"Math is Cool" Masters -- 2024-25

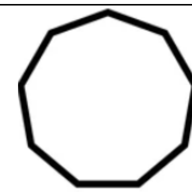
High School

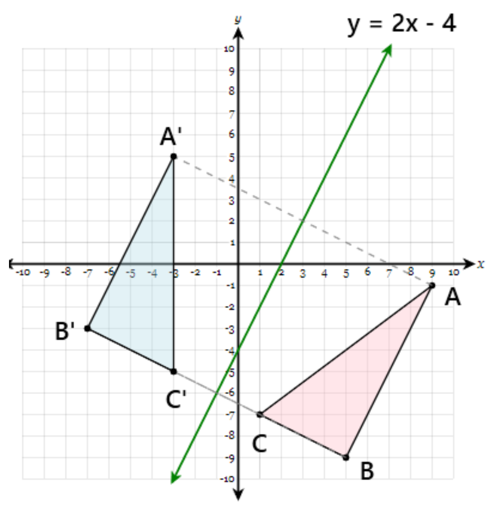
Mental Math Solutions

	Answer	Solution
1	16 [gummy worms]	Keke has a bag of gummy worms. She gives half of them to her sister Coco, then she gives two of them to their dog Lulu. Now she has six gummy worms left. How many did she start with? 6 + 2 = 8 8*2 = 16
2	250 [bricks]	What is the minimum number of rectangular bricks, each measuring twelve inches by eighteen inches, needed to completely cover five flat rectangular surfaces, each measuring sixty inches by one hundred eighty inches? 12x18 covering 60x180 60/12 = 5, 180/18 = 10, need 5*10 = 50 for one surface. Therefore need 5*50 to cover five surfaces.
3	5	What is the sum of the coordinates of the point at which y equals x minus three and y equals negative two x plus nine intersect? $x - 3 = -2x + 9$ $3x = 12$, $x = 4$, $y = 1$ 4 + 1 = 5
4	5	What positive integer can be added to the set of integers: one, two, four, and eight, such that the new set of five integers has a median that is equal to its mean? 1, 2, 4, 5, 8 Sum = 20, mean = 4, median = 4
5	1	What is the units digit of three raised to the thirty-sixth power? $3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$, then the units digit repeats in a cycle of 4. 36 is divisible by 4, so 3^{36} will end in 1.
6	-8 [= y cubed]	A line with a slope of two passes through the points two comma six and negative two comma y. What is the value of y cubed? From (2, 6), go down 2 and left 1, repeat, until arriving at (-2, -2). $(-2)^3 = -8$.
7	20	What integer is closest to pi-squared times two? π^2 is about 9.9, so 2 times that is going to be closest to 20.
8	17 [= A + B]	The first term of an arithmetic (pronounced air-ith-MET-ic) sequence is one-fourth, and the fifth term is one-half. As a reduced common fraction, the sum of the second, third and fourth terms is A over B. What is A + B? To get from ¼ to ½ in 4 steps, must add 1/16 each time. Therefore the missing terms are: 5/16, 6/16, 7/16. Sum = 18/16 = 9/8, and 9 + 8 = 17.

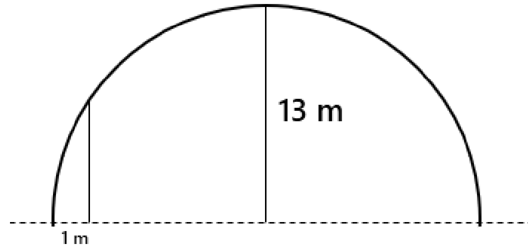
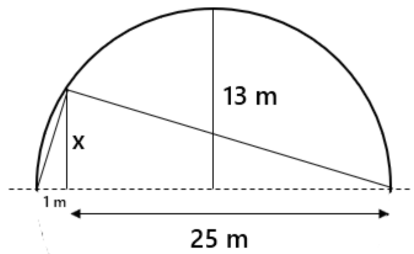
“Math is Cool” Masters -- 2024-25
High School
Individual Test Solutions

<table>
<tr><th></th><th>Answer</th><th>Solution</th></tr>
<tr><td>1</td><td>8 [= x]</td><td>Solve for x: 2(2x – 10) = 12
$2(2x - 10) = 12$
$2x - 10 = 6$
$2x = 16$, $x = 8$</td></tr>
<tr><td>2</td><td>20</td><td>When the following subtraction is performed, what is the sum of the digits in the resulting difference?
1,020,945 – 199,621
1,020,945 – 199,621 = 821,324
8+2+1+3+2+4 = 20</td></tr>
<tr><td>3</td><td>3 [units]</td><td>Two sides of a triangle are twelve and fourteen units. What is the shortest possible integer length for the third side, in units?
Two units is just enough to make 12 + 2 = 14, the third side. By the triangle inequality it has to be longer than that, so 3 units is the minimum integer length.</td></tr>
<tr><td>4</td><td>156 [= 10th term]</td><td>What is the tenth term of the arithmetic sequence that begins as follows?
-15, 4, 23, …
Common difference = +19
$a_n = a_1 + (n - 1)d$
$a_{10} = -15 + 9(19) = 156$</td></tr>
<tr><td>5</td><td>28 [%]</td><td>The following table summarizes Dr. Bartrand’s calculus students at Columbia Basin College by whether or not they have a laptop and a graphing calculator. What is the probability in percent that a randomly selected student does not have a graphing calculator?
<table><tr><th></th><th>Has a laptop</th><th>Does not have a laptop</th></tr><tr><td>Has a graphing calculator</td><td>12</td><td>6</td></tr><tr><td>Does not have a graphing calculator</td><td>5</td><td>2</td></tr></table>There are a total of 25 students. 7 of them do not have a graphing calculator. 7/25 = 28/100 = 28%</td></tr>
<tr><td>6</td><td>1,000,000</td><td>In the final step of a calculation, Hasan incorrectly divided by 1000 instead of multiplying by 1000. What number does he need to multiply his result by to get the correct answer?
He did x/1000 instead of doing x*1000. Now needs to multiply by 1000*1000 = 1,000,000.</td></tr>
</table>

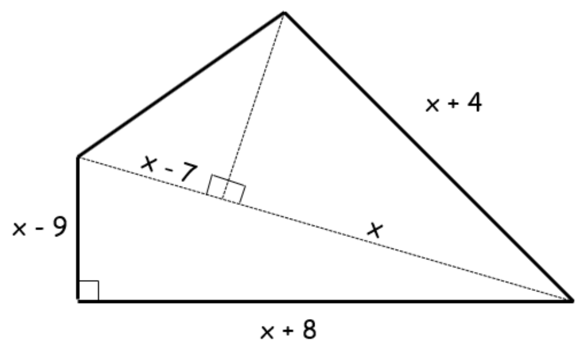
7	36 [= sum]	What is the sum of the distinct prime factors of 2024? $2024 = 2^3 11^1 23^1$ 2+11+23 = 36
8	40 [°]	The figure shown here is a regular polygon. In degrees, what is the smallest possible clockwise rotation around its center that will result in the figure being mapped to itself?  It is a nonagon, with 9 sides. 360/9 = 40.
9	7 [= the average]	If a and b are positive integers, and $(12^a)^b = 12^{13}$, what is the average of a and b? $(12^a)^b = 12^{ab}$ Therefore, ab = 13, so one of them = 1, and the other = 13. The average is (1 + 13)/2 = 7.
10	60 [base 10]	What is the positive difference, in base 10, between the largest three-digit base 5 number and the smallest four-digit base 4 number? 444_5 = 4x25 + 4x5 + 4 = 124 $1000_4 = 1\text{x}4^3 = 64$ 124 – 64 = 60
11	7,562,500	As an integer, what is the value of the following: $(2.75 \times 10^3)^2$ $(2.75 \times 10^3)^2 = 2.75^2 \times 10^6$ = 7.5625 x 1,000,000 = 7,562,500
12	2880 [ways]	At a math competition, a 1st, 2nd and 3rd place trophy will be given to the top three Geometry students and the top three Algebra students. If there are four Geometry students and six Algebra students in the competition, how many different ways could the six trophies be handed out? 4P3 x 6P3 = 24 x 120 = 2880
13	15 [= difference]	What is the positive difference between the range and the median of the set S consisting of the 10 smallest prime numbers? Ten smallest primes are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 Median = 12, range = 27, 27 – 12 = 15
14	432 [°]	In degrees, what is the sum of four of the interior angles of a regular pentagon? Sum of all interior angles = 180(n – 2) = 180(3) = 540. Each angle = 540/5 = 108 Four angles = 108*4 = 432

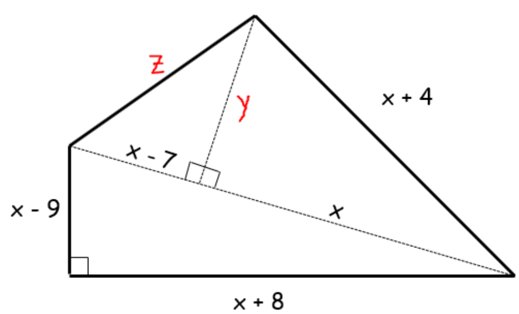
15	6 [= integer values of x]	The solution to the following compound inequality includes how many integer values of x? -4x + 3 < -9 and -4x + 3 > -37 -4x + 3 < -9 -4x + 3 > -37 -4x < -12 -4x > -40 x > 3 and x < 10 The integer solutions are 4, 5, 6, 7, 8, 9
16	57 [= sum]	What is the sum of all positive integers less than 25 that cannot be written as the sum of two (not necessarily distinct) prime numbers? The positive integers less than 25 that cannot be written as the sum of two primes are: 1, 2, 3, 11, 17 and 23. 1+2+3+11+17+23 = 57
17	-3 [= sum of y-coordinates]	Triangle ABC has vertices A (9, -1), B (5, -9) and C (1, -7). After the triangle is reflected over the line y = 2x – 4, to new vertices A', B' and C', what is the sum of the y-coordinates of A', B' and C'? The line we are reflecting over has m = 2, therefore the lines to reflect each point along have m = -1/2. A' (-3, 5), B' (-7, -3), C' (-3, -5). 5 + (-3) + (-5) = -3. 
18	-7 [= f(-3) + f(-6)]	Given the following function, find the value of f(-3) + f(-6). $f(x) = \frac{10}{x+1}$ $f(-3) = \frac{10}{-3+1} = -5$ $f(-6) = \frac{10}{-6+1} = -2$ -5 + (-2) = -7

19	256 [is Ana's integer]	A group of friends divide up the following set of numbers among themselves, and then make the following statements. When Ana's numbers are arranged to make the smallest possible integer, what is her resulting integer? {1, 2, 2, 3, 4, 4, 5, 6, 6} Ana: Each of us has 3 numbers, and each of us has an odd sum. Beto: The product of my 3 numbers is the same as the product of Cesar's 3 numbers. Cesar: The sum of my 3 numbers is 2 more than the sum of Beto's 3 numbers. Ana: 2, 5, 6 Beto: 2, 3, 4 Cesar: 1, 4, 6
20	-9 [= a + b]	The vertex of the parabola described by the following equation is at the point (a, b). What is a + b? $y = 7x^2 + 56x + 107$ x-coordinate of the vertex = -b/(2a) = -56/(2*7) = -4 $y = 7(-4)^2 + 56(-4) + 107$ = 7(16)-224+107 = -5 Vertex = (-4, -5), -4 + (-5) = -9
21	20 [%]	An integer from 10 to 99, inclusive, is chosen at random. As a percentage, what is the probability that the integer contains at least one digit that is a 4? There are 90 total integers to choose from. There are 18 that contain at least one 4: 14, 24, 34, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 54, 64, 74, 84, 94. P = 18/90 = 2/10 = 20%
22	9 [= median]	An arithmetic sequence of integers with n terms has first term a_1 = 11, and nth term a_n = 75. What is the median of all possible values of n? 75 – 11 = 64, therefore the common difference d must be a factor of 64. Factors of 64 are: 1, 2, 4, 8, 16, 32, 64. If d = 64, n = 2. If d = 32, n = 3. If d = 16, n = 5. If d = 8, n = 9. If d = 4, n = 17. If d = 2, n = 33. If d = 1, n = 65. Median = middle number = 9.

23	500 [cm]	A semicircular arch has a height of 13 meters at its center point. In centimeters, what is the height of the arch exactly 1 meter from the edge of the base?  Drawing the two lines as shown creates two similar triangles. The diameter of the semicircle is 13x2 = 26 m, therefore the base of the larger triangle is 256 – 1 = 25 m. Using similarity: $\frac{1}{x} = \frac{x}{25}$ x^2 = 25, x = 5 meters = 500 cm 
24	455 [ways]	A manned space mission to Mars will consist of 4 astronauts chosen from a short-list of 14 available astronauts. Five of the 14 astronauts are scientists. If the mission requires at least 2 scientists, in how many different ways can the crew of 4 be selected? If "at least 2" scientists are required, that could mean 2, or 3, or 4. Calculate for each, and add them together. For each, we are pulling from 2 sub-groups: 5 scientists, and 9 non-scientists. 5C2 x 9C2 = 360 5C3 x 9C1 = 90 5C4 x 9C0 = 5 360 + 90 + 5 = 455
25	11 [= n]	What is the smallest number of terms n in the following infinite series, that when added together gives a sum greater than 3? $$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots$$ Rounding each to 3 decimal places: 1 + 0.5 + 0.333 + 0.25 + 0.2 + 0.167 + 0.143 + 0.125 + 0.111 + 0.1 + 0.091 = 3.02

26	-12 [=a]	The line that contains the points (2, 3) and (-6, 1) also contains the point $\left(a, -\frac{1}{2}\right)$. What is a? $$m = \frac{3-1}{2-(-6)} = \frac{2}{8} = \frac{1}{4}$$ $$y - 3 = \frac{1}{4}(x-2)$$ $$y = \frac{1}{4}x + \frac{5}{2}$$ y $= -\frac{1}{2}$: $-\frac{1}{2} = \frac{1}{4}x + \frac{5}{2}$ $$-3 = \frac{1}{4}x$$ x = -12
27	185	What is the smallest positive integer multiple of 37 that leaves a remainder of 3 when divided by 13? We want to find n such that 37n ≡ 3(mod 13). But 37 is congruent to 11 (mod 13). Therefore, have 11n ≡ 3(mod 13). This works for n = 5, because 55 = 3(mod 13). Therefore we want the 5th multiple of 37: 37x5 = 185.

<table>
<tr>
<td>28</td>
<td>66 [units]</td>
<td>In the following quadrilateral, some of the lengths are given (in units) in terms of x, and right angles are as indicated. In units, what is the perimeter of the quadrilateral?


Start by solving for x using the lower triangle:
$(x-9)^2 + (x+8)^2 = (x-7+x)^2$
Simplifies to: $x^2 - 13x - 48 = 0$
$(x-16)(x+3) = 0$
Therefore, x = 16.
On the upper right triangle:
$y^2 + 16^2 = 20^2$
y = 12
On the upper left triangle:
$z^2 = 9^2 + 12^2$
z = 15
Perimeter = 7 + 15 + 20 + 24 = 66

</td>
</tr>
<tr>
<td>29</td>
<td>5 [sets]</td>
<td>How many distinct sets of three positive integers have a mean of 8, a median of 9, and a range that is at least 4?
The sum of the three integers equals 24, and the middle number is 9.
List:
6 9 9 (doesn't work, range = 3)
5 9 10
4 9 11
3 9 12
2 9 13
1 9 14
Five sets work.</td>
</tr>
</table>

<table>
<tr><td>30</td><td>10 [= k]</td><td>For what real value of k is the following polynomial divisible by the binomial x + 1?

$$P(x) = x^{100} + kx + 9$$
If the synthetic division is set up, it looks like this:

$$\begin{array}{l|cccccccc} \text{term:} & x^{100} & x^{99} & x^{98} & x^{97} & \dots & x^2 & x^1 & x^0 \\ & 1 & 0 & 0 & 0 & \dots & 0 & k & 9 \\ \hline -1 & 1 & -1 & 1 & -1 & \dots & 1 & k-1 & -k+1+9 \end{array}$$

The last value needs to equal 0, or:
$-k + 1 + 9 = 0$
$k = 10$</td></tr>
<tr><td>31</td><td>695 [$]</td><td>A math competition (not this one!) has 16 schools competing, and a total of $8000 to award in prize money. The 16th place school will receive $275 in prize money, and the award increases by the same amount for each successive finishing place. In dollars, how much will the 2nd place school receive?
$a1 = 275$
$S_{16} = 8000 = n/2(a1 + a_{16}) = 8(275 + a_{16})$
Solve for $a_{16} = 725$, the amount that the 1st place team gets.
Therefore, $d = (725 – 275)/15 = 30$
Therefore, 2nd place award = 725 – 30 = 695</td></tr>
<tr><td>32</td><td>6 [triples]</td><td>How many integer triples (x, y, z) satisfy the following properties:
1. x, y and z are positive integers less than 30, and
2. $xy^2z^3 = 10000$
$10000 = 2^4 5^4$
Need x, y, z < 30, such that $xy^2z^3 = 10000$
$$\begin{array}{lll} x & y^2 & z^3 \\ \hline 10 & 1^2 & 10^3 \\ 20 & 2^2 & 5^3 \\ 5 & 4^2 & 5^3 \\ 2 & 25^2 & 2^3 \\ 16 & 25^2 & 1^3 \\ 25 & 20^2 & 1^3 \end{array}$$</td></tr>
<tr><td>33</td><td>420 [points]</td><td>Brent got a score of 775 on a standardized test that had a mean of 600 and a standard deviation of 100. A second test has a mean of 350 and a standard deviation of 40. What score (in points) does Brent need on the second test to do equivalently well as he did on the first test? All units are in points, and assume that the scores on both tests are normally distributed.
First test: $z = \frac{775-600}{100} = 1.75$
Second test: $1.75 = \frac{x-350}{40}$
Solve for x = 420 points to have the same z-score as the first test.</td></tr>
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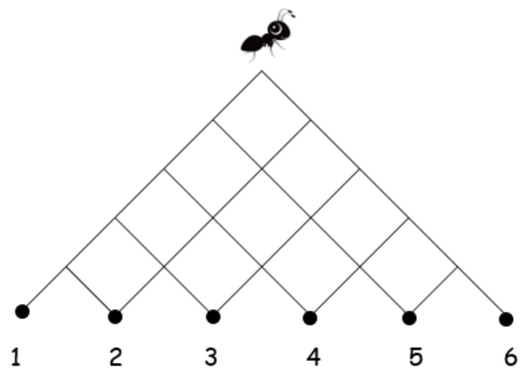
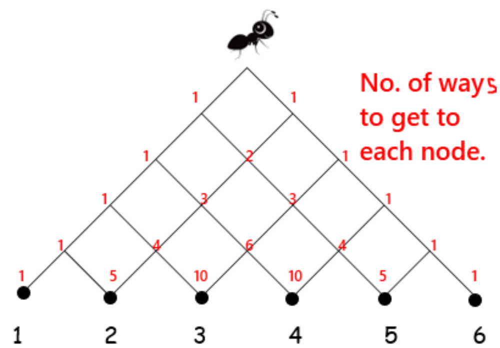
<table>
<tr>
<td>34</td>
<td>13 [= A + B]</td>
<td>An ant is located at the top of a triangular grid, and will walk down along the lines to one of the points labeled 1 through 6. At each intersection, starting at the very top, the ant will randomly decide with equal probability whether to turn left or right, but always moving downward. The probability that the ant ends up at point 3 or 4 can be written as a reduced common fraction A/B. What is A + B?

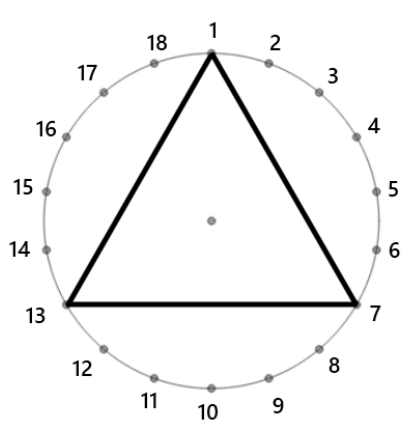


The grid can be labeled with the number of ways to get to each point. There are a total of $2^5 = 32$ different paths. Ten of them land at C, and ten of them land at D. 10/32 = 5/8, 5 + 8 = 13.



</td>
</tr>
<tr>
<td>35</td>
<td>810 [combinations]</td>
<td>Eighteen dots are evenly spaced around the circumference of a circle. How many combinations of three dots can be selected from the 18 that do not form an equilateral triangle?

There are 18C3 = 816 total ways to choose 3 dots and form a triangle. In order to form an equilateral triangle, the three vertices need to be equally spaced around the circle. 18/3 = 6, therefore from any given vertex, move forward 6 points for the next vertex, then forward another 6 points for the third vertex. There are only 6 different triangles that can be formed in this manner, starting at points 1, 2, 3, 4, 5 or 6. Therefore, 816 – 6 = 810 triangles that are not equilateral.



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</table>

36	3	A cube contains an inscribed sphere, and also has a sphere circumscribed about it. As an integer, what is the ratio of the surface area of the circumscribed sphere to the surface area of the inscribed sphere? Assume that the side length of the cube = 1. Therefore, the radius of the inscribed sphere = $\frac{1}{2}$. The space diagonal of the cube = $\sqrt{3}$, therefore the radius of the circumscribed sphere = $\frac{\sqrt{3}}{2}$. SA = $4\pi r^2$. Therefore the ratio will be $\frac{\left(\frac{\sqrt{3}}{2}\right)^2}{\left(\frac{1}{2}\right)^2} = 3$
37	12 [= xy]	Given the following two equations, where x and y are real numbers, what is the value of xy? $$x^2 + xy = 20$$ $$y^2 + xy = 30$$ Add the two equations: $x^2 + 2xy + y^2 = 50$, $(x + y)^2 = 50$ Subtract the two equations: $x^2 - y^2 = -10$ $(x + y)(x - y) = -10$, square both sides $(x + y)^2(x - y)^2 = 100$ $(50)(x - y)^2 = 100$ $(x - y)^2 = 2$ Also, know that $(x + y)^2 - (x - y)^2 = 4xy$, after multiplying out and simplifying. Therefore, $4xy = 50 - 2 = 48$, $xy = 48/4 = 12$

38

258

The digits 1 through 9 are to be placed in a 3 by 3 grid, with each digit being used exactly once. Three sets of clues are given as follows. As a 3-digit integer, what number appears in the middle row?

5	6	
	4	7
2		

Four numbers are in the correct columns but are in the incorrect squares. No numbers are in the correct rows.

		4
3	5	

Two numbers are in the correct columns but are in the incorrect squares. One number is in the correct row, but in the incorrect square.

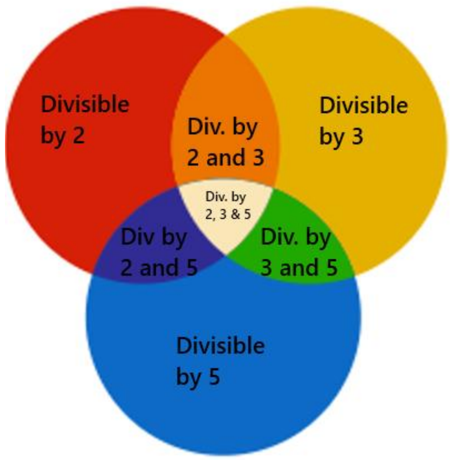
8	2	5
7	1	4

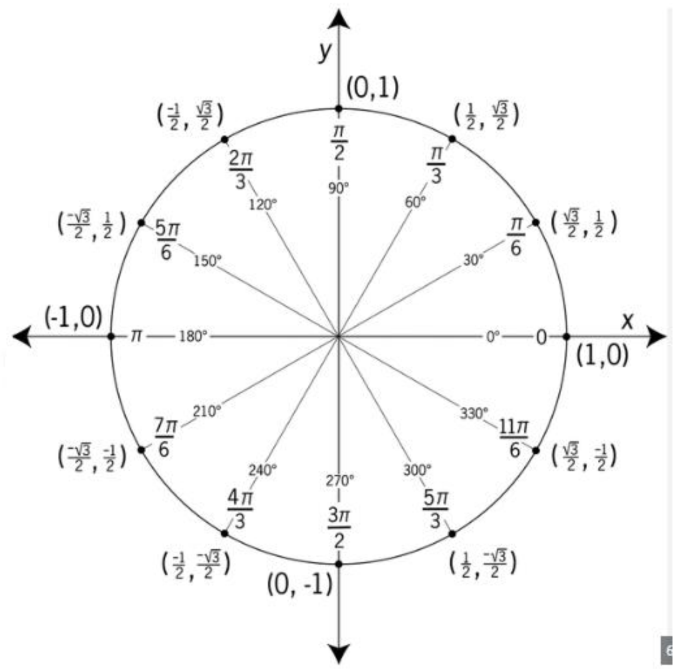
No numbers are in the correct columns. Five numbers are in the correct row but are in the incorrect squares.

?	?	?

From Clue 1: “No numbers are in the correct rows”. Cross out all occurrences of those numbers in each row. (red)
From Clue 3: “No numbers are in the correct columns”. Cross out all occurrences of those numbers in each column. (blue)
From Clue 2: “Two numbers are in the correct columns”. It’s not 4 (from Clue 3), therefore 3 and 5 are in the correct columns, 1^{st} and 2^{nd} columns respectively. Cross out all occurrences in other columns. (lime) Also, 3 and 5 are not in the correct squares. Also, 4 is in the correct row.
From Clue 3: Therefore, all numbers except for the 4 are in the correct row, so cross out occurrences in the other rows. (aqua)
At this point, enough eliminations have been made to cross out enough numbers and solve for the middle row, which is 258. (black)

~~1~~	~~2~~	3	~~1~~	~~2~~	~~3~~	~~1~~	~~2~~	~~3~~
~~4~~	~~5~~	~~6~~	4	~~5~~	~~6~~	~~4~~	~~5~~	~~6~~
~~7~~	~~8~~	~~9~~	~~7~~	~~8~~	~~9~~	~~7~~	~~8~~	9
~~1~~	2	~~3~~	~~1~~	~~2~~	~~3~~	~~1~~	~~2~~	~~3~~
~~4~~	~~5~~	~~6~~	~~4~~	5	~~6~~	~~4~~	~~5~~	~~6~~
~~7~~	~~8~~	~~9~~	~~7~~	~~8~~	~~9~~	~~7~~	8	~~9~~
1	~~2~~	~~3~~	~~1~~	~~2~~	~~3~~	1	~~2~~	~~3~~
~~4~~	~~5~~	6	~~4~~	~~5~~	6	~~4~~	~~5~~	6
~~7~~	~~8~~	~~9~~	7	~~8~~	~~9~~	7	~~8~~	~~9~~

39	265 [integers]	How many integers are there from 1 to 1000 inclusive whose smallest prime factor is at least 7? If their smallest prime factor is at least 7, then they do not have prime factors of 2, 3 or 5. Add up the numbers that are divisible by 2, 3 or 5: 1000/2 + 1000/3 + 1000/5 = 500 + 333 + 200 = 1033 Subtract off the "overcount", which is the numbers divisible by 6, 10 and 15: 1000/6 + 1000/10 + 1000/15 = 166 + 100 + 66 1033 – (166 + 100 + 66) = 701 But, have to add back in the numbers that are divisible by 30: 1000/30 = 33 701 + 33 = 734. Therefore, subtract this total from 1000, plus an additional 1 for the number 1: 1000 – 734 – 1 = 265. 
40	30	Julia has a bag of numbers. The bag contains one set of integers from 1 through 9 inclusive, plus some extra 5s and some extra 8s. The mean of all of the numbers in the bag is 6.4. What is the smallest possible number of numbers in the bag? **m = number of extra 5s** **n = number of extra 8s** **Total = 9 + m + n numbers** **Sum = 45 + 5m + 8n** $$mean = \frac{45+5m+8n}{9+m+n} = 6.4$$ $$\frac{450+50m+80n}{9+m+n} = 64$$ **450+50m+80n=64(9+m+n)** **Simplify to:** $n = \frac{7(9+m)}{8}$ **(9 + m) must be a multiple of 8, and the smallest possible value is 16.** **Therefore, m = 16 – 9 = 7, and therefore n = 14.** **Total (smallest) number of numbers = 9 + 7 + 14 = 30.**

41	300 [°]	In degrees, what is the solution for 'x' in the following equation, on the interval given in radians [π, 2π]? cos x = 0.5 Using the unit circle trigonometry, cos x = ½ when x = 300°, where x is between 180 and 360°. 
42	6 [= sum of zeros]	Find the sum of all zeros of the following function. $$P(x) = x^4 - 6x^3 + 14x^2 - 14x + 5$$ Possible rational zeros are ±1, ±5. Using synthetic division, 1 is a zero: P(x) = (x – 1)(x^3 – 5x^2 + 9x – 5) Possible rational zeros are ±1, ±5. Using synthetic division, 1 is a zero (multiplicity 2): Q(x) = (x – 1)(x^2 – 4x + 5) Use quadratic formula to find zeros of x^2 – 4x + 5: 2 + i, 2 – i Sum of zeros = 1 + 1 + 2 + i + 2 – i = 6
43	32	Given the following functions, find: $(f^{-1} \circ g^{-1})(1)$ $$f(x) = \frac{1}{8}x - 3 \qquad g(x) = x^3$$ $$f^{-1}(x) = 8(x+3)$$ $$g^{-1}(x) = \sqrt[3]{x}$$ $$g^{-1}(1) = \sqrt[3]{1} = 1$$ $$f^{-1}(g^{-1}(1)) = 8(1+3) = 32$$

44	4 [= m + b]	The equation of the tangent line to the graph of the following function at the point (0, 1) can be written as: y = mx + b. What is m + b? $y = e^{3x}$ $$y = e^{3x}$$ $$y' = 3e^{3x}$$ $$y'(0) = 3e^{3(0)} = 3$$ Tangent line: y – 1 = 3(x – 0) y = 3x + 1 3 + 1 = 4
45	2 [= x]	Solve for the value of x that makes the following equation true: $\int_1^x \frac{3}{t}\,dt = \int_{1/4}^x \frac{1}{t}\,dt$ $$\int_1^x \frac{3}{t}\,dt = \int_{1/4}^x \frac{1}{t}\,dt$$ $$3 \cdot \ln t \Big\|_1^x = \ln t \Big\|_{1/4}^x$$ 3*(ln x – ln 1) = ln x – ln (1/4) 3 ln x = ln x – ln (1/4) 2 ln x = - ln(1/4) ln x = (-1/2)ln(1/4) = (-1/2)ln(4^{-1}) = (1/2)ln(2^2) = ln 2 ln x = ln 2, therefore x = 2

"Math is Cool" Masters -- 2024-25

High School

Multiple Choice Solutions

9/10th	11/12th	Answer	Solution
1		B	Given: $8^a \cdot 8^b = \frac{8^c}{8^d}$ What is an expression for d, in terms of a, b, and c? A) $\frac{c}{ab}$ B) $c-a-b$ C) $a+b-c$ D) $c-ab$ E) None of the above. $8^a \cdot 8^b = \frac{8^c}{8^d}$ $8^{a+b} = 8^{c-d}$ a+b = c – d d = c – a – b
	1	A	Simplify: $2\sqrt[3]{8x^5} \div \sqrt[4]{16x^8}$ A) $\frac{2}{\sqrt[3]{x}}$ B) $\frac{1}{2x}$ C) $2x$ D) $\sqrt[3]{x}$ E) None of the above. $2\sqrt[3]{8x^5} \div \sqrt[4]{16x^8}$ $= \frac{2 \cdot 2 \cdot x\sqrt[3]{x^2}}{x} = \frac{2}{\sqrt[3]{x}}$
2		C	If x is a real number, and 0 < x < 1, then which of the following orderings is correct? A) $x < x^2 < x^3 < x^4$ B) $x^3 < x < x^2 < x^4$ C) $x^4 < x^3 < x^2 < x$ D) $x < x^3 < x^4 < x^2$ E) None of the above. For example, if x = ½: 1/16 < 1/8 < ¼ < ½

	2	D	If x is a real number, and -1 < x < 0, then which of the following orderings is correct? A) $x < x^2 < x^3 < x^4$ B) $x^3 < x < x^2 < x^4$ C) $x^4 < x^3 < x^2 < x$ D) $x < x^3 < x^4 < x^2$ E) None of the above. For example, if x = $-\frac{1}{2}$: -1/2 < -1/8 < 1/16 < 1/4
3		A	Let S equal the set of all positive numbers n such that 1 < n < 100, and $\sqrt{n}$ is an integer. What is the median of the members of set S? A) 30.5 B) 35 C) 35.5 D) 40.5 E) None of the above. S = {4, 9, 16, 25, 36, 49, 64, 81} The median will be the average of the two center elements. (25 + 36)/2 = 30.5
	3	C	Let S equal the set of all positive numbers n such that 1 < n < 100, and $\sqrt{n}$ is an integer. What is the mean of the members of set S? A) 30.5 B) 35 C) 35.5 D) 40.5 E) None of the above. S = {4, 9, 16, 25, 36, 49, 64, 81} Mean = 284/8 = 35.5
4		C	The point (-6, 11) is rotated 630° clockwise about the origin. What are the coordinates of the new point after the rotation? A) (11, -6) B) (6, -11) C) (-11, -6) D) (-6, -11) E) None of the above. 630 – 360 = 270° rotation, which will take it to (-11, -6) in Q3.
	4	C	What is the value of: $sin(-30°) + cos(-30°)$ A) $\frac{1}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $\frac{\sqrt{3}-1}{2}$ D) $\frac{-\sqrt{3}+1}{2}$ E) None of the above. sin 30 = $\frac{1}{2}$ cos 30 = $\frac{\sqrt{3}}{2}$ All students take calculus! Cos is + in Q4, sin is -. Therefore, $sin(-30°) + cos(-30°)$ = $-\frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{3}-1}{2}$

<table>
<tr>
<td>5</td>
<td></td>
<td>B</td>
<td>Biff and Eho decide to make up their own base 6 system of numeration. The table shows how to translate their base 6 characters into our base 10 digits. Convert the number 472 (base 10) to its Biff & Eho equivalent.

<table>
<tr><td>Biff & Eho</td><td>%</td><td>@</td><td>$</td><td>?</td><td>#</td><td>&</td></tr>
<tr><td>Base 10</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
</table>
A) $ % # & B) $ @ % # C) @ % @
D) @ # ? E) None of the above.
Base 6 will have place values:
6^3 6^2 6^1 6^0, or
216 36 6 1
Therefore, 472 will convert to: 2104, which is: $ @ % #</td>
</tr>
<tr>
<td></td>
<td>5</td>
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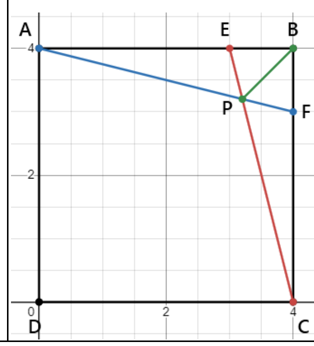
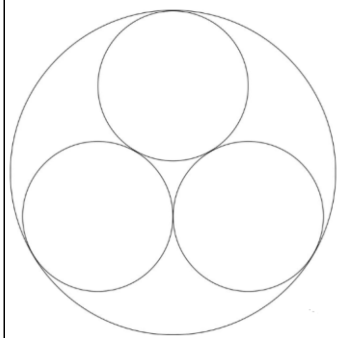
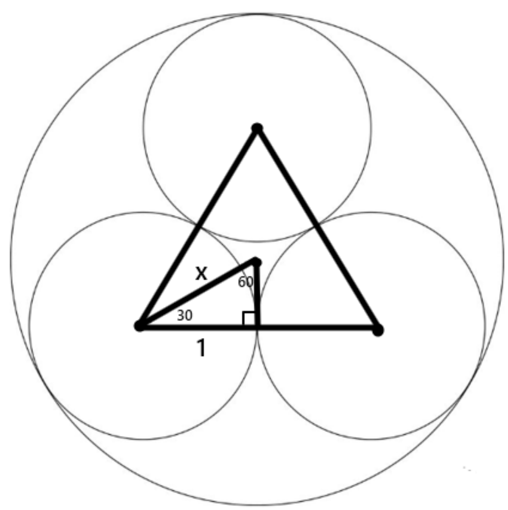
6^3 6^2 6^1 6^0, or
216 36 6 1
Therefore, 472 will convert to: 2104, which is: $ @ % #</td>
</tr>
</table>

6		*C*	Let ABCD be a square with side length 4 units, with vertex D located at the origin on the coordinate plane. Let point E lie on AB and point F lie on BC, so that BE = BF = 1 unit. Let AF and CE meet at point P. What is the length of BP? A) $\sqrt{2}$ B) $\frac{2}{5}\sqrt{2}$ C) $\frac{4}{5}\sqrt{2}$ D) $\frac{4}{5}$ E) Answer not given. Point P lies on the line y = x, and also on the line containing CE, which is y = -4x + 16. -4x + 16 = x, 5x = 16, x = 16/5, so P is (16/5, 16/5). Use the distance formula to find BP: $d = \sqrt{\left(4-\frac{16}{5}\right)^2+\left(4-\frac{16}{5}\right)^2}$ = $\frac{4}{5}\sqrt{2}$ 

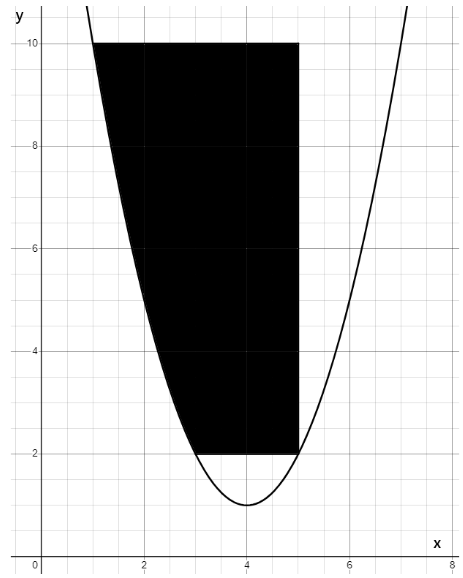
<table>
<tr><td></td><td>6</td><td>A</td><td>Three smaller congruent circles are externally tangent to each other, and internally tangent to the larger circle. The smaller circles each have a radius of one unit. What is the radius of the larger circle?

A) $\frac{3+2\sqrt{3}}{3}$ B) $\frac{2}{3}\sqrt{3}$ C) $2\sqrt{3}$ D) $\sqrt{3}+1$ E) Answer not given.
Connecting the centers of the three circles, and then dropping a perpendicular, forms a 30-60-90 triangle with one leg = 1. Therefore the other leg will equal $\frac{1}{\sqrt{3}}$ and the hypotenuse = $\frac{2}{\sqrt{3}}$. Thus the radius of the large circle = $1+\frac{2}{\sqrt{3}}=\frac{3+2\sqrt{3}}{3}$

</td></tr>
<tr><td>7</td><td></td><td>C</td><td>Sahil and Mir play a series of three games. In each game, Sahil's probability of winning is 2/3. What is Sahil's probability of winning at least 2 of the 3 games?

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{20}{27}$ D) $\frac{3}{4}$ E) None of the above.
There are 4 ways for Sahil to win either 2 or 3 games: SSS, SSM, SMS, MSS. P = (2/3)(2/3)(2/3) + (2/3)(2/3)(1/3) + (2/3)(1/3)(2/3) + (1/3)(2/3)(2/3) = 8/27 + 4/27 + 4/27 + 4/27 = 20/27</td></tr>
</table>

	7	B	An infinite sequence of numbers begins as follows: 1, 3, 9, 27, 81, 242, … Gregg takes the log base x of each number and ends up with an arithmetic sequence that has a common difference of 2. What is the value of x? A) $\sqrt{2}$ B) $\sqrt{3}$ C) 2 D) 3 E) None of the above. The original sequence is: $3^0, 3^1, 3^2, \ldots$ Taking the log base x results in: $\log_x 3^0, \log_x 3^1, \ldots$, or $0 \cdot \log_x 3, 1 \cdot \log_x 3, \ldots$ This simplifies to $0, \log_x 3, \ldots$ Therefore, $\log_x 3 = 2$, $3 = x^2$, $x = \sqrt{3}$
8		B	A "Fibonacci-type" sequence begins as follows: 1.5, a, b, 5.9, c, 15.5, d, … where the first two terms are given, and every term from the third term on is the sum of the previous two terms. What is a + b + c + d? A) 25.1 B) 40.6 C) 50.1 D) 63 E) None of the above. 1.5 + a = b a + b = 5.9 Solve for a = 2.2, and b = 3.7. b + 5.9 = c, c = 9.6. c + 15.5 = d, d = 25.1. 2.2 + 3.7 + 9.6 + 25.1 = 40.6.

<table>
<tr>
<td></td>
<td>8</td>
<td>A</td>
<td>

Find the area of the shaded region in the graph:



A) $25\frac{1}{3}\ units^2$

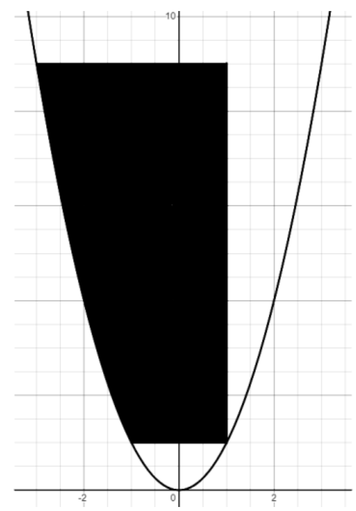
B) $26\frac{2}{3}\ units^2$

C) $28\ units^2$

D) $34\frac{2}{3}\ units^2$

E) None of the above.

Notice that the curve is just y = x^2, translated over 4 units and up 1 unit. So we can simplify it by just shifting the curve back:



In the translated curve, the area is between y = 9 and y = x^2, from x = -3 to x = 1, minus the area between y = 1 and y = x^2, from x = -1 to x = 1.

Set up the integration:

$$= \int_{-3}^{1} (9 - x^2)dx \ - \int_{-1}^{1} (1 - x^2)dx$$

$$= \left(9x - \frac{1}{3}x^3\right)\Big|\begin{matrix}1\\-3\end{matrix} - \left(x - \frac{1}{3}x^3\right)\Big|\begin{matrix}1\\-1\end{matrix}$$

= 9(1 - -3) – (1/3)(1 - -27) – [(1 - -1) – (1/3)(1 - -1)]

= 80/3 – 4/3 = 76/3 = 25 1/3

</td>
</tr>
</table>

9	**9**	D	Ana, Bryson and Catalina are each given a positive integer. They do not know each other's integers, but they know that the sum of their three integers is 14. Ana says: Bryson and Catalina have different numbers. Bryson then says: I already knew that we each have different numbers. Catalina then says: Now I know what all three numbers are. What integer does Catalina have? A) 1 B) 2 C) 4 D) 6 E) None of the above. The sum is 14, which is even. So it is either e+e+e, or o+o+e. If A knows that B and C have different numbers, then A must have an odd number. If B already knew that they had 3 different numbers, then B must have 7, 9 or 11, because any other number would allow 2 of the numbers to be the same. The remaining possibilities are listed as follows. The only one where Catalina can be confident of knowing the three numbers is when Catalina has the number 6.

B	A	C
7	1	6
7	3	4
7	5	2
9	1	4
9	3	2
11	1	2

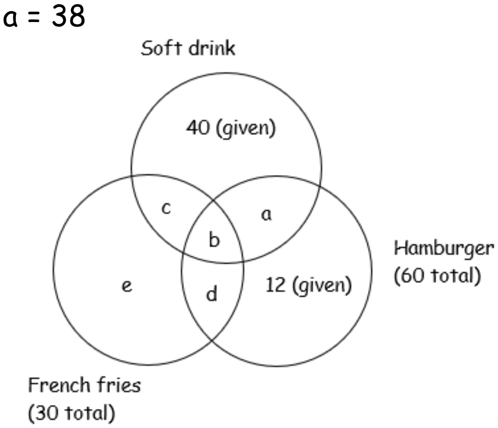
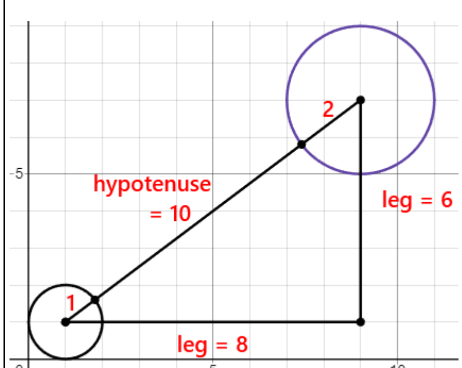
10	**10**	*C*	Two numbers, a and b, are chosen (with replacement) from the set of integers from 1 to 100, inclusive. What is the probability that the value of $3^a + 7^b$ has a units digit of 8? A) $\frac{1}{16}$ B) $\frac{1}{8}$ C) $\frac{3}{16}$ D) $\frac{3}{8}$ E) None of the above. The powers of 3 and 7 both repeat in a cycle of 4, as far as their units digits: $3^1 = 3$ $3^2 = 9$ $3^3 = 27$ $3^4 = 81$, then repeat $7^1 = 7$ $7^2 = 49$ $7^3 = 343$ $7^4 = 2401$, then repeat Therefore, out of the possible values for a, 25 each will result in 3^a ending in 3, 9, 7 or 1. Out of the possible values for b, 25 each will result in 7^b ending in 7, 9, 3 or 1. Therefore, there are 4x4 = 16 possible pairs of units digits for the addition. The pairs that will give a resulting units digit of 8 are (1, 7), (7, 1) and (9, 9), so P = 3/16.

"Math is Cool" Masters -- 2024-25

High School

Team Test Solutions

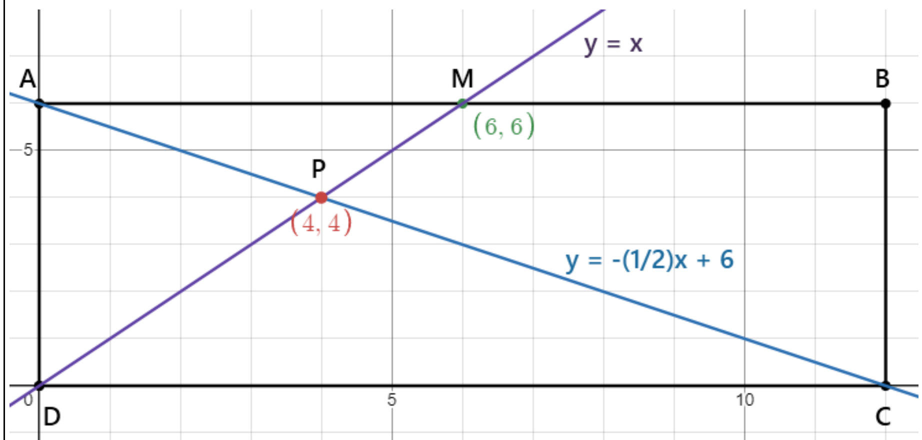
9/10	11/12	Answer	Solution
1	**1**	12 [minutes]	Biff and Eho live 1.08 miles away from each other. From their respective homes, they walk towards each other at a constant rate, with Biff walking at 2.5 miles per hour and Eho walking at 2.9 miles per hour. How many minutes will they each walk before meeting? Walking towards each other, their closing speed is 2.5 + 2.9 = 5.4 miles per hour. T = D/R = 1.08 miles/5.4 miles per hour = 0.2 hours x 60 minutes/hour = 12 minutes.
2		201	What is the smallest positive difference between the squares of two distinct positive three-digit integers? $101^2 - 100^2 = 201$
	2	2001	What is the smallest positive difference between the squares of two distinct positive four-digit integers? $1001^2 - 1000^2 = 2001$

3	**3**	38 [customers]	The Hot Mess Burgers food truck sells only hamburgers, french fries and soft drinks. One day, exactly 120 customers bought something at Hot Mess. Half of the customers bought at least a hamburger, one-fourth of the customers bought at least french fries, and one-third of the customers bought only a soft drink. Of the customers who bought a hamburger, four-fifths of them bought at least one other item. How many customers bought a hamburger and soft drink, but not french fries? One-third bought only a soft drink: (120)(1/3) = 40 Half bought at least a hamburger: (120)(1/2) = 60 Of those, 4/5 bought at least another item: (60)(4/5) = 48, therefore 12 bought only a hamburger. Assign variables to the other unknowns as shown. a + b + c + d + e = 120 – 40 – 12 = 68 b + c + d + e = 30 Subtract the second equation: a = 38 
4		7 [units]	What is the shortest distance, in units, between circle P and circle Q, where the circles are defined as follows: P: $(x-9)^2+(y-7)^2=4$ Q: $(x-1)^2+(y-1)^2=1$ The shortest distance will be a line between the two centers. A 6-8-10 right triangle is formed, so the distance is 10. Subtracting off the radii, the shortest distance is 7. 

<table>
<tr><td></td><td>4</td><td>35</td><td>Let f(0) = 5, and f(n) = f(n – 1) + 2.
What is the value of P, where P is defined as follows?
$$P = f^{-1}\left(f\left(f\left(f(5)\right)\right)\right)$$
<u>n</u> <u>f(n)</u>
0 5
1 7
… …
It is describing a linear function which can be written as: y = 2x + 5.
Therefore, f(5) = 15
f(15) = 35
f(35) = 75
$f^{-1}(75) = 35$</td></tr>
<tr><td>5</td><td>5</td><td>7458</td><td>A secret code consists of four digits in a row, where the digits are from 0 to 9 inclusive, and no two digits are the same. Given the following clues, what is the correct 4-digit number?
<table><tr><td>2</td><td>3</td><td>4</td><td>5</td><td>Two digits are correct but are in the wrong positions.</td></tr><tr><td>4</td><td>5</td><td>6</td><td>7</td><td>Three digits are correct but are in the wrong positions.</td></tr><tr><td>6</td><td>9</td><td>0</td><td>1</td><td>Nothing is correct.</td></tr><tr><td>3</td><td>4</td><td>1</td><td>5</td><td>Two digits are correct, and one is in the correct position.</td></tr><tr><td>7</td><td>3</td><td>5</td><td>2</td><td>Two digits are correct, and both are in the correct positions.</td></tr></table>Start with a list of the digits 0 – 9, and use process of elimination. From the third clue, 6, 9, 0 and 1 can be crossed off. From the second clue, therefore, 4, 5 and 7 must be correct. Then from the first clue, can eliminate 2 and 3, since 4 and 5 are correct, which leaves 8 as the 4th digit. Use the 5th and 4th clues to get the correct positions.</td></tr>
<tr><td>6</td><td></td><td>415 [ping-pong balls]</td><td>Ping-pong balls are numbered as follows. One ping-pong ball is numbered ‘1’, two ping-pong balls are numbered ‘2’, and so one, through 50 ping-pong balls being numbered ‘50’. All of the balls are put into a box. The balls are then drawn at random from the box, without replacement. What is the minimum number of ping-pong balls that must be drawn from the box to ensure that at least 10 of them are labeled with the same number?

Worst case scenario: the balls numbered 1 – 9 can all be drawn, since there is no possibility of getting 10 of them. The sum of 1 through 9 = 45. After that, 9 of each of the other numbers can be drawn. There are 50 – 9 = 41 numbers left, times 9 = 369. So 369 + 45 = 414. The next ball will be the 10th of some number, so the total required is 415.</td></tr>
</table>

	6	252 [3-digit positive integers]	How many three-digit positive integers contain at least one 4? Using casework: Three 4's: 1 way, 444 Two 4's: 44_, 9 ways 4_4, 9 ways _44, 8 ways One 4: 4_ _, 9x9 = 81 ways _4_, 8x9 = 72 ways _ _ 4, 8x9 = 72 ways 1+9+9+8+81+72+72 = 252
7	7	19 [= a+b+c+d+e]	A new function is defined as follows, where the inputs a, b, c, d and e are each positive integers. If ★a, b, c, d, e★ = $\frac{44}{389}$, what is the value of a + b + c + d + e ? $\bigstar a, b, c, d, e\bigstar = \frac{1}{a+\frac{1}{b+\frac{1}{c+\frac{1}{d+\frac{1}{e}}}}}$ $\frac{44}{389} = \frac{1}{a+\frac{1}{b+\frac{1}{c+\frac{1}{d+\frac{1}{e}}}}}$ Therefore, $\frac{389}{44} = 8\frac{37}{44} = a + \frac{1}{b+\frac{1}{c+\frac{1}{d+\frac{1}{e}}}}$ a = 8, and: $\frac{44}{37} = 1 + \frac{7}{37} = b + \frac{1}{c+\frac{1}{d+\frac{1}{e}}}$ b = 1, and: $\frac{37}{7} = 5 + \frac{2}{7} = c + \frac{1}{d+\frac{1}{e}}$ c = 5, and: $\frac{7}{2} = 3 + \frac{1}{2} = d + \frac{1}{e}$ d = 3, e = 2 a + b + c + d + e = 8 + 1 + 5 + 3 + 2 = 19

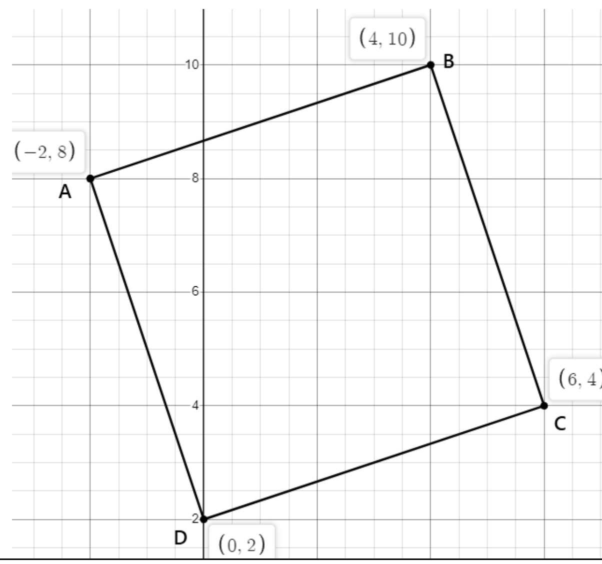
8	**8**	20 [= x^2]	Let ABCD be a rectangle, with AB = 12 units and BC = 6 units. Let M be the midpoint of AB, and let P be the intersection of MD and AC. If x is the length of AP, what is the value of x^2? Position the rectangle on the coordinate plane such that D is at the origin. MD is on the line y = x, and AC is on the line y = -1/2 x + 6. Set them equal and solve to find they intersect at P (4, 4). Find the distance from A to P: $d = \sqrt{(4-0)^2 + (4-6)^2} = \sqrt{20} = x$ Therefore, x^2 = 20. 
9	**9**	60 [%]	A bag contains five marbles, three are blue and two are green. Marbles are randomly removed one at a time without replacement until either all of the blue marbles are removed or all of the green marbles are removed. As a percentage, what is the probability that the last marble removed is green? Consider drawing all 5 marbles out of the bag. There are 5!/(3!2!) = 10 total ways to do so. Out of these 10 ways, there are 6 that will result in having 2 green marbles removed before 3 blue marbles are removed: GG GBG BGG GBBG BGBG BBGG 6/10 = 60%

10	**10**	85 [integers]	How many distinct three-digit positive integers can be written as a sum of a three-digit positive integer and its reversal (containing one, two, or three digits)? Comment: any reversals that result in leading zeros can just ignore the zeros. For example, the reversal of 100 is 001 = 1. ABC = DEF + FED E + E = 2E = B mod 10 There are 5 options for B: 0, 2, 4, 6, 8. C = D + F, but D cannot equal 0. C can be from 1 to 9. 2 cases: 2E < 10, so A = C, or 2E > 8, so A = C + 1. Case 1: A has 9 choices, 1 to 9 Case 2: A has 8 choices, 2 to 9, since the min(C + 1) = 2. 9*5 = 45 numbers for Case 1. 8*5 = 40 numbers for Case 2. 45 + 40 = 85 total

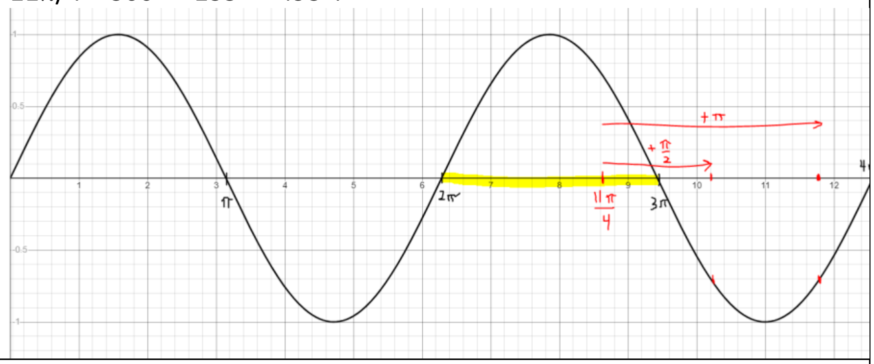
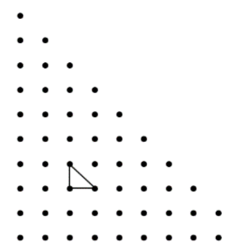
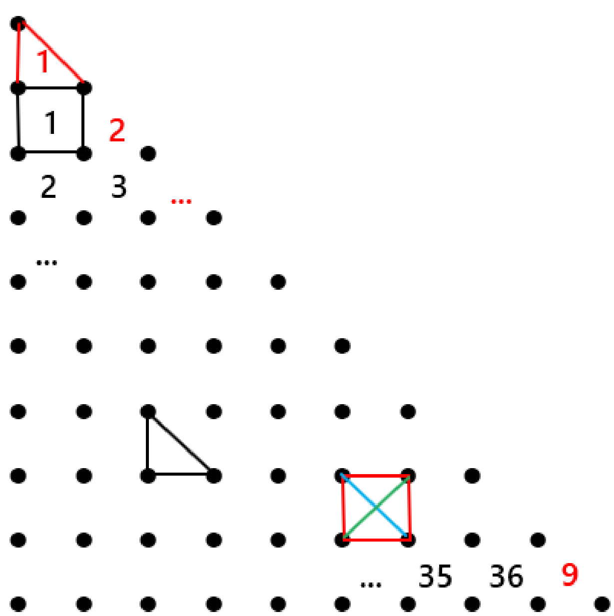
"Math is Cool" Masters -- 2024-25

High School

Pressure Round Solutions

9/10th	11/12th	Answer	Solution
1	1	16 [= $x_1 + x_2 + y_1 + y_2$]	Square ABCD has vertices A (x_1, y_1), B (4, 10), C (x_2, y_2) and D (0, 2). What is the value of $x_1 + y_1 + x_2 + y_2$? The slope of the line between B and D (diagonal) = 2, therefore the slope between A and C will equal -1/2. Find the midpoint of BD = (4/2, 12/2) = (2, 6), which is the center of the square. From the center, use the slope of -1/2 to count over to point A (-2, 8) and C (6, 4). 
2		9 [= abc]	A parabola defined by $y = ax^2 + bx + c$ has its vertex at the point (6, 15), and contains the point (0, -3). What is the product abc? Knowing (0, -3) is on the parabola, solve for c = -3. x = 6: 15 = 36a + 6b – 3 x = 6: 6 = -b/2a, 12a + b = 0 Solve the two equations for a = -1/2, b = 6. abc = (-1/2)(6)(-3) = 9

	2	31 [= A + B]	In the following equation, the value of x can be written as a reduced common fraction A/B. What is the value of A + B? $\sqrt[x]{4\sqrt{2\sqrt{2}}} = 32$ $4\sqrt{2\sqrt{2}} = 32^x$ $16 \cdot 2\sqrt{2} = 32^{2x}$ $16 \cdot 2\sqrt{2} = 32^{2x} = 2^{10x}$ $\sqrt{2} = 2^{10x-5}$ $2^1 = 2^{2(10x-5)}$ 1 = 20x – 10, 20x = 11, x = 11/20
3	**3**	27 [= sum of digits]	When the last digit (units place) of a positive 6-digit integer is moved to the first position (hundred thousands), and the other digits all shift one place to the right, the new 6-digit integer is exactly one-third of the original number. What is the sum of the six digits that make up the numbers? Let original number be ABCDEF. ABCDEF = 3xFABCDE Or, FABCDE x 3 ABCDEF The only possibilities for F are 1, 2 and 3, otherwise multiplying it by 3 will result in a 7-digit number. If we start by assuming that F = 1, then E must = 7, to give 7x3 = 21: 1ABCD7 x 3 ABCD71 Continue moving from right to left with the multiplication, and filling in the numbers: 142857 x 3 428571 4+2+8+5+7+1 = 27
4		43 [=a + b + c + d + e]	Let a, b, c, d, and e be distinct integers such that: (10 – a)(10 – b)(10 – c)(10 – d)(10 – e) = 175 What is a + b + c + d + e? $175 = 5^2 7^1$ The only way to multiply five different integers to get 175 is if they equal: 1, -1, 5, -5, 7 Therefore, a = 9, b = 11, c = 5, d = 15, e = 3. 9+11+5+15+3 = 43

	4	495 [°]	If x is measured in radians, and $sin(x+\pi) = sin\left(x+\frac{\pi}{2}\right)$ for $2\pi < x < 3\pi$, then what is the measure of x, in degrees? A graphical solution shows that for x between 2π and 3π, it must be $11\pi/4$, because adding either $\pi/2$ or π will result in the same sin value. $11\pi/4$ = 360° + 135° = 495°. 
5	5	153 [triangles]	In the grid shown here, all dots are equally spaced both horizontally and vertically. One isosceles 45-45-90 triangle has been drawn. Including this triangle, how many triangles congruent to this one, in any orientation, can be drawn using the dots in the grid?  Each unit square can be split into 4 congruent triangles, and there are 36 squares. 36*4 = 144. Additionally, there are 9 congruent triangles going down the diagonal. 144 + 9 = 153 

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High School

College Bowl Round #1 Solutions

	Answer	Solution
1	800 [%]	The radius of a circle is tripled. What is the percentage increase in the area of the circle? Suppose r = 1, area = π. Triple r, so r = 3, area = 9π. Percent increase = $(9\pi - \pi)/\pi \times 100 = 800\%$
2	0 [= the remainder]	When an integer n is divided by twelve, the remainder is six. What is the remainder when n is divided by six? The remainer is divisible by 6, therefore the original integer is divisible by 6.
3	50 [%]	Biff won some goldfish at a carnival. During the first week, one-fifth of the fish died, and during the second week, three-eighths of the remaining fish died. What percentage of the original goldfish were still alive after two weeks? 4/5 = alive after 1 week (5/8)(4/5) = ½ = 50% alive after 2 weeks.
4	30 [= largest integer]	The sum of eight consecutive integers is two hundred twelve. What is the largest of the eight integers? $x + (x+1) + (x+2) + (x+3) + (x+4) + (x+5) + (x+6) + (x+7) = 212$ $8x + 28 = 212$, $x = 23$ Largest = $x+7 = 30$
5	36 [%]	What is the probability as a percentage that a randomly selected integer from one to one hundred inclusive contains the digit five or the digit six? 5, 6, 15, 16, 25, 26, 35, 36, 45, 46, 50 – 69 (20 numbers), 75, 76, 85, 86, 95, 96. 36 total numbers, 36/100 = 36%
6	9 [sides]	The difference in the degree measure of an interior and exterior angle of a regular polygon is one hundred degrees. How many sides does the polygon have? A regular nonagon has interior angles of 140°, and exterior angles of 360/9 = 40°.
7	64 [ways]	For one full week, Nate will do exactly one of the following activities per day: running, swimming or biking, and will not do the same activity on two consecutive days. He is going to swim on Wednesday. In how many different ways can he schedule his activities? Wednesday is already chosen. Every other day has 2 ways to choose. 2^6 = 64 ways.

8	150 [cents]	Two hot dogs and a soda cost three dollars and twenty-five cents. Three hot dogs and a soda cost four dollars and fifty cents. In cents, how much do two sodas cost? 2H + S = 3.25 3H + S = 4.50 H = 1.25, therefore S = 0.75 2S = $1.50 = 150 cents
9	1600	What is the next number in the sequence that begins as follows: Ten, fifty, twenty, one hundred, seventy, three hundred fifty, three hundred twenty, and so on. Pattern is multiply by 5, subtract 30.
10	56 [% times greater]	If a is thirty percent greater than x, and b is twenty percent greater than y, then a times b is what percent greater than x times y? a = 1.3x b = 1.2y ab = (1.3x)(1.2y) = 1.56xy

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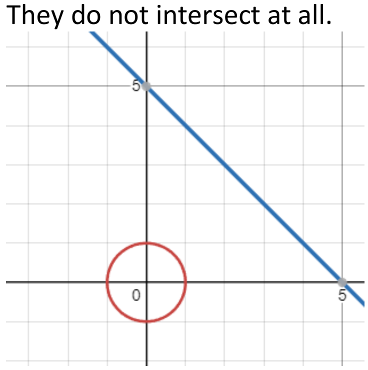
College Bowl Round #2 Solutions

	Answer	Solution
1	5	What is the value of six factorial divided by the quantity five factorial plus four factorial? $$\frac{6!}{5!+4!} = \frac{6!}{4!\,(5+1)} = \frac{5!}{4!} = 5$$
2	0 [points]	The graphs of the equations x-squared plus y-squared equals one, and x plus y equals five, intersect in how many points? They do not intersect at all. 
3	2 [= xy]	If x and y are negative integers, and x minus y equals one, what is the least possible value of x times y? $-1-(-2) = 1$ $(-1)(-2) = 2$
4	240	What is the coefficient on the x-squared y-to the fourth term after expanding the quantity x minus two y raised to the sixth? The solution used the binomial theorem. This term would be 6C4 $a^2 b^4$ $= 15\, x^2 (-2y)^4 = 15 \times 16\, x^2y^4 = 240\, x^2y^4$
5	20 [code words]	How many different six-letter code words can be made from the letters in the word eleven, spelled E-L-E-V-E-N, if the V must be in the first position? Without the V, because it is already fixed, there are 5 letters, and 3 of them are Es. Therefore, $5!/3! = 20$.
6	2 [values of x]	How many real numbers x satisfy the following equation: three raised to the x equals six x minus three $3^x = 6x - 3$ The intersection of an exponential graph and a line can only have a maximum of two points. By inspection, the equation is true for x = 1 or 2.

7	44 [= minimum possible value]	For the expression: one blank two blank three blank four blank five blank six blank seven blank eight blank nine, each blank will be filled in with a plus sign or a multiplication sign. What is the minimum possible value that can result? 1x2+3+4+5+6+7+8+9 = 44
8	5 [= x + y]	x and y are positive integers. The mean of four, twenty and x is equal to the mean of y and sixteen. What is the smallest possible value of x plus y? $(24 + x)/3 = (y + 16)/2$ $48 + 2x = 48 + 3y$ $2x = 3y$, smallest values that make it true are $x = 3$, $y = 2$
9	250 [%]	The value of x is forty percent of y. What is the value of y as a percent of x? $x = 0.4y$ $y = x/0.4 = (10/4)x = (5/2)x = 2.5x$
10	555 [= next number]	What is the next number in the sequence that begins as follows: nine hundred fifty-one, eight hundred fifty-two, seven hundred fifty-three, six hundred fifty-four, and so on. 951, 852, 753, 654, 555 Each time the first digit is going down by 1, and the last digit is going up by 1.

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High School
College Bowl Round #3 Solutions

	Answer	Solution
1	220 [yards]	One-eighth of a mile is how many yards? 5280 feet/8 = 660 feet 660/3 = 220 yards
2	2012 [in]	A rectangle has side lengths of two thousand inches and two thousand twenty-four inches, resulting in the same perimeter as square S. In inches, what is the side length of square S? 2(2000 + 2024) = 8048 8048/4 = 2012
3	60 [%]	A spinner is divided into ten equal regions, numbered one through ten. When it is spun one time, what is the probability in percent that it does not land on a prime number? 1, 4, 6, 8, 9, and 10 are not prime.
4	24 [$]	I have exactly five bills, worth one dollar, two dollars, five dollars, six dollars and ten dollars, respectively. What is the sum of the whole number dollar amounts from one dollar to twenty dollars inclusive, that I cannot pay exactly using one of more of these bills? Have: 1, 2, 5, 6, 10 The only two amounts from 1 – 20 that cannot be paid are $4 and $20. 4+20 = 24
5	3 [= x]	The mean of the integers seven, three, eleven, thirteen, five and x is four more than the mode of the integers. What is the value of x? 3, 5, 7, 11, 13 and x Sum of first 5 = 39. If we add x = 3, the sum = 42, mean = 42/6 = 7, and mode = 3.
6	6 [= n]	If twenty-one is written as a sum of n consecutive positive integers, what is the greatest possible value of n? 1+2+3+4+5+6 = 21
7	171 [trips]	There are nineteen stations on the Ginza subway line in Tokyo, traveling from west to east. If a trip is defined as starting at one station and finishing at a different station, always moving eastward, how many total trips are possible on the Ginza line? From the 1st station, there are 18 possible trips. From the 2nd station, there are 17 possible trips, and so on, down to 1 trip. The sum of 1 through 18 = (18)(19)/2 = 171

8	20 [%]	Seventeen is what percent of eighty-five? 17/85 = 0.2
9	4 [meters]	A circle with radius r has a circumference of at least twenty meters. In meters, what is the smallest possible integer value of the radius? $$C \geq 20, 2\pi r \geq 20, r \geq \frac{10}{\pi}$$ Approximating π as 3, r must be ≥ 10/3, so the least integer value is 4.
10	48	If one-half of a number is eight less than two-thirds of the number, what is the number? (1/2)x + 8 = (2/3)x 8 = (1/6)x, x = 48

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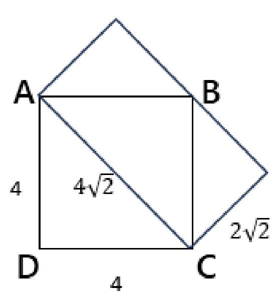
College Bowl Round #4 Solutions

	Answer	Solution
1	80 [liters]	One hundred liters of a salt and water solution contains one percent salt. After some of the water evaporated, the solution contains five percent salt. How many liters of water evaporated? Out of 100 liters, 1 is salt and 99 are water, 1% salt. To get 5% salt, there needs to be 1 liter of salt and 19 liters of water, 1/20 = 5%. Therefore, 99 – 19 = 80 liters evaporated.
2	30	What is six times the sum of the distinct prime factors of one hundred forty-four? $144 = 2^4 3^2$ 2 + 3 = 5, 5 x 6 = 30
3	9 [values of n]	For how many positive integers n is it possible to have a triangle with side lengths five, twelve and n? By the triangle inequality, the possible side lengths are 8, 9, 10, 11, 12, 13, 14, 15 or 16.
4	143	If x minus twelve times x plus twelve equals zero, what is the value of x minus one times x plus one? $(x - 12)(x + 12) = 0$ $x = 12$ or -12 $(12 - 1)(12 + 1) = 143$ $(-12 - 1)(-12 + 1) = 143$
5	16 [square units]	ABCD is a square with side length four units, and AEFC is a rectangle with point B on side EF. In square units, what is the area of rectangle AEFC? The length of the rectangle is the diagonal of the square, which is $4\sqrt{2}$. The width of the rectangle is half of that. The area = $4\sqrt{2} \cdot 2\sqrt{2} = 16$. 
6	10	Six positive integers have a mean of six, and a median of eight. What is the largest possible value of one of the six integers? 1, 1 8, 8, 8, 10 are the numbers that give the largest possible integer.

7	51 [= sum=	What is the sum of the finite series that begins with one minus two plus three minus four, continues in this manner, and ends with plus ninety-nine minus one hundred plus one hundred one? (1 – 2) + (3 – 4) + … + (99 – 100) + 101 Each consecutive pair results in -1, and there are 50 pairs, so -50 total. -50 + 101 = 51
8	0 [solutions]	How many solutions to the following equation exist where x and y are positive integers: two raised to the two x minus two raised to the two y equals fifty-five $2^{2x} - 2^{2y} = 55$ The powers of 2 raised to a positive integer start with: 2, 4, 8, 16, 32, 64, … They are always even, and even minus even = even, so it is impossible to get 55.
9	4 [= x + y + z]	If x plus two y plus three z equals six, two x plus three y plus z equals eight, and three x plus y plus two z equals ten, what is the value of x plus y plus z? x + 2y + 3z = 6 2x + 3y + z = 8 3x + y + 2z = 10 Therefore, 6x + 6y + 6z = 24 x + y + z = 4
10	12 [fractions]	How many positive proper fractions in lowest terms are there that have a denominator of twenty-six? 26 = 2x13 Therefore, the numerators that will result in a proper fraction in lowest terms are: 1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23 and 25.

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College Bowl Round #5 Solutions

	Answer	Solution
1	9 [= digit in tens place]	If seventeen over x equals eleven over three hundred nineteen, what digit is in the tens place of x? 17/x = 11/319 x = 319*17/11 = 493
2	120 [°]	The ratio of the angles of a quadrilateral is three to four to five to six. How many degrees are in the largest angle? 3x + 4x + 5x + 6x = 360 18x = 360, x = 20 6(20) = 120
3	6 [students]	Some students are in Mrs. Casto’s classroom. Six new students enter the classroom, and two leave. Now there are three times as many students as there were originally. How many students are in the classroom now? x = original number x + 4 = new number x + 4 = 3x, x = 2
4	195 [= a + b]	Given the sequence that starts as follows, what is the value of a plus b? three, nine, twelve, twenty-one, thirty-three, a, eighty-seven, b, and so on. Starting with the 3^{rd} term, each term is the sum of the previous 2. 12 + 33 = 54 = a 54 + 87 = 141 = b 54 + 141 = 195
5	8 [ways]	Foster has ten nickels, ten dimes and ten quarters. In how many different ways can he make exactly forty-five cents? QDD QDNN QNNNN DDDDN DDDNNN DDNNNNN DNNNNNNN NNNNNNNNN

6	10 [= mean]	The mean of a set of n numbers is twenty-five, and the mean of a set of three n numbers is five. What is the mean when the two sets are combined? Let n = 1, so the first set is {25}. Therefore the second set could be {5, 5, 5}. The mean of the combined set is (25 + 5 + 5 + 5)/4 = 40/4 = 10
7	14 [= A + B}	When two six-sided dice are rolled, the probability that the sum of the numbers rolled is a multiple of three or four is a reduced common fraction A over B. What is A + B? 20 of the 36 outcomes are multiples of 3 or 4. 20/36 = 5/9, 5 + 9 = 14
8	27	What is the least possible sum of two positive integers whose product is one hundred eighty-two? Factors of 182: 1, 2, 7, 13, 14, 26, 91, 182 Least sum is 13+14 = 27
9	29,160 [$]	A Ford Expedition is currently valued at forty thousand dollars. Its value decreases by the same percentage every year. At the end of one year it will be worth thirty-six thousand dollars. How many dollars will it be worth at the end of three years? It decreases by 10% each year. 40000 – 4000 = 36000 36000 – 3600 = 32400 32400 – 3240 = 29160
10	-5 [= minimum function value]	What is the minimum function value of y equals three x-squared plus six x minus two? $3x^2 + 6x - 2$ The vertex will be at $x = -6/(2x3) = -1$. At $x = -1$, $y = 3(-1)^2 + 6(-1) - 2 = 3 - 6 - 2 = -5$

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College Bowl Round #6 Solutions

	Answer	Solution
1	34,225	What is one hundred eighty-five squared? $185^2 = 34{,}225$
2	2 [= the integer]	A positive integer plus four times its reciprocal is equal to the product of the integer and four times its reciprocal. What is the integer? $x + 4/x = x(4/x) = 4$ $x^2 + 4 = 4x$ $x^2 - 4x + 4 = 0$ $(x - 2)^2 = 0$, $x = 2$
3	25 [%]	Tan flips a fair coin four times. As a percent, what is the probability that the coin comes up heads exactly one time? There are $2^4 = 16$ possible outcomes. Four of them have exactly 1 head: HTTT, THTT, TTHT, TTTH
4	11 [= n]	What is the smallest integer n where n is greater than three, and seven n plus four is a perfect square? $7(11) + 4 = 81$
5	6 [cm]	When the length of each edge of a cube is increased by one centimeter, the cube’s total surface area increases by seventy-eight square centimeters. In centimeters, what is the length of an edge on the original cube? x = original length, $SA = 6x^2$ $x + 1$ = new length, $SA = 6(x+1)^2 = 6x^2 + 78$ $6x^2 + 12x + 6 = 6x^2 + 78$ $x = 6$
6	67 [= last number in 5th row]	The positive integers are written in order, in rows of different lengths. The first row contains the number one. For every following row, the number of entries in the row is the sum of the numbers in the previous row. For example, row two contains the number two, and row three contains the numbers three and four. What is the last number in the fifth row? 1 2 3,4 $(3 + 4 = 7)$ 5,6,7,8,9,10,11 $(5 + 6 + 7 + 8 + 9 + 10 + 11 = 56)$ x = last number in next row $x - 11 = 56$, $x = 67$

7	9 [integers]	How many of the integers from ten through fifty inclusive have the sum of their digits equal to a perfect square? The possible squares that could be achieved are 1, 4 or 9. 1: 10 4: 13, 22, 31, 40 9: 18, 27, 36, 45 That is a total of 9 integers.
8	7 [= range]	What is the range of the following set of numbers? one hundred five over nine, twenty-eight thirds, four and two-thirds, ten and two-ninths $\frac{105}{9} = 11\frac{2}{3}$, $\frac{28}{3} = 9\frac{1}{3}$ max - min = $11\frac{2}{3} - 4\frac{2}{3} = 7$
9	6 [= x + y]	The point x comma y lies at the intersection of the lines y equals x and y equals negative two-thirds x plus five. What is x plus y? y = x, y = (-2/3)x + 5 x = (-2/3)x + 5 (5/3)x = 5, x = 3, y = 3, x + y = 6
10	5	What is the value of the quantity twenty squared minus fifteen squared divided by the quantity eighteen squared minus seventeen squared? $\frac{20^2 - 15^2}{18^2 - 17^2} = \frac{(20+15)(20-15)}{(18+17)(18-17)}$ $= \frac{35 \cdot 5}{35 \cdot 1} = 5$

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College Bowl Extra Questions Solutions

	Answer	Solution
1	15129 [= sum]	What is the sum of the first one hundred twenty-three positive odd integers? Sum of first positive 'n' odd integers = n^2. $123^2 = 15129$
2	233	What is the thirteenth number in the Fibonacci sequence that starts with one, one, two, and so on? 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233
3	90 [°]	Three angles of a convex pentagon are one hundred, one hundred twenty and one hundred forty degrees. The remaining angles are congruent to each other. What is the measure of one of the remaining angles, in degrees? Total of interior angles = 540°. 540 – 100 – 120 – 140 = 180. 180/2 = 90
4	-8 [= f inverse of 3]	The function f of x equals the quantity two x minus five divided by the quantity x plus one. What is f inverse of three? $f(x) = (2x-5)/(x+1) = 3$ Solve for x = -8 If $f(-8) = 3$, then $f^{-1}(3) = -8$
5	2209 [sq cm]	What is the area in square centimeters of a square with a perimeter of one hundred eighty-eight centimeters? 188/4 = 47 cm side length $47^2 = 2209$ sq cm area
6	8 [%]	Biff buys a sandwich that costs twelve dollars and twenty-five cents, and pays thirteen dollars and twenty-three cents total with the tax. As a percentage, what was the tax rate? \$13.23 - \$12.25 = 0.98 0.98/12.25 = 0.08 = 8%
7	211 [base 10]	The hexadecimal number D three is equal to what base ten number? D3 means 13x16 + 3x1 = 211