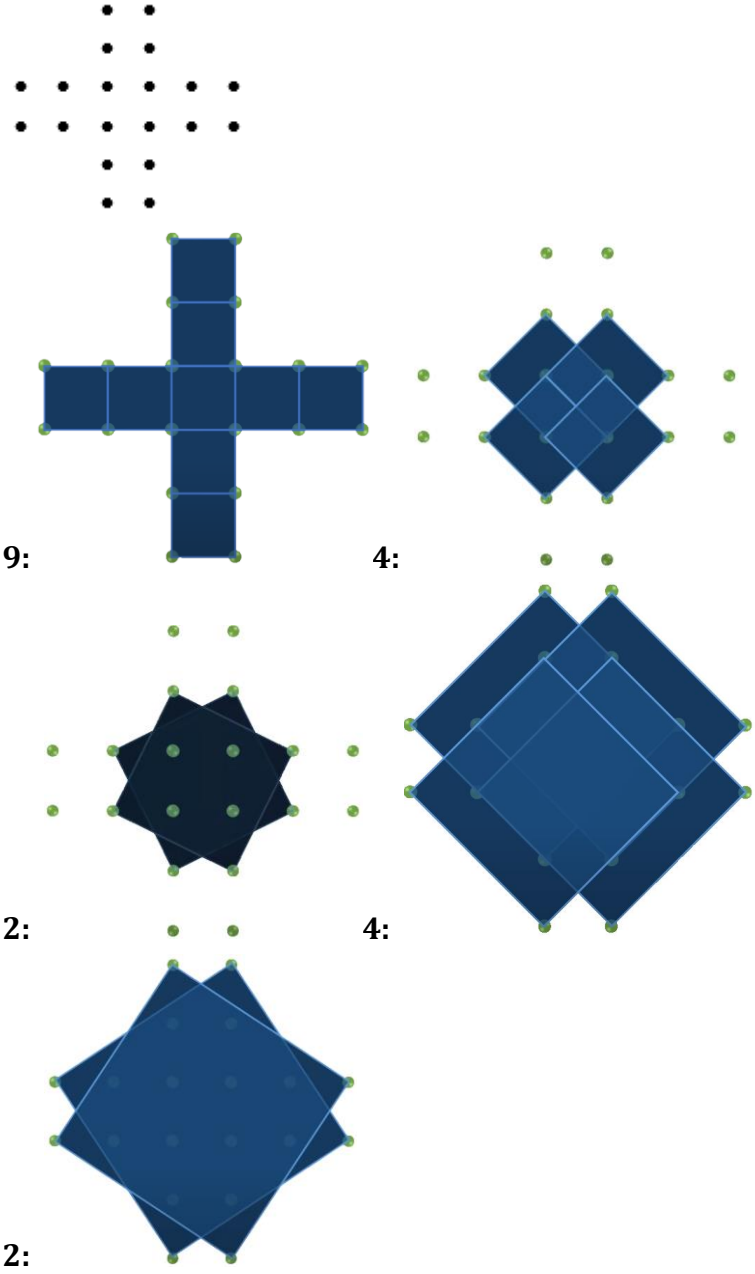
8	648 [5-digit palindromes]	A palindrome is a number that reads the same forwards and backwards, like one hundred twenty-one. How many five-digit palindromes are there that do not include the digit nine? The first digit can be 1 – 8. The second and third digits can be 0 – 8. The fourth and fifth digits are determined by the 1st and 2nd. 8x9x9x1x1 = 648 palindromes

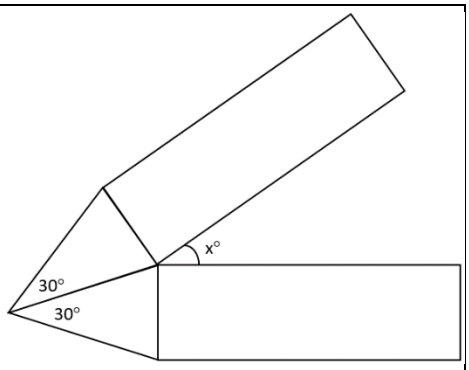
"Math is Cool" Championships -- 2024-25
High School
Individual Test Solutions

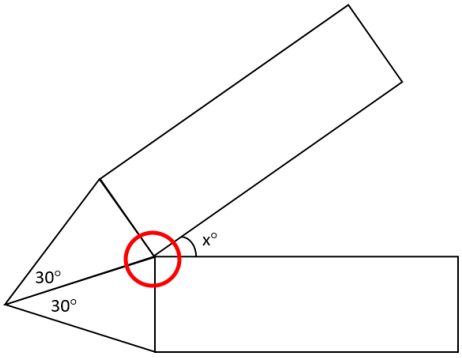
	Answer	Solution
1	56 [pints]	How many pints are in seven gallons? 1 gallon = 4 quarts, 1 quart = 2 pints 7x4x2 = 56
2	4 [units]	The points A, B, C and D all lie on the same line segment, in that order from left to right. The ratio of AB:BC:CD is equal to 2:1:3. If AD = 6 units, what is the length of BD in units? AB = 2, BC = 1, CD = 3. Therefore, BD = BC + CD = 1 + 3 = 4.
3	70 [ways]	If the Southridge High School math team consists of 8 students, in how many ways can a team of 4 students be selected for the Math Is Cool Masters competition? $8C4 = \frac{8!}{4!\,4!} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} = 70$
4	6 [values of x]	How many of the following values for 'x' make the inequality true? $-3x < 60$ {-30, -21, -17, -25, -20, -15, -23, -19, -10, 0, 15} $-3x < 60$ x > -20 Six of the values are greater than -20: -17, -15, -19, -10, 0, 15.
5	20 [= sum]	A vendor at a farmer's market puts 18 potatoes into 'n' bags such that each bag contains the same number of potatoes. There is more than one potato and fewer than 18 potatoes in each bag. What is the sum of all possible values of 'n'? Each value of 'n' must be a divisor of 18, but not 1 or 18. Divisors are 2, 3, 6, 9. 2 + 3 + 6 + 9 = 20
6	850	What is the next number in the following sequence? 3, 10, 31, 94, 283, … The pattern is x3 + 1 283x3 + 1 = 850
7	22 [= A + B]	A spinner is divided into 15 equal sections, numbered 1 through 15. When the spinner is spun one time, the probability that it lands on a multiple of 5 or a multiple of 3 can be written as a reduced common fraction A/B. What is A + B? Multiple of 3 or 5: 3, 5, 6, 9, 10, 12, 15 (7 ways) P(mult of 3 or 5) = 7/15 7 + 15 = 22

8	18 [= x]	In rhombus ABCD, the measure of angle ABE equals 34°. If the measure of angle CBE equals (2x – 2)°, what is the value of x?  ABE = CBE 34 = 2x – 2 x = 18
9	20425 [cents]	A school district orders 10 boxed lunches from Panera for a total of $107.50. If each boxed lunch costs the same amount, how much do 19 boxed lunches cost, in cents? $107.50/10 = $10.75 each $10.75 x 19 = 204.25 $ = 20425 ¢
10	11 [students]	Mr. Tosch gathered data from his Statistics students on how many social media posts each student had made the previous day. How many students made at least 3 social media posts? Add up the frequencies for 3 or more posts. 1+2+2+4+1+1 = 11 
11	88	Evaluate: $8 + 6 \times 4^2 - 16$ $8 + 6 \times 4^2 - 16 = 8 + 6 \times 16 - 16$ $= 8 + 96 - 16 = 88$

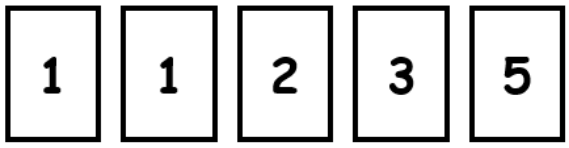
12	2 [is false]	Exactly one of the following statements is false, and the rest are true. Which statement is false? Answer with the statement number: 1, 2, 3 or 4. 1. Statement 4 is true. 2. Statement 1 is false. 3. Statement 2 is false. 4. Statement 3 is true. Check each case. Assume 1 is false: Therefore 4 is false, but that is two false statements ➔ doesn't work. **Assume 2 is false: Therefore 1 is true, meaning 4 is true, meaning 3 is true. No contradictions.** Assume 3 is false: Therefore 2 is true, meaning 1 is false, that is two false statements ➔ doesn't work. Assume 4 is false: Therefore 3 is false, that is two false statements ➔ doesn't work.
13	5 [=x]	Solve for x in the following equation: $\frac{8}{3x+9} = \frac{3}{3x-6}$ 8(3x – 6) = 3(3x + 9) 24x – 48 = 9x + 27 15x = 75 x = 5
14	75 [%]	Zhanlu spends $450 of his $1800 monthly income on food. What percent of his monthly income does not go towards food? 450/1800 = ¼ = 25% on food 75% is NOT on food
15	-822 [= 77th term]	Find the 77th term of the arithmetic sequence that begins: 14, 3, -8, … d = -11 $a_n = a_1 + (n - 1)d = 14 + (76)(-11) = -822$
16	1 [= gcd]	What is the greatest common divisor of 2024! and 2024! + 1? In general, the GCD of two consecutive integers must be 1. Suppose some integer 'n' divides evenly into 2024!. Therefore, the next smallest integer which 'n' divides is 2024! + n.

17	21 [squares]	The following dots are equally spaced on a grid of horizontal and vertical lines. How many squares can be created by connecting any four of the dots?  **9:** **4:** **2:** **4:** **2:** **Total = 9+4+2+4+2 = 21**
18	0 [x-intercepts]	How many x-intercepts does the following parabola have? $y = x^2 - 6x + 13$ The discriminant is: $\sqrt{(-6)^2 - 4(1)(13)} = \sqrt{-16}$ Because this is imaginary, the parabola is above the x-axis, and has 0 x-intercepts.
19	26 [points]	On a test that has a normal (bell-shaped) distribution, a score of 38 points falls two standard deviations above the mean, and a score of 14 points falls two standard deviations below the mean. In points, what was the mean of the test? The mean will be in the middle of the two values: (38 + 14)/2 = 26

<table>
<tr>
<td>20</td>
<td>30 [°]</td>
<td>In the following diagram, two congruent isosceles triangles share a side, and have one angle measuring 30°. Attached to each triangle is a rectangle that shares a side with the base of the triangle. In degrees, what is the measure of angle x?
 

The angles inside the circle must add up to 360°. Each of the base triangle angles is 75°, and each of the rectangle angles is 90°. Therefore, 360 – 2(75) – 2(90) = 30°
 
</td>
</tr>
<tr>
<td>21</td>
<td>80 [%]</td>
<td>Mrs. Stephenson has 20 students in her Statistics class. On Wednesday she gave a test, and also an assignment was due. Ten students passed the test, and 14 students completed the assignment. There were 4 students who failed the test and also did not complete the assignment. As a percentage, what is the probability that a student completed the assignment, given that they passed the test?
Given information:
<table>
<tr><td></td><td>Passed test</td><td>Failed test</td><td>Total</td></tr>
<tr><td>Completed assignment</td><td></td><td></td><td>14</td></tr>
<tr><td>Did not complete</td><td></td><td>4</td><td></td></tr>
<tr><td>Total</td><td>10</td><td></td><td>20</td></tr>
</table>
Can complete table:
<table>
<tr><td></td><td>Passed test</td><td>Failed test</td><td>Total</td></tr>
<tr><td>Completed assignment</td><td>8</td><td>6</td><td>14</td></tr>
<tr><td>Did not complete</td><td>2</td><td>4</td><td></td></tr>
<tr><td>Total</td><td>10</td><td>10</td><td>20</td></tr>
</table>
8 out of 10 students who passed the test also completed the student = 80%.</td>
</tr>
</table>

22	82	The following pattern of numbers continues infinitely. What number is exactly in the middle of Row 18? The even numbered rows have a number exactly in the middle, and the pattern starts as: 1, 2, 5, 10, … The pattern is: +1, +3, +5, … Continuing the pattern, the middle number in Row 18 is 82. 1 Row 0 1 1 Row 1 1 2 1 Row 2 1 3 3 1 … 1 4 5 4 1 1 5 7 7 5 1 1 6 9 10 9 6 1 … … … … … … … …
23	-54 [= f(4)]	Given the function f(x), such that: $f(x) = x^2 + bx + c$ $f(-1) = 1$ $f(-3) + f(2) = -1$ Find the value of f(4). $(-1)^2 + b(-1) + c = 1$ $1 - b + c = 1$ $b = c$ $f(x) = x^2 + bx + b$ $(-3)^2 + (-3)b + b + (2)^2 + 2b + b = -1$ $b + 13 = -1$ $b = -14$ $f(4) = 4^2 + (-14)(4) + (-14) = -54$
24	-9 [= x + y]	On the coordinate plane, the point (4, -1) is reflected over the point (0, -3) to a point (x, y). What is x + y? Using a graphical method, to get from (4, -1) to (0, -3), move down 2 units and left 4 units. Moving down 2 and left 4 again brings us to the point (-4, -5).
25	8 [sum =]	Find the sum of all possible values of x that make the following equation true: $\log(2x + 3) + \log(x - 4) = \log(x^2 + 12)$ $\log[(2x + 3)(x - 4)] = \log(x^2 + 12)$ $(2x + 3)(x - 4) = x^2 + 12$ $2x^2 - 5x - 12 = x^2 + 12$ $x^2 - 5x - 24 = 0$ $(x - 8)(x + 3) = 0$ $x = 8$ or $x = -3$ CHECK! $x = -3$ is NOT in the domain of the problem! $x = 8$ is the only solution

26	30 [%]	Gibson has the following set of cards, which he turns face downward, shuffles, and then randomly selects three of the cards. As a percentage, what is the probability that the sum of the numbers on the three cards is a Fibonacci number? In other words, a number in the Fibonacci sequence that begins: 0, 1, 1, 2, and so on, where each term from the third term on is the sum of the previous two terms.  Fibonacci numbers: 0, 1, 1, 2, 3, 5, 8, 13, … There are 5C3 = 10 ways to select 3 cards. Three of the 10 sums are Fibonacci numbers. 3/10 = 30% **Selected cards** / **Sum** 1, 1, 2 / 4 1, 1, 3 / 5 1, 1, 5 / 7 1, 2, 3 / 6 1, 2, 5 / 8 1, 3, 5 / 9 1, 2, 3 / 6 1, 2, 5 / 8 1, 3, 5 / 9 2, 3, 5 / 10
27	5 [units]	In a non-equilateral triangle, the length of each side of the triangle is an integer number of units. In units, what is the smallest possible perimeter of the triangle? A triangle with sides 2-2-1 gives the smallest possible perimeter equal to 5 units.

28 | 383125 [$]

A real estate agent is trying to predict the average selling price of a 3-bedroom house based on its distance from a certain city, using the following data:

Distance from city (miles)	Selling Price ($)
10	755,000
50	580,000

Using a linear model based on these two data point, what is the predicted selling price in dollars for a 3-bedroom house that is 95 miles away from the city?
Use the two points to calculate the slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{580000 - 755000}{50 - 10} = \frac{-175000}{40} = -4375$$

Don't know the y-intercept, so pick either of the points and use the point-slope form of the equation:

$y - y_1 = m(x - x_1)$
$y - 755000 = -4375(x - 10)$
$y - 755000 = -4375x + 43750$
$y = -4375x + 798750$ or, $S(d) = -4375d + 798750$

Substitute the value d = 95 into the equation:
$S(95) = -4375(95) + 798750 = \$383125$

29 | 26 [factors]

When the following expression is fully expanded, how many of the resulting terms have at least two variable factors that are vowels? Reminder that the vowels are: a, e, i, o and u.

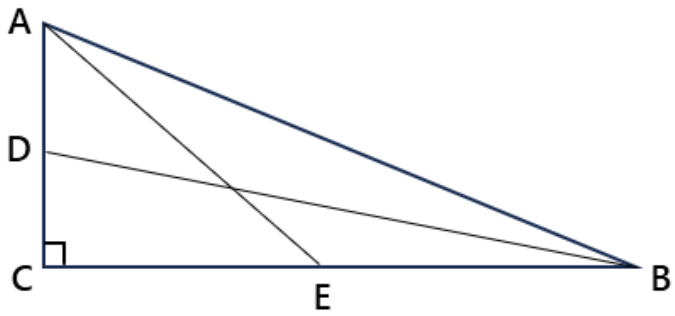
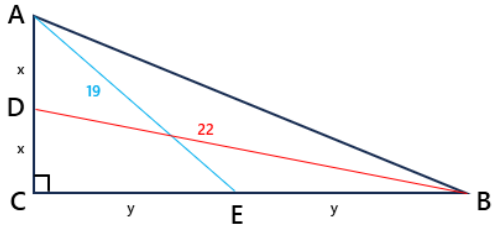
$$(a + p)(e + q)(i + r)(o + s)(u + t)$$

There will be a total of $2^5 = 32$ terms. It will be easiest to subtract off the ones that have either 0 vowels or 1 vowel. There will be only one term that has 0 vowels, pqrst. There will be 5 terms that have 1 vowel: aqrst, perst, pqist, and pqrsu. The rest will have at least two variable factors that are vowels, so 32 – 1 – 5 = 26.

30 | -12

Steve has a calculator that only has one operation. When given a number, the calculator adds 6 to it. Jordan similarly has a calculator that does one thing: it takes a number and squares it. Alice gives Steve a number 'a'. Steve plugs 'a' into his calculator and gives the result to Jordan who plugs it into his calculator. He gives Alice the result. Similarly, Adam gives another number 'b' to Steve who computes it, passes the result to Jordan, who computes it and gives the result to Adam.

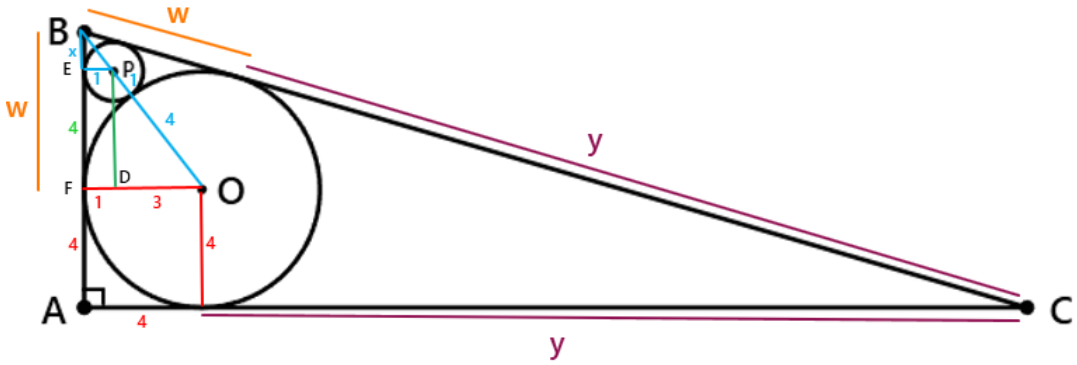
If Alice and Adam both got the same number back and $a \neq b$, what is $a + b$?
$(a + 6)^2 = (b + 6)^2$ but $a \neq b$, therefore one must = 0 and one must = -12, therefore the sum is -12.

31	26 [units]	In right triangle ABC, points D and E are the midpoints of sides AC and BC, respectively. The length of AE = 19 units, and the length of BD = 22 units. In units, what is the length of the hypotenuse, AB?  Let AD = CD = x, and let BE = CE = y. $x^2 + (2y)^2 = 22^2$ $(2x)^2 + y^2 = 19^2$ Add the two equations: $5x^2 + 5y^2 = 845$ $x^2 + y^2 = 169$ $(AC)^2 + (BC)^2 = (AB)^2$ $(2x)^2 + (2y)^2 = 4(x^2 + y^2) = 4(169) = (AB)^2$ Therefore, AB = 2(13) = 26 

32	68 [= A + B]	If three people are selected at random, the probability that at least two of them were born on the same day of the week can be written as a reduced common fraction A/B. What is A + B? None of the people are related to each other. P(at least 2) = P(2) + P(3). There are 343 (7x7x7) total outcomes, and 7 outcomes where all 3 are born on the same day: MMM, TTT, etc. For 2 people the same, you have to consider the different orders. For example if the first two people are the same, you have: MM__ (6 ways) TT __ (6 ways) etc., for a total of 6x7 = 42 ways Same if the first and 3rd person are the same (42 ways), and if the 2nd and 3rd person are the same (42 ways). Therefore, total number of ways = 7 + 42x3 = 133 P = 133/343 = 19/49 19 + 49 = 68
33	458,745 [= sum]	Find the sum of the following finite arithmetic series. $7 + 14 + 28 + \cdots + 229{,}376$ Sum of a finite geometric series is: $S_n = \frac{a_1 - a_1 \cdot r^n}{1 - r}$ $a_1 = 7$ r = 2 n = 16: n a_n 1 $7 \cdot 2^0$ 2 $7 \cdot 2^1$ … … n $7 \cdot 2^{n-1}$ 16 $7 \cdot 2^{15} = 229376$ $S_n = \frac{7 - 7 \cdot 2^{16}}{1 - 2} = 458745$

34	-1	Let r and s be the roots of the following equation: $x^2 + x + 1 = 0$ Find the value of: $r^2 + s^2 + r + s + rs$ For a quadratic equation $y = ax^2 + bx + c$, the sum of the roots = -b/a, and the product of the roots = c/a. Therefore, r + s = -1/1 = -1 rs = 1/1 = 1 $(r + s)^2 = r^2 + 2rs + s^2$ $r^2 + s^2 = -2rs = -2(1) = -2$ Therefore, $r^2 + s^2 + r + s + rs$ = -2 -1 +1 = -1
35	5625 [integers]	How many positive 5-digit integers have the property that every pair of adjacent digits have different parity? Note: parity indicates the oddness or evenness of the digits. The integer needs to be either eoeoe or oeoeo. eoeoe: 4x5x5x5x5 = 2500 ways oeoeo: 5x5x5x5x5 = 3125 ways Total = 2500 + 3125 = 5625
36	66666	A 5-digit whole number has the following feature. If the digit '1' is added to the beginning of the number, the new number is four times smaller than if the digit '4' is added to the end of the original number. What is the original 5-digit number? Let x = original 5-digit number 100,000 + x = original 5-digit number with a '1' added in front 10x + 4 = original 5-digit number with a '4' added in back 4(400000 + x) = 10x + 4 x = 66666
37	10 [= the most fun]	On a scale from 0 to 10, where 0 = the least fun and 10 = the most fun, how much fun are you having on this test? Note: your answer must be an integer, and the answer is > 9 and < 11. Freebie! Because, why not? How many of you read through all of the Challenge Questions?

38	448 [=N]	Triangle ABC has a right angle at vertex A, and contains circles O and P. Circle O has a radius of 4 units, and is tangent to each side of the triangle. Circle P has a radius of 1 unit, and is tangent to two sides of the triangle and to circle O. The area of triangle ABC can be written as a reduced common fraction $\frac{N}{3}$. What is N?



Want to find AB and AC. By creating triangle POD, see that it is a 3-4-5 right triangle, so the length of PD = EF = 4.

Triangle BEP is similar to triangle BFO, therefore: x/1 = (x+4)/4, solve for x = 4/3.

AB = 4+4+4/3 = 28/3

w = 4+4/3 = 16/3

Now we need to find the length of y:

Use the Pythagorean Theorem:

$(AB)^2 + (AC)^2 = (BC)^2$

$$\left(\frac{28}{3}\right)^2 + (4+y)^2 = \left(\frac{16}{3}+y\right)^2$$

$$\left(\frac{28}{3}\right)^2 = \left(\frac{16}{3}+y\right)^2 - (4+y)^2$$

Use the fact that: $a^2 - b^2 = (a+b)(a-b)$

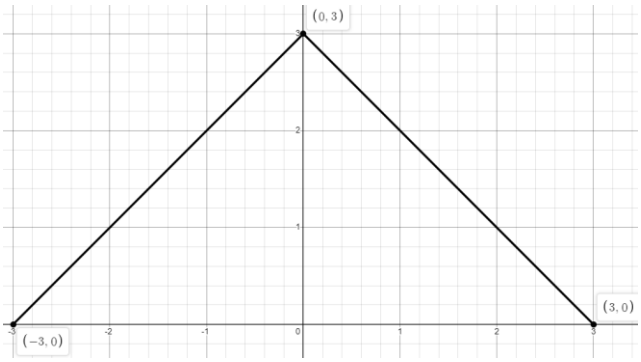
$$\left(\frac{28}{3}\right)^2 = \left(\frac{16}{3}+y+4+y\right)\left(\frac{16}{3}+y-4-y\right)$$

$$\left(\frac{28}{3}\right)^2 = \left(\frac{28}{3}+2y\right)\left(\frac{4}{3}\right)$$

2y = 168/3 = 56, ➔ y = 28

AC = 4 + 28 = 32

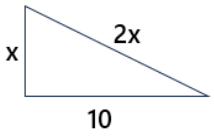
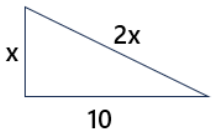
Area of triangle ABC = (1/2)(AB)(AC) = (1/2)(28/3)(32) = 448/3

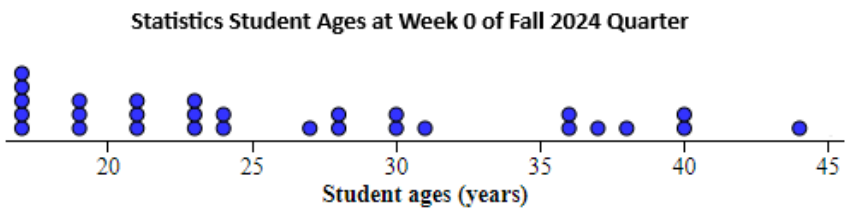
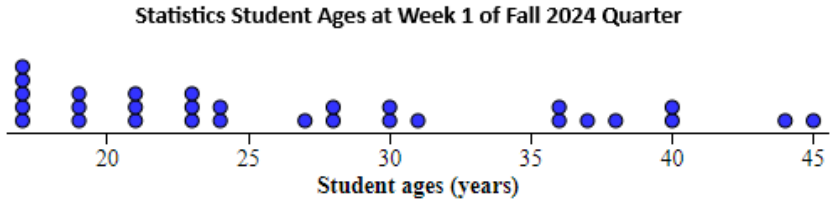
39	9	Evaluate the following definite integral: $$\int_{-3}^{3} (3 - |x|)\, dx$$ The value of the definite integral will be the area between the graph of the function and the x-axis. In this case the function 3 - |x| forms a triangle with the x-axis, with base = 6 and height = 3. A = 1/2(6)(3) = 9. 
40	14 [divisors]	How many positive divisors of 360 have fewer than 7 divisors themselves? $360 = 2^3 3^2 5^1$ Any divisor will be of the form $2^a 3^b 5^c$, with $0 \le a \le 3$, $0 \le b \le 2$, and $0 \le c \le 1$. Any such divisor will have (a + 1)(b + 1)(c + 1) divisors. Using casework, the possible values for (a, b, c) are: (0, 0, 0), (0, 1, 0), (0, 2, 0), (1, 0, 0), (1, 1, 0), (1, 2, 0), (2, 0, 0), (2, 1, 0), (3, 0, 0), (0, 0, 1), (0, 1, 1), (0, 2, 1), (1, 0, 1), and (2, 0, 1). This results in 14 divisors that meet the requirements.

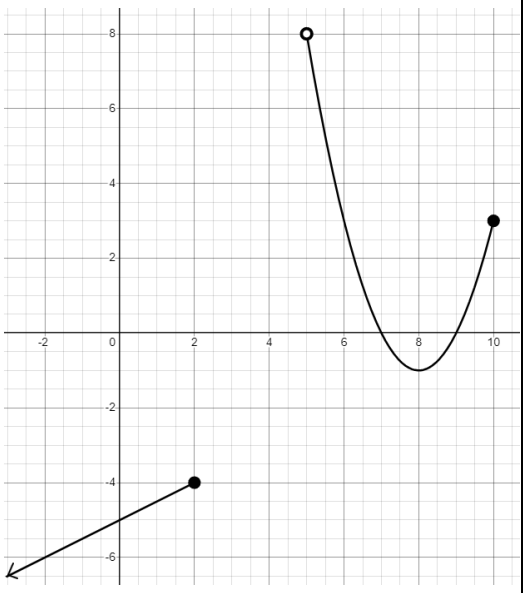
"Math is Cool" Championships -- 2024-25

High School

Multiple Choice Solutions

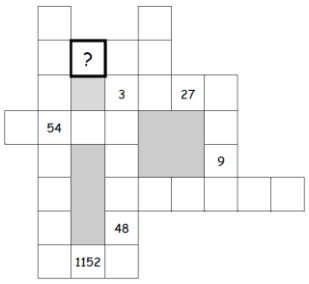
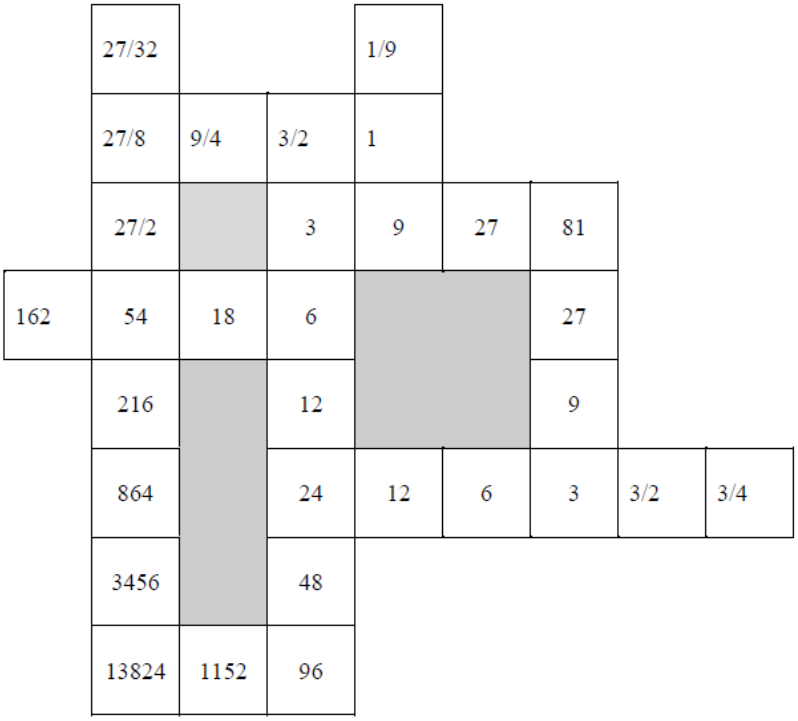
9/ 10th	11/ 12th	Answer	Solution
1		***A***	The hypotenuse of a right triangle is twice the length of one of the legs, and the other leg has length 10 cm. What is the length of the shorter leg? A) $\frac{10\sqrt{3}}{3}$ cm B) $2\sqrt{5}$ cm C) $\frac{20\sqrt{3}}{3}$ cm D) $4\sqrt{5}$ cm E) $10\sqrt{5}$ cm $x^2 + 10^2 = (2x)^2$ $x^2 + 100 = 4x^2$ $3x^2 = 100$ $x^2 = 100/3$ $x = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$ 
	1	***C***	The hypotenuse of a right triangle is twice the length of one of the legs, and the other leg has length 10 cm. What is the length of the hypotenuse? A) $\frac{10\sqrt{3}}{3}$ cm B) $2\sqrt{5}$ cm C) $\frac{20\sqrt{3}}{3}$ cm D) $4\sqrt{5}$ cm E) $10\sqrt{5}$ cm $x^2 + 10^2 = (2x)^2$ $x^2 + 100 = 4x^2$ $3x^2 = 100$ $x^2 = 100/3$ $x = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$  $2x = \frac{20\sqrt{3}}{3}$

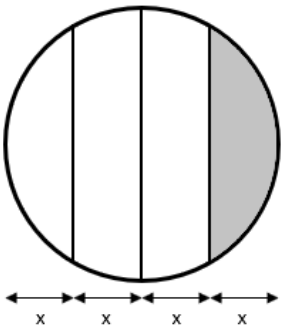
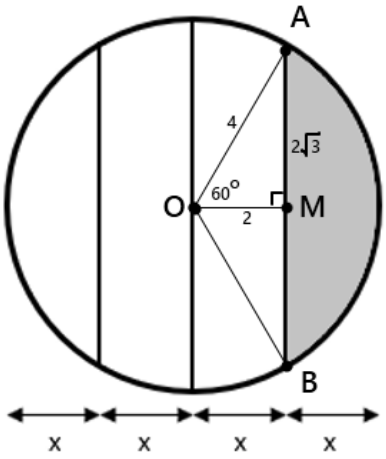
2	**2**	D	In Mrs. Stephenson's Fall 2024 Statistics class, she recorded the following ages in integer years for her students during Week 0, where each dot represents one student's age:  During the first week of class, another student enrolled, who happened to be 45 years old, resulting in an updated plot:  For these data sets, which of the following values were increased in Week 1, compared to Week 0 of the quarter? A) Mean, median, and range B) Median, mode and range C) Median and range D) Mean and range E) Answer not given The mean is increased because a number was added that was higher than all previous numbers, no need to calculate. The range goes up for the same reason. The mode is still 17. The median is still 24.
3		E $\left[=\frac{8}{25}\right]$	Five-eighths is the same fraction of two-thirds as three-tenths is of what number? A) $\frac{9}{32}$ B) $\frac{15}{16}$ C) $\frac{18}{25}$ D) $\frac{12}{19}$ E) Answer not given. $\frac{\frac{5}{8}}{\frac{2}{3}}=\frac{\frac{3}{10}}{x}$ $\frac{5}{8}x=\frac{3}{10}\cdot\frac{2}{3}=\frac{1}{5}$ $x=\frac{1}{5}\cdot\frac{8}{5}=\frac{8}{25}$

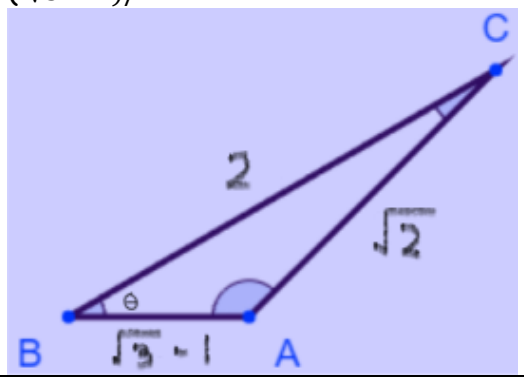
	3	E (-4/5)	If $tan\,\theta = \frac{3}{4}\ and\ sin\,\theta < 0$, what is $cos\,\theta$? A) $\frac{4}{5}$ B) $\frac{3}{5}$ C) $-\frac{3}{5}$ D) $-\frac{4}{3}$ E) Answer not given. With tan + and sin -, must be in QIII, where cos is -. From the given tan value, it is a 3-4-5 right triangle, therefore cos = -4/5.
4		A	Simplify the following expression: $-\boldsymbol{3m} - \boldsymbol{3}[\boldsymbol{2m} - \boldsymbol{3}(\boldsymbol{m} + \boldsymbol{5})]$ A) 45 B) $-6m - 45$ C) $-3m + 45$ D) $3m + 45$ E) Answer not given. = –3m – 3[2m – 3m – 15] = –3m – 3[–m – 15] = –3m – 3(–m) – 3(–15) = –3m + 3m + 45 = 45
	4	C	What is the range of the following function? A) (–∞, 10] B) (–∞, 2] ∪ (5, 10] C) (–∞,–4] ∪ [–1, 8) D) (–∞,8) E) Answer not given. The y-values extend from -infinity up to -4, then continue from -1 up to but not including 8. 
5		B	Which of the following linear equations is perpendicular to the line through the points (-3, 2) and (5, -1)? A) 3x – 8y = -22 B) 8x – 3y = 10 C) x – 2y = -6 D) 8x + 3y = 10 E) Answer not given. The slope through the 2 points is: m = (-1 – 2)/(5 - -3) = -3/8 Therefore the slope of the perpendicular line will = 8/3, which is only true for equation B.

	5	*A*	What is the equation of the line that is the perpendicular bisector of the line segment whose endpoints are (9, 1) and (-1, 5)? A) $y = \frac{5}{2}x - 7$ B) $y = \frac{2}{5}x + 2$ C) $y = -\frac{2}{5}x - 4$ D) $y = \frac{5}{2}x - 4$ E) Answer not given. The slope through the 2 points is: m = (5 – 1)/(-1 - 9) = 4/-10 = -2/5 Therefore the slope of the perpendicular line will = 5/2. The midpoint of the segment is (9-1)/2, (1+5)/2 = (4, 3) y – y1 = m(x – x1) y – 3 = 5/2(x – 4) y = (5/2)x - 7
6		*C*	A 3x3x3 cube is painted blue, then cut into 27 1x1x1 cubes. One of the cubes is chosen at random, then rolled like a die. What is the probability that it lands with a blue side facing up? A) $\frac{1}{27}$ B) $\frac{1}{9}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) Answer not given. There are 27x6 = 162 total cube faces. There are 9x6 = 54 that are painted blue. Therefore, P(blue) = 54/162 = 1/3
	6	D	Gregg rolls a standard 6-sided die until he rolls a 6. What is the probability that he rolled the die exactly 'n' times? A) $\frac{1}{n}$ B) $\frac{1}{6}$ C) $\frac{n \cdot 5^{n-1}}{6^n}$ D) $\frac{5^{n-1}}{6^n}$ E) $\frac{5^n}{6^n}$ So in other words, for the first 'n – 1' rolls, he did NOT roll a 6, which would have a probability of (5/6) each time, or $(5/6)^{n-1}$ for the first 'n – 1' rolls. On the nth roll, the probability of a 6 is 1/6. Therefore the total probability is $(1/6)(5/6)^{n-1} = \frac{5^{n-1}}{6^n}$

7	7	D	The local time in Auckland, New Zealand is 19 hours ahead of the local time in Seattle, WA. In other words, it is 19 hours later in Auckland compared to Seattle. Suppose you are visiting friends in Auckland, and want to call your family in Seattle at 8 p.m. Seattle time. What time should it be in Auckland when you call? A) 1 a.m. B) 9 a.m. C) 1 p.m. D) 3 p.m. E) Answer not given. Say it is midnight in Seattle. Then it is +19 hours in Auckland, or 7 p.m. To reach your family at 8 pm, which is 4 hours earlier than midnight, then call 4 hours earlier than 7 pm, so call at 3 pm.

8	**8**	*C*	Each row and column in the following puzzle contains a geometric sequence, with the progression going either up, down, left or right. What number belongs in the box with the question mark?  A) $\frac{27}{2}$ B) $\frac{9}{2}$ C) $\frac{9}{4}$ D) 1 E) Answer not given 

9		*A*	A circle of radius 4 centimeters is divided into 4 regions of equal width by 3 vertical lines. What is the area of the shaded region?  A) $\frac{16}{3}\pi - 4\sqrt{3}$ B) $12\sqrt{3} - \frac{16}{3}\pi$ C) $16\pi - 12\sqrt{3}$ D) $6\sqrt{3}$ E) 4π The points of the intersection of the right-most vertical line are A and B, and the center of the circle is O. M is the midpoint of AB. Therefore OA = 4 and OM = 2, which makes OAM a 30-60-90 triangle. The entire sector enclosed by angle AOB is 1/3 of the circle, or $(1/3)\pi \cdot 4^2 = 16\pi/3$. Subtract off the area of the 2 triangles AOM and BOM: $$\frac{16}{3}\pi - 4\sqrt{3}$$ 

	9	A	What is the area of a triangle with side lengths 2, $\sqrt{2}, and\ \sqrt{3}-1$? A) $\frac{\sqrt{3}-1}{2}$ B) 1 C) $\frac{\sqrt{3}+1}{2}$ D) $\sqrt{2}$ E) $2\sqrt{2}$ Start with the Law of Cosines, where θ is the angle between the sides of length 2 and $\sqrt{3}-1$. $c^2 = a^2 + b^2 - 2abcos\theta$ $\sqrt{2}^2 = 2^2 + (\sqrt{3}-1)^2 - 2(2)(\sqrt{3}-1)(cos\theta)$ $4(\sqrt{3}-1)(cos\theta) = 6 - 2\sqrt{3}$ $$cos\theta = \frac{6-2\sqrt{3}}{4(\sqrt{3}-1)}$$ After simplifying and rationalizing the denominator: $$cos\theta = \frac{\sqrt{3}}{2}$$ Therefore, $\theta = 30°$. Area of the triangle = $\frac{1}{2}$ ab $sin\theta$ = $\frac{1}{2}$ (2)$(\sqrt{3}-1)(1/2) =$ $(\sqrt{3}-1)/2$ 
10	**10**	E	An urn contains 4 silver coins and 3 gold coins. If the coins are drawn one-by-one, without replacement, what is the probability that the 5th coin drawn is silver? A) $\frac{5}{14}$ B) $\frac{3}{7}$ C) $\frac{1}{2}$ D) $\frac{9}{14}$ E) $\frac{4}{7}$ The problem is equivalent to saying: if the coins are randomly arranged in a row, looking from left to right, what is the probability that the 5th one is silver? Each individual coin has the same chance to be in the 5th spot, therefore the probability = 4/7.

"Math is Cool" Championships -- 2024-25

High School

Team Test Solutions

9/ 10th	11/ 12th	Answer	Solution
1	**1**	135 [jumps]	Packard's pet rabbit, Kirby, jumps 15 centimeters with each jump. Kirby is currently 2024 centimeters away from Packard, and jumps directly towards Packard on a straight line. How many jumps will it take Kirby to reach Packard? Assume that on the last jump, Kirby can jump exactly the remaining distance needed, if it is less than 15 cm. 2024/15 = 134.933… Therefore it will take a total of 135 jumps.
2		78 [= A]	A circular swimming pool is 6 feet in diameter, and it is surrounded by a ring of tiling that is 3 inches wide. The outer circumference of the tile is $A\pi$ inches. What is A? The radius of the circle = 3 ft = 36 inches. Add 3 inches for the tile, total radius = 39 inches. $C = 2\pi r = 2\pi(39) = 78\pi$.
	2	20 [= x + y]	Triangle ABC has vertices A (0, 0), B (30, 0) and C (0, 15) on the coordinate plane. Point D, which lies on BC, is the foot of the angle bisector from A to BC. The coordinates of point D are (x, y). What is x + y? The angle bisector through A is the line y = x. Additionally, BC is on the line y = -1/2 x + 15. Equating the two: x = -1/2 x + 15 3/2 x = 15 x = 10, and also y = 10, since point D lies on the line y = x.
3		405	What is the 3rd largest divisor of 2025? $2025 = 3^4 \cdot 5^2$ Therefore the 3rd largest divisor will be 2025/9 = 405

<table>
<tr><td></td><td>3</td><td>6 [perfect squares]</td><td>How many perfect squares are divisors of 2025?

$2025 = 3^4 \cdot 5^2$
Therefore, the perfect squares that divide it will be 1, 3^2, 3^4, 5^2, $3^2 \cdot 5^2$ and $3^4 \cdot 5^2$</td></tr>
<tr><td>4</td><td></td><td>46 [=A + B]</td><td>A positive integer less than 40 is randomly selected. The probability that the integer is a multiple of 5 can be written as a reduced common fraction A/B. What is A + B?

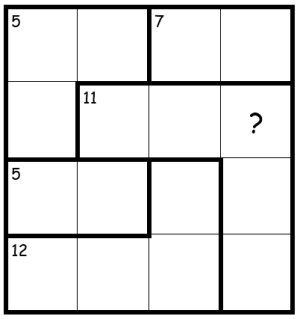
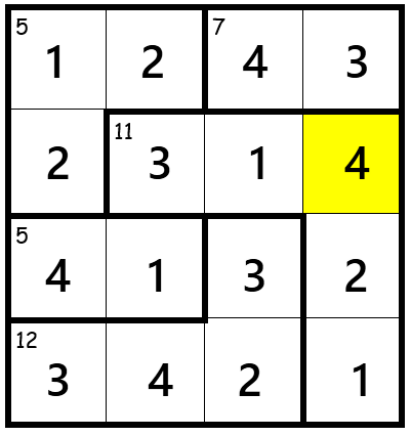
There are 39 positive integers less than 40 (1 – 39), and there are 7 multiples of 5 less than 40 (5*1 through 5*7). Therefore: P = 7/39 = A/B
A + B = 7 + 39 = 46</td></tr>
<tr><td></td><td>4</td><td>44 [%]</td><td>Let A = {1, 2, 3, 4, 5} and B = {15, 16, 17, 18, 19}. One number is randomly selected from set A, and one number is randomly selected from set B. As a percentage, what is the probability that dividing the number from B by the number from A (B/A) results in an integer?

There are 25 total outcomes in the table, and 11 of them are integers. P = 11/25 = 44/100

<table>
<tr><th></th><th>15</th><th>16</th><th>17</th><th>18</th><th>19</th></tr>
<tr><td>1</td><td>15/1 = 15</td><td>16/1 = 16</td><td>17/1 = 17</td><td>18/1 = 18</td><td>19/1 = 19</td></tr>
<tr><td>2</td><td>15/2, non-int</td><td>16/2 = 8</td><td>17/2, non-int</td><td>18/2 = 9</td><td>19/2, non-int</td></tr>
<tr><td>3</td><td>15/3 = 5</td><td>16/3, non-int</td><td>17/3, non-int</td><td>18/3 = 6</td><td>19/3, non-int</td></tr>
<tr><td>4</td><td>15/4, non-int</td><td>16/4 = 4</td><td>17/4, non-int</td><td>18/4, non-int</td><td>19/4, non-int</td></tr>
<tr><td>5</td><td>15/5 = 3</td><td>16/5, non-int</td><td>17/5, non-int</td><td>18/5, non-int</td><td>19/5, non-int</td></tr>
</table></td></tr>
<tr><td>5</td><td></td><td>27 [= average]</td><td>Four numbers are written in a row. The average of the first two numbers is 10. The average of the middle two numbers is 3. The average of the last two numbers is 20. What is the average of the first and last numbers?

Call them: a b c d
a + b = 20
b + c = 6
c + d = 40
a – c = 14
a + d = 54, therefore average is 54/2 = 27</td></tr>
<tr><td></td><td>5</td><td>36</td><td>Evaluate: $25^{\log_5 6}$
$25^{\log_5 6} = (5^2)^{\log_5 6} = 5^{2\log_5 6}$
$= 5^{\log_5 6^2} = 6^2 = 36$</td></tr>
</table>

6		20 [units]	What is the sum, in units, of the minimum and maximum distances from the point (4, -3) to the following circle? $x^2 + y^2 + 4x - 10y - 7 = 0$ Put the circle into standard form: $(x + 2)^2 + (y - 5)^2 = 36$ Center is (-2, 5), radius = 6 Use the distance formula to find the distance from the center to the point (4, -3). $d = \sqrt{(4-(-2))^2 + (-3-5)^2} = 10$ Therefore, the minimum distance from (4, -3) to the circle is 10 – 6 = 4, and the maximum distance is 4 + 12 = 16. 16+4 = 20
	6	2 [polynomials]	How many polynomials are there of the following form, such that c and d are real numbers, and the three roots of the polynomial are distinct positive integers? $x^3 - 8x^2 + cx + d$ By Vieta's formula, the sum of the roots r, s, t will be -b/a = -(-8)/1 = 8. If all three roots are positive integers, then: r + s + t = 8. The only way to accomplish this is: 1, 2, 5, or 1, 3, 4.
7	**7**	324 [4-digit integers]	How many positive 4-digit integers have three of the same digit and a different fourth digit (in any order)? Consider the case where there are 3 0's. There can't be a leading 0, so there are only 9 ways. For any other digit, for example 1, there are 4 cases: 111x – 9 ways 11x1 – 9 ways 1x11 – 9 ways x111 – 8 ways (no leading 0) where x is any other digit. There are a total of 9+9+9+8 = 35 ways, times 9 digits = 315. 315+9 (from above) = 324 total.

8	**8**	4	In the following grid, the integers 1, 2, 3 and 4 appear exactly once in each row and each column. The sum of the integers in each heavily outlined region is equal to the number given in the upper left corner of the region. What number goes in the square with the question mark?  
9	**9**	96	The number 89 can be represented as the product plus the sum of the same two positive integers, as follows: $89 = (5 \times 14) + (5 + 14)$ What is the smallest integer greater than 89 that cannot be represented in this manner? The integer is being represented as: n = ab + (a + b) = a(b + 1) + b Add 1 to both sides: n + 1 = a(b + 1) + (b + 1) = (a + 1)(b + 1) Therefore, n + 1 must be factorable. For example in this case, n = 89, so n + 1 = 90 which factors as 6 x 15, or (5 + 1)(14 + 1), so a = 5 and b = 14. Therefore, 89 = $(5 \times 14) + (5 + 14)$ The next largest prime number is 97, therefore we cannot write the preceding integer 96 in this manner.

<table>
<tr>
<td>10</td>
<td></td>
<td>11</td>
<td>Let N = 1234567891011…998999 be the integer formed by writing the inters 1, 2, 3, …, 999 in order. What is the 2-digit integer formed by the 2024th and 2025th digits from the left, in that order?

1…9 = 9 digits
10…99 = 180 digits
2024 – 189 = 1835
1835/3 = 611r2, so the 2024th and 2025th digits are 11.</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>480</td>
<td>For a real number x, the floor function $\lfloor x \rfloor$ is defined as the greatest integer less than or equal to x.

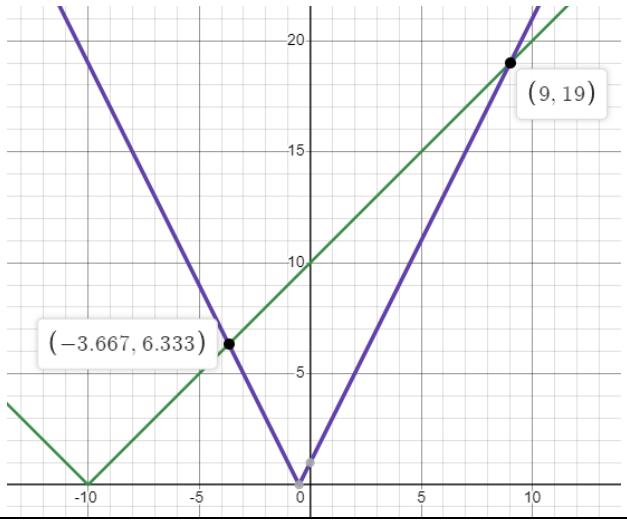
Evaluate the following:
$$\sum_{n=1}^{100} \lfloor log_2 n \rfloor$$
For the function $y = \log_2 x$, the exponential form is $2^y = x$. Therefore, for x = 1 through 100, the output values will be 0 through 6, since $2^7 = 128$. Count how many times each output value occurs:
<table>
<tr><td>x</td><td>floor($\log_2 x$)</td><td># of times</td><td>Sum</td></tr>
<tr><td>1</td><td>0</td><td>1</td><td>0</td></tr>
<tr><td>2…3</td><td>1</td><td>2</td><td>2</td></tr>
<tr><td>4…7</td><td>2</td><td>4</td><td>8</td></tr>
<tr><td>8…15</td><td>3</td><td>8</td><td>24</td></tr>
<tr><td>16…31</td><td>4</td><td>16</td><td>64</td></tr>
<tr><td>32…63</td><td>5</td><td>32</td><td>160</td></tr>
<tr><td>64…100</td><td>6</td><td>37</td><td>222</td></tr>
<tr><td></td><td></td><td></td><td>480</td></tr>
</table>
</td>
</tr>
</table>

"Math is Cool" Championships -- 2024-25

High School

Pressure Round Solutions

9/ 10th	11/ 12th	Answer	Solution
1	1	1025 [= a + b]	Sahil and Mir are solving a math problem which asks for the sum of the first ten powers of 2. Sahil assumes this means the powers of 2 starting with $2^0 = 1$, and Mir assumes that it means the powers of 2 starting with $2^1 = 2$. Let 'a' equal the positive difference of their answers, and let 'b' equal Mir's answer divided by Sahil's answer. What is a + b? Sahil: 1+2+4+8+16+32+64+128+256+512 = 1023 Mir: 2+4+8+16+32+64+128+256+512+1024 = 2046 a = 2046 – 1023 = 1023 b = 2046/1023 = 2 1023 + 2 = 1025
2		2 [= n]	Solve for n, where n > 0: $9^{3n} = 27^{n^2}$ $9^{3n} = 27^{n^2}$ $(3^2)^{3n} = (3^3)^{n^2}$ $3^{6n} = 3^{3n^2}$ $6n = 3n^2$ $2n = n^2$ $n^2 - 2n = 0$ $n(n - 2) = 0$ $n = 2$
	2	10 [= x]	Solve for x: $2^{38} + 4^{19} + 8^{13} = 16^x$ $2^{38} + (2^2)^{19} + (2^3)^{13} = (2^4)^x$ $2^{38} + 2^{38} + 2^{39} = 2^{4x}$ $2^{38}(1 + 1 + 2) = 2^{4x}$ $4 \cdot 2^{38} = 2^{4x}$ $2^{38} = 2^{4x-2}$ $4x - 2 = 38$ $4x = 40$ $x = 10$

3	**3**	50 [%]	A 2-digit positive integer is randomly chosen. As a percentage, what is the probability that the product of the digits in the number is a positive multiple of 3? There are a total of 90 2-digit positive integers. If you make a partial list, there is a pattern: all of the numbers 31-39, 61-69 and 91-99 are divisible when the digits are multiplied. For the rest of the numbers, there are 3 in each column, for example 23, 26 and 29. Therefore, a total of 9x3 + 3x6 = 45, and 45/90 = $\frac{1}{2}$ = 50%.
4	**4**	12 [integers]	How many integers x satisfy the following inequality? $\lvert x+10\rvert > \lvert 2x+1\rvert$ One way to solve would be by making a rough sketch of each function and investigating the boundary values. |2x + 1| is less than |x + 10| for x = -3 through 8, or a total of 12 x values. 
5		19607	Evaluate: $7^5 + 7^4 + 7^3 + 7^2 + 7$ 16807 + 2401 + 343 + 49 + 7 = 19607
	5	66429	Evaluate: $9^5 + 9^4 + 9^3 + 9^2 + 9$ 59049 + 6561 + 729 + 91 + 9 = 66429

"Math is Cool" Championships -- 2024-25

High School

College Bowl Round #1 Solutions

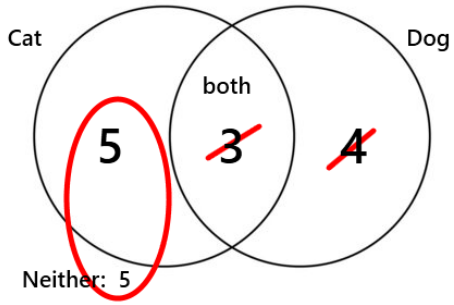
	Answer	Solution
1	-4 [=x]	Solve for x: Negative two over seventeen equals 'x' over thirty-four -2/17 = x/34 x = -4
2	30 [square units]	The hypotenuse of a right triangle with integer side lengths is thirteen units. In square units, what is the area of the triangle? It is a 5-12-13 right triangle, therefore the area = 1/2 (5)(12) = 30
3	10000 [= n] [ten thousand]	For which positive integer 'n' does four hundred squared times four hundred squared equal sixteen squared times n-squared? $400^2 x 400^2 = 16^2 x 10000^2 = 16^2 x n^2$ n = 10000
4	19 [values of n]	Quinn has 'n' pieces of candy that she is trying to split between five people equally. However, after splitting them into five equal piles, two pieces are left over. If Quinn has between five and one hundred pieces of candy, how many possible values of 'n' are there? The possibilities start with 5·1 + 2 = 7, and continue up to 5·19 + 2 = 97, for a total of 19 possible values.
5	450 [even three digit positive integers)	How many even three digit positive integers are there? 9·10·5 = 450
6	20 [m]	A soccer ball is dropped from a height of ten meters, and each time it bounces it goes up to one-third of its previous height. How many meters does the ball travel in total before coming to rest? The first drop = 10, then each bounce up and return after that is multiplied by 1/3. $10 + 2(10/3) + 2(10/3^2) + \ldots$ $= 10 + 2 \cdot \frac{\frac{10}{3}}{1 - \frac{1}{3}} = 20$

7	10 [%]	Leon has five black shirts, seven green shirts and eight yellow shirts. He also has four pairs of black pants and six pairs of khaki pants. If he randomly chooses one shirt and one pair of pants, what is the probability in percent that they are both black? 5+7+8 = 20 shirts 4+6 = 10 pants P(black & black) = (5/20)(4/10) = 1/10 = 10%
8	3 [= sum]	What is the sum of all values of 'x' that satisfy the following equation: x squared minus ten equals three 'x' $x^2 - 10 = 3x$ $x^2 - 3x - 10 = 0$ $(x - 5)(x + 2) = 0$ $x = 5, -2$
9	336 [square units]	Rectangles A and B are similar, with a side length of two point one units on rectangle B corresponding to a side length of twenty-one units on rectangle A. What is the area of rectangle A in square units, if the area of rectangle B is three point three six square units? The scale factor for the side lengths is 21/2.1 = 10. Therefore the scale factor for the area will be $10^2 = 100$. Area of A = 100(3.36) = 336.
10	40 [minutes]	Julio lives one mile away from his school, and walks at a constant rate of three miles per hour. How many minutes will it take him to walk to school and then back home? He walks a total of 2 miles at a rate of 3 miles/hour. Time = 2/3 hours = 40 minutes

"Math is Cool" Championships -- 2024-25

High School

College Bowl Round #2 Solutions

	Answer	Solution
1	-3 [= x]	What is the largest real number x that satisfies the inequality: two 'x' plus three is greater than or equal to four 'x' plus nine $2x + 3 \geq 4x + 9$ $-6 \geq 2x$ $-3 \geq x$, or $x \leq -3$
2	1464 [hours]	How many hours are in the months of October and November combined? 31 + 30 = 61 days 61x24 = 1464
3	4 [= d]	In an arithmetic (pronounced air-ith-MET-ic) sequence, the first term is twenty-three and the seventh term is forty-seven. What is the value of the common difference, 'd'? Adding 4 each time we have: 23, 27, 31, 35, 39, 43, 47
4	7 [people]	Diego is throwing a party for at least four other friends and buys a bag of five hundred forty-six pieces of candy. He distributes all of the candy, with each person including himself getting an equal number of pieces. If the number of people present is odd, what is the smallest number of people that could be present? 546 not divisible by 5. 546/7 = 78, therefore the least number is 7.
5	50 [%]	In a class of seventeen students, eight have a cat, seven have a dog, and three have both a cat and a dog. What is the probability as a percent that a student has a cat, given that they do not have a dog? There are 10 students who do NOT have a dog, therefore P(cat | no dog) = 5/10 = 50% 

6	39 [= least possible sum]	Four friends each picked a different non-prime positive integer. The greatest common factor of each pair of integers was one. What is the least possible sum of the four integers? The numbers are: 1, 4, 9, 25, because none of them share a factor other than 1. 1+4+9+25 = 39
7	900 [5-digit palindromes]	How many positive five digit palindromes are there, where a palindrome is a number that reads the same forwards and backwards? There are 9 ways to choose the 1st digit (1 – 9), 10 ways to choose the 2nd digit (0 – 9) and 10 ways to choose the 3rd and middle digit (0 – 9). The 4th and 5th digits are determined by the 1st and 2nd. Therefore, 9x10x10 = 900.
8	2024	If the quantity 'x' minus one times the quantity 'y' minus one equals two thousand twenty-four, what is the value of quantity one minus 'x' times the quantity one minus 'y'? If $(x – 1)(y – 1) = 2024$, then $(1 – x)(1 – y) = (-1)(x – 1)(-1)(y – 1) = 2024$
9	50 [square units]	ABCD is a square with side length 10 units, and point P lies on side AB. In square units, what is the area of triangle PCD? Area = ½(b)(h) = ½(10)(10) = 50
10	5 [days]	If five cows can produce five gallons of milk in five days, how many days will it take six cows to produce six gallons of milk? It takes 5x5 = 25 cow-days/5 gallons or 5 cow-days/1 gallon. Therefore we need 30 cow-days/6 gallons, and 30 cow-days/6 cows = 5 days.

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High School

College Bowl Round #3 Solutions

	Answer	Solution
1	2 [values of x]	How many integer values of 'x', where 'x' is from zero to seven inclusive, satisfy the following inequality: negative two 'x' plus four is greater than or equal to one $-2x + 4 \geq 1$ $3 \geq 2x$ $x \leq 3/2$ $x = 0$ or 1
2	50000 [= fifty thousand]	As an integer, what is two times ten to the negative six divided by four times ten to the negative eleven? $2x10^{-6}/4x10^{-11} = 0.5x10^{5} = 5x10^{4} = 50000$
3	1 [= positive difference]	What is the positive difference between the median and mean of the following data set: Five, fifty, forty-six, thirty, ten, twenty-nine, thirty-three In order: 5, 10, 29, 30, 33, 46, 50 Middle = 30 = median Mean = 203/7 = 29 Difference = 30 – 29 = 1
4	7 [positive divisors]	How many positive divisors does seven hundred twenty-nine have? $729 = 3^{6}$, therefore it is divisible by 3^{0}, 3^{1}, …, 3^{6}.
5	2 [minutes]	How many minutes does it take a clock's hour hand to move through one degree of arc around the clock face? A clock's hour hand moves 30° in one hour, or 30°/60 minutes = 1°/2 minutes
6	1250 [mm]	Zeno is forty meters from his car, and every minute he walks half the remaining distance to the car. During the first minute he walks twenty meters towards the car. During the second minute he walks ten meters towards the car. How many millimeters does he walk during the fifth minute? 40m, 20m, 10m, 5, 2.5m, 1.25m 1.25m = 1250 mm
7	35 [%]	A jar is filled with red, white and blue marbles. If one marble is selected at random, the probability that it is red is one-fourth, and the probability that it is white is two-fifths. As a percentage, what is the probability that it is blue? ¼ = 25% 2/5 = 40% 100 – 25 – 40 = 35%

8	20 [= n, = no. of people]	You are going to buy either one hundred hot dogs or sixty hamburgers for a group of 'n' people, including yourself. With either choice, the items can be distributed equally between the 'n' people. What is the largest possible value of n? Find gcd(60, 100) $60 = 2^2 3^1 5^1$ $100 = 2^2 5^2$ Therefore gcd = $2^2 5^1 = 20$
9	64 [= A]	A cone has a diameter of four units and a volume of eight pi cubic units. A cone that is similar to this one has a diameter of eight units and a volume of A times pi cubic units. What is A? Two similar 3-D figures with a similarity ratio of 'x' have a volume ratio of x^3. In this case the similarity ratio is 8/4 = 2, and $2^3 = 8$, so the volume is 8 times the volume of the smaller cone.
10	12 [= n]	For what integer 'n' are the roots of the following equation consecutive integers? x-squared minus 7x plus 'n' equals 0 $x^2 - 7x + n = 0$ $(x - 3)(x - 4) = x^2 - 7x + 12$ n = 12

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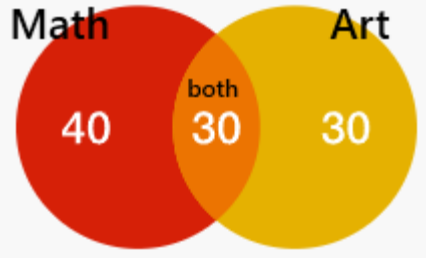
High School

College Bowl Round #4 Solutions

	Answer	Solution
1	3 [unique prime factors]	How many distinct prime factors does eight-four have? $84 = 2^2 3^1 7^1$, so it has 3 distinct prime factors.
2	48	What is two raised to the sixth power minus two raised to the fourth power? $2^6 - 2^4 = 64 - 16 = 48$
3	65,536	Find the next term in the sequence that begins: one, sixteen, two hundred fifty-six, four thousand ninety-six, and so on. Pattern is powers of 16.
4	0	Let f of 'x' equal 'x' to the fourth minus seven 'x' to the third minus three 'x' plus ten. How many negative numbers 'a' are there such that f of 'a' equals zero? $f(x) = x^4 - 7x^3 - 3x + 10$ Consider putting ANY negative number in: $f(neg) = (neg)^4 - 7(neg)^3 - 3(neg) + 10$ = pos + pos + pos + 10
5	6 [square units]	Three circles with radii of one, two and three units are each externally tangent to the other two. A triangle is formed by joining the three centers of the circles. What is the area of the triangle, in square units? Connecting the centers will make a 3-4-5 right triangle, so the area = (1/2)(3)(4) = 6.
6	40 [students]	There are one hundred students at a school, each of whom takes at least a math or an art class. Seventy of the students take a math class, and thirty students take both a math and an art class. How many students take a math class but not an art class? 

7	22 [units]	The length of a rectangle is two times its width. When its length is decreased by four units and its width is increased by six units, a new rectangle with the same area is created. In units, what is the perimeter of the new rectangle? Original rectangle is x by 2x New rectangle is x + 6 by 2x – 4 $2x^2 = (x + 6)(2x – 4) = 2x^2 + 8x – 24$ x = 3, therefore new rectangle is 9 by 2, perimeter = 22
8	83 [%]	A technical college has four thousand students in their freshman class, and three thousand three hundred and twenty of them are majoring in a STEM-related field. What percent of freshmen are majoring in a STEM-related field? 3320/4000 = 0.83 = 83%
9	9 [square cm]	In square centimeters, what is the maximum possible area of a rectangle with a perimeter of twelve centimeters? A square always has the maximum area. 12/4 = a side length of 3.
10	6 [= x times y]	The following two lines intersect at the point 'x' comma 'y'. What is 'x' times 'y'? three 'x' plus three 'y' equals fifteen two 'x' plus six 'y' equals twenty-two 3x + 3y = 15 (2x + 6y = 22)/-2 -x – 3y = -11 Add equations: 2x = 4, x = 2 Solve for y = 3, 2x3 = 6

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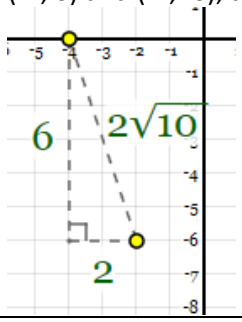
High School

College Bowl Round #5 Solutions

	Answer	Solution
1	6440 [cents]	At Wolffy's Hamburgers, your check comes to fifty-six dollars, and you leave a fifteen percent tip. In cents, how much total money do you pay? 56(1.15) = 64.4 = 6440 cents
2	110	What is the smallest three-digit positive integer that can be written as the product of two consecutive positive integers? 10x11 = 110
3	12 [= A + B]	On the coordinate plane, the distance between the points negative four comma zero and negative two comma negative six can be written in simplest radical form as A times the square root of B. What is A plus B? (-4, 0) and (-2, -6), and use the Pythagorean Theorem. 
4	20 [= b – c]	A, b, c and d are four numbers, with a less than b and c less than d. The average of a and b is c. The average of c and d is b. If d minus a equals sixty, what is the value of b minus c? The information gives us 3 equations: a + b – 2c = 0 2b – c – d = 0 -a + d = 60 Add the 3 equations: 3b – 3c = 60 b – c = 20
5	84 [square units]	In square units, what is the area of a triangle with side lengths of thirteen, fourteen and fifteen units? Use Heron's formula. The semi-perimeter = (13+14+15)/2 = 21 A = $\sqrt{21(8)(6)(7)} = 7 \cdot 3 \cdot 4 = 84$

6	16 [= 10^{th} term]	In a sequence with a first term of five, each succeeding term is the sum of the digits of the square of the previous term. For example, five squared equals twenty-five, and two plus five equals seven which is the second term. What is the tenth term in the sequence? 5, 5^2 = 25, 2+5 = 7 7, 7^2 = 49, 4+9 = 13 13, 13^2 = 169, 1+6+9 = 16 16, 16^2 = 256, 2+5+6 = 13 13 16 13 16 13 16 is the 10^{th} term
7	50 [%]	If a fair coin is flipped ten times, what is the probability in percent of getting an odd number of heads? The number of ways to get an odd number of heads is the same number of ways to get an even number of heads, so the probability is 50% for both.
8	12 [units]	On the coordinate plane, points A, B and C are at one comma one, five comma negative two, and one comma negative 2. In units, what is the perimeter of triangle ABC? A 3-4-5 triangle is formed, which has a perimeter of 3+4+5 = 12.
9	24 [inches]	A square piece of paper is folded in half horizontally to form a rectangle. The rectangle has a perimeter of eighteen inches. In inches, what is the perimeter of the original square? Let x = the side of the square The sides of the rectangle will be x, x, x/2, x/2. Perimeter of the rectangle = 3x = 18, therefore x = 6.
10	-50	If x-squared plus four 'x' plus five equals zero, what is the value of ten x-squared plus forty 'x'? $x^2 + 4x + 5 = 0$ $x^2 + 4x = -5$ $10(x^2 + 4x = -5)$ $10x^2 + 40x = -50$

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High School

College Bowl Round #6 Solutions

	Answer	Solution
1	1200 [12 hundred]	What is one thousand two hundred thirty-seven point seven eight five rounded to the nearest hundred? 1237.785
2	1375	What is two thousand minus 'x' to the fourth, when 'x' equals five? $5^4 = 625$ 2000 – 625 = 1375
3	7 [= x-coordinate of point C]	On the coordinate plane, point A is at two comma five and point B is at three comma four. If point C is co-linear with points A and B, and lies on the x-axis, what is the x-coordinate of point C? The slope of the line is -1, so can count down and over to reach point C at (7, 0).
4	4 [= x]	The greatest common divisor of 'x' squared plus fifty and four 'x' plus two is six. What is the smallest positive integer that 'x' could be? x = 1: 51, 6 x = 2: 54, 10 x = 3: 59, 14 x = 4: 66, 18 – first case where gcd = 6
5	6 [orders]	Jimmy and Seth are playing a best-of-five card game series, where the first player to win three games wins the series, and there are no ties. If the series lasts five games, with Seth winning, in how many different orders could the outcomes of the games have gone? It must be __ __ __ __ S, since Seth won the 5th game. The first 4 games are SSJJ in some order, so 4!/(2!2!) = 6.
6	-1 [= 10th term]	In a sequence that begins with one, three, two, and so on, each term starting with the third term is equal to the term preceding it minus the term preceding that one. What is the tenth term in the sequence? The first 10 terms will be: 1, 3, 2, -1, -3, -2, 1, 3, 2, -1
7	46 [%]	Apple tells the truth sixty percent of the time and Bailey tells the truth seventy percent of the time, independently of each other. It is true that they are both sixteen years old. They are each asked: are you sixteen years old? As a percent, what is the probability that they give different answers? Either Apple tells the truth and Bailey lies, or Apple lies and Bailey tells the truth. P(TL or LT) = (0.6)(0.3) + (0.4)(0.7) = 0.18 + 0.28 = 0.46 = 46%

8	3 [= n]	For what positive integer 'n' are both 'n' and n-squared plus two prime? 3 and 3^2 + 2 are both prime
9	20 [cubic meters]	What is the volume, in cubic meters, of a cylinder with a height of eighty over pi meters, and a base radius of fifty centimeters? h = 80/pi meters radius = 50 cm = ½ m $V = \pi r^2 h = \pi(1/2)^2(80) = 20$
10	7 [= x]	Solve for x: Negative four over three plus the quantity 3x plus seven over twelve equals 1 $-4/3 + (3x + 7)/12 = 1$ $(12)(-4/3) + 3x + 7 = 12$ $-16 + 3x + 7 = 12$ $3x = 21$ $x = 7$

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High School

College Bowl Extra Questions Solutions

	Answer	Solution
1	216 [miles]	A car drives six hundred thirty miles on thirty-five gallons of gas. How many miles can it drive on twelve gallons of gas? 630/35 = x/12 x = 216
2	51 [$]	Three friends spent a combined total of one hundred twelve dollars at the county fair. Ross spent half the amount Joey spent, and Chandler spent ten dollars more than Ross spent. How many dollars did Joey spend? R = ½ J C = R + 10 R + J + C = 112 R + 2R + R + 10 = 112 4R = 102 R = 25.5, J = 2R = 51
3	1 [= a + b]	The line 3x plus four equals y is reflected over the y-axis to the line ax plus b equals y. What is the sum of a and b? Slope becomes negative, b remains same.
4	16 [= n]	Anita rolls two fair n-sided dice, where n is greater than five. What is n, if the most probable sum of the numbers obtained is seventeen? n = 17-1 Pattern is n+ 1 = most probable sum
5	36 [socks]	There are fifty unpaired socks in a sock drawer. Fifteen are pink, fifteen are purple, and the rest are yellow. How many socks do you have to take out of the drawer to have guaranteed that at least one sock of each color was taken out? You would have to pull all of the yellow socks, and then pink (or purple) socks. After that, then you would finally get one of the last color.
6	11 [= n + d]	The number zero point three seven five can be expressed as a fraction n over d, where n and d are relatively prime integers. What is n plus d? 1/8 = 0.125; 3/8 = 0.375 3+8 = 11