"Math is Cool" Masters-1999-00

April 29, 2000

Individual Contest, 7th and 8th Grade

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record your answers on the green sheet.

1. If the profit of an item is $18 and the sum of the cost and the profit is $34, what is the cost of the item in dollars?

2. MIC, Ltd. sweatshirts cost $16 each. If Jenny buys 6 MIC, Ltd. sweatshirts at once, how much, in dollars, does she pay?

3. How many lines of symmetry does a regular hexagon have?

4. What is the square root of 81?

5. A circle's radius is 13. What is its circumference?

6. 2 prime numbers differ by 5. What is the larger of the two numbers?

7. Nadav has a bunch of goldfish in his aquarium. For every goldfish, there are 8 gallons of water in his aquarium. If the aquarium has 480 gallons of water, how many goldfish does Nadav have?

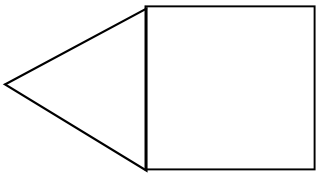
8. What is the sum of the first 25 positive integers?

9. How many positive primes have a remainder of zero when divided by 13?

10. The sum of five consecutive numbers is 35. What is the largest number?

11. Robert spent 1 day, 10 hours, and 20 minutes in Budapest. How many seconds was Robert there?

12. The figure below is formed when an equilateral triangle is placed on a square with area of $49cm^2$. What is the perimeter of the figure?



13. What is the center of a circle containing the points (3,3), (2,4), (3,5) and (4,4)?

14. Spencer makes 9 out of 13 shots in the first half of a basketball game. How many consecutive shots must he make in the second half of the game to raise his percentage to 96%?

15. How many factors does 210 have?

16. The mean of x and y is 7.5. The mean of x, y and z is 9. What is the value of z?

17. Nicole opened her math book and multiplied the two page numbers that were facing her. Their product was 40200. What was the smaller page number facing her?

18. Find the number of sides in a polygon whose interior angles have a sum of 1980E.

19. A ball is dropped from a height of 81 feet. Each time it drops *h* feet, it rebounds $\frac{1}{3}h$ feet. Find the total distance traveled by the ball.

20. A large group of Gregorian monks are standing in a circle, evenly spaced. The 112^{th} monk is directly opposite the 888^{th} . How many monks are there altogether?

21. If $\left(\frac{1}{9}\right)^{2x-1} = 3^{(1-x)}$, what is the value of x?

22. Katie takes a 4 question true/false quiz and randomly guesses on every problem. What is the probability that Katie will score exactly 50% on her quiz?

23. Joel has three different types of cheese and two different types of crackers. If he must alternate eating cheese and crackers, and no type of cheese or cracker can be eaten twice, how many ways can Joel eat his cheese and crackers if he eats all of the different types?

24. On Drew's planet, the probability of being struck by lightning on any given day is 2/9. The probability of being struck by a meteor on any given day is 9/10. What is the probability of being struck by a meteor but not by lightning on April 29th, 2000?

25. A crew of pirates is stranded on a desert island with a stash of food. In the night, Redbeard Robert takes 1/3 of the food and hides it. Scurvy Silas takes 1/3 of what's left and hides it. Grungy Greg takes 1/3 of what's left and hides it. In the morning, there are 16 pounds of food left. How much food, in pounds, was originally in the stash?

26. The same number is added to the numerator and the denominator of $\frac{7}{11}$ to make it equal to $\frac{4}{5}$. What is the number?

27. Divide $\frac{2001}{11}$ and round to four decimal place.

28. On a 30 question math test, there are 8 calculus problems, and the rest are trigonometry. What is the ratio of calculus problems to trigonometry problems?

29. Cory is twice as old as Roy. In 6 years, Roy will be 2/3 as old as Cory. How old will Cory be in 10 years?

30. In the annual two-hand touch football championship game, the Denominators are beating the Lone Prime Rangers. The Lone Prime Rangers team makes a touchdown (worth six points). If they make one extra point, they will have $\frac{1}{4}$ of the Denominators' score. If they make the two point conversion, they will have $\frac{2}{7}$ of the Denominators' score. How many points do the Denominators have?

31. How many two digit numbers are divisible by 3, but not divisible by 6?

32. In the Prodigies of America Gifted Program, 18 students are taking theoretical physics, 27 are taking Asian history, and 20 are taking nuclear thermodynamics. If 4 are taking all three, 8 are taking only physics and history, 7 are taking only history and thermodynamics, and 6 are taking only physics and thermodynamics, how many students are enrolled at the Prodigies of America Gifted Program (assuming everyone takes at least one course)?

33. What is the millions digit of 400^3?

34. When gold was discovered, the population of Pascal's Triangle increased by 40%. When a tornado hit, the population decreased by 20%, leaving 3360 people. How many people originally lived in Pascal's Triangle?

35. In a triangle, one leg is twice the length of the other leg. If the length of the hypotenuse is $4\sqrt{5}$, what is the sum of the legs?

36. How many points with integer coordinates are exactly 9 units away from (0,0)?

37. Find the perimeter of a rectangle with area 60 and diagonal of length 13.

38. If 6 mathletes can eat 20 Cheetos in 6 seconds, how many seconds are needed for 10 mathletes to eat 10 Cheetos?

39. Ryan's Caustic Special uses 3 ounces of tabasco sauce for every 5 ounces of taco meat. When making 4 pounds of his Caustic Special, he accidentally uses 5 ounces of tabasco sauce for every 3 ounces of taco meat. How many ounces of meat must he add to create the proper ratio of tabasco sauce to taco meat?

40. Jenny drove from Proton to Electron traveling 70 mph. On her return trip, she traveled along the same road going 80 mph. What was Jenny's average speed, for the entire trip, to the nearest tenth of a mph?

"Math is Cool" Masters-1999-00

April 29, 2000

Individual Multiple Choice Contest, 7^{th} and 8^{th} Grade

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1. If a is the number of positive perfect squares less than 100, and b is the number of positive perfect cubes less than 100, what is the value of a^b ?

A) 1771561 B) 6561 C) 59149 D) 10,000 E) 100,000 F) Answer not given

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2. A bag of marbles contains 6 red marbles and 4 blue marbles. If two are selected simultaneously, what is the probability that at least 1 blue marble is drawn?

A) 2/3 B) 2/5 C) 4/25 D) 9/25 E) 1/3 F) Answer not given

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3. Evaluate: $2^{2^2} \times \frac{2^{2^2}}{2^2 + 2^2}$

A) 32 B) 16 C) 64 D) 8 E) 128 F) Answer not given

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4. Sampson draws a square with side length 8. Then he draws a new square using the diagonal of the first square as a side. What is the area of the new square?

A) $128\sqrt{2}$ B) 64 C) $64\sqrt{2}$ D) 128 E) 256 F) Answer not given

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5. How many three digit numbers start with a prime digit, have an even digit in the middle, and an odd digit at the end?

A) 90 B) 99 C) 125 D) 80 E) 100 F) Answer not given

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6. Dan rows up a river in 4 hours and down the river, at the same pace, in 2 hours. If the distance he traveled in one direction was eight miles, what is the rate of the current?

A) 1 B) 6 C) 2.5 D) 2 E) 3 F) Answer not given

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7. A number is chosen at random between 1 and 25, inclusive. Express as a percent, the probability that it is prime.

A) 30% B) 32% C) 28% D) 40% E) 25% F) Answer not given

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8. What is the smallest integer divisible by all the numbers 1 through 10?

A) 720 B) 2520 C) 360 D) 5040 E) 3628800 F) None given

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9. If x is a positive number, what is the least possible value of the sum of x and its reciprocal?

A) 0 B) 2.5 C) 1 D) 2 E) Answer not given

"Math is Cool" Masters-1999-00

April 29, 2000

Team Contest, 7th and 8th Grade

Express all answers as reduced fractions unless stated otherwise.
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1. Amy leaves for Mexico at ten in the morning going 70 miles per hour. Anne leaves at eleven going 80 miles per hour. If they drive along all the same roads, at what time will Anne pass Amy?

2. There are fifteen pool balls in a set. Seven are classified as solids and seven are striped; one is classified as the black ball. If the balls are arranged randomly in a triangle, what are the chances that the black ball is in the center and there is one solid and one striped in the back corners?

3. Drew brings his lunch from home every other school day; one week he brings his lunch Monday, Wednesday, and Friday, and the next he brings it Tuesday and Thursday. On Tuesday and Wednesday he eats three sandwiches, and on Monday, Thursday, and Friday he eats two. After 17 weeks, (there are no days off and he only eats sandwiches for lunch at school) how many sandwiches will he eat?

4. x, y, and z are prime numbers. When averaged, the result, q, is another prime number. What is the lowest possible product of x, y, z, and q?

5. What is the 15th number in the sequence: 1, 5, 13, 25, 41...?

6. Four of the exterior angles of a regular polygon add up to 288 degrees. How many sides does the polygon have?

7. When the square of a positive number is added to 9 times the reciprocal of the same number, the result is 6. What is the number?

8. What is the area of a square inscribed in a circle with radius 2?

9. Find all real values of x for which $x^4 - 16 = 0$.

10. A line segment is drawn connecting the origin and the point (8,6). A line segment of length 8 is then drawn from (8,6) to point B. What is the least possible distance from the origin to B?

"Math is Cool" Masters-1999-00

April 29, 2000

Pressure Round Contest, 7th and 8th Grade

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1. A cone with base radius of 4 units and a height of 24 units is turned upside down and filled with water at the rate of 4π units per second. How many seconds will it take for the cone to fill to a mark halfway up the side?

2. What is the sum of the first ten prime numbers?

3. All the four digit numbers possible using 1, 2, 3, and 4 exactly once are listed in order from least to greatest. What is the 10th number on the list?

4. How many integer values of x satisfy the inequality: $4 < |X| < 18$?

5. Expand: $[(x+1)^2+1]^2$

"Math is Cool" Masters-1999-00

April 29, 2000

Mental Math Contest, 7th and 8th Grade

Express all answers as reduced fractions unless stated otherwise.
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Person A

1. What is the area of a circle with a diameter of 14?
2. What is the least common multiple of 10 and 16?
3. What is the volume of a pyramid with base area 12 and height 1/2?
4. Three of the angles in a quadrilateral measure 70, 80, and 90 degrees. What is the measure of the fourth angle?

Person B

1. 1000 grams plus 1 decagram is equal to how many decagrams?
2. What is the seventh smallest prime number?
3. Evaluate: 3! times 2!
4. What is the perimeter of a regular pentagon with side length 12?

Person C

1. What is the median of 6, 5, 8, 10, and 16?
2. Find the radius of a circle with circumference 96π.
3. The sum of two number is 32. The difference is 8. What is the smaller of the two numbers?
4. List the prime factors of 24.

Person D

1. What is the sum of the first 7 odd numbers?
2. What is the ones digit of the product of the first 9 primes?
3. What is the length of the smaller diagonal of a rhombus with one angle equal to 60 degrees and side length 8?
4. Evaluate: $64^{2/3}$ (Read as "Evaluate 64 to the two-thirds power")

"Math is Cool" Masters-1999-00

April 29, 2000

7^{th} and 8^{th} Grade

College Knowledge Bowl Questions #1

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1. A Faultfinder randomly analyzes 5% of a math test for mistakes. If there are two mistakes on a 40 question math test, what is the probability the Faultfinder will find both mistakes?

Answer: 1/780

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2. A box of walnuts can be split evenly among groups of 2, 3, and 4 people, but there is 1 walnut left over when split among 5 people. What is the least possible number of walnuts in the box?

Answer:36

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3. What is the hypotenuse of a right triangle with legs of length $\sqrt{2}$ and $\sqrt{6}$?

Answer: $2\sqrt{2}$

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4. What is the sum of the first 11 odd numbers?

Answer: 121

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5. What is the probability of rolling at least one 6 when two six sided dice are rolled?

Answer:11/36

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6. What is the probability of getting at least one head when a coin is flipped 8 times?

Answer:255/256

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7. If trophies are awarded to the top three finishers in a race with 8 people, how many ways can the trophies be awarded?

Answer:336

Extra Question: What is the 6^{th} term in the Fibonacci sequence?

Answer: 8

"Math is Cool" Masters-1999-00

April 29, 2000

7th and 8th Grade

College Knowledge Bowl Questions #2

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1. On the L.C. Distance team, 7 people run the 800 meter race, 6 people run the 3200 meter race, 2 run both, and everybody runs the 1600 meter race. How many people run the 1600 meter race?

Answer:11

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2. During the first half of a basketball game, Gonzaga scored 16 points and the Lone Prime Rangers score 4 points. In the second half of the basketball game, the Lone Prime Rangers scored 28 points while Gonzaga scored 16 points, what is the ratio of the score of the Lone Prime Rangers to the score of Gonzaga's?

Answer:1:1

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3. If the sum of two numbers is 37 and their difference is 18, what is the larger number?

Answer:55/2 or 27.5

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4. A plane flies 800 mph against a wind of 30 mph. How far will it travel in 2.5 hours?

Answer: 1925 miles

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5. Evaluate: (999x)(1001x)

Answer: $999{,}999\,x^2$

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6. How many square numbers are less than 1000?

Answer:31

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7. Bryceson's favorite number has 5 digits. They are all odd numbers, and none of them are the same. What is the difference between the greatest and smallest possible values of Bryceson's favorite number?

Answer:83952

Extra Question: The midpoints of an equilateral triangle with perimeter 18 are connected to create a smaller equilateral triangle. What is the perimeter of the smaller equilateral triangle?

Answer:9

"Math is Cool" Masters-1999-00

April 29, 2000

7^{th} and 8^{th} Grade

College Knowledge Bowl Questions #3

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1. The sum of eight prime numbers is 239. What is the smallest number that can be used in this sum?

Answer:2

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2. An integer is chosen at random, and the next greatest integer is added to it. That sum is then squared, then divided by two. Is the remainder always one, always 2, or possibly either?

Answer: Always one

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3. Two sides of an isosceles triangle are 2 and 7. What is the sum of all the possible lengths for the third side?

Answer:7

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4. Solve for x: $x^2 - 12 = x$

Answer: 4 and -3

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5. How many 3-digit area codes are possible if the first digit cannot be zero and numbers can not be repeated.

Answer:648

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6. What is the remainder when 14641 is divided by 11?

Answer:0

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7. The fraction a/b is in reduced form, and a is less than b. If a+b =6, what is a/b?

Answer:1/5

Extra Question: When a certain number is divided by 17, the remainder is 8 and the quotient is 9. What is the number?

Answer:161

"Math is Cool" Masters -- 1999-00

Grades 7-8 - April 129,2000

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Full Name:_____

1st Score

Out of 40

Individual Contest - Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer		
1	($)16		
2	($)96		
3	6		
4	9		
5	26π		
6	7		
7	60(fish)		
8	325		
9	1		
10	9		
11	123600(sec)		
12	35(cm)		
13	(3,4)		
14	87(shots)		
15	16(factors)		
16	12		
17	200		
18	13(sides)		
19	162		
20	1552(monks)		

	Answer		
21	1/3		
22	3/8		
23	12		
24	7/10		
25	54(lbs)		
26	9		
27	181.9091		
28	4/11 or 4:11		
29	22		
30	28		
31	15		
32	36(students)		
33	4		
34	3000(people)		
35	12		
36	12		
37	34		
38	5(sec)		
39	128/3 or 42 2/3(oz)		
40	74.7(mph)		

"Math is Cool" Masters -- 1999-00

Grades 7-8 - April 129,2000

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Individual Multiple Choice Contest-Score Sheet

1st Score

Out of 18

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is worth 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer		
1	B		
2	A		
3	A		
4	D		
5	E		
6	A		
7	F		
8	B		
9	D		

"Math is Cool" Masters -- 1999-00

Grades 7-8 - April 129,2000

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Team Contest-Score Sheet

1st Score

Out of 10

DO NOT WRITE IN SHADED REGIONS

	Answer		
1	6:00 P.M.		
2	7/65		
3	103		
4	525		
5	421		
6	5		
7	-3		
8	88		
9	2,-2		
10	2		

"Math is Cool" Masters -- 1999-00

Grades 7-8 - April 129,2000

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Mental Math - Score Sheet

1	49π
2	80
3	2
4	120
1	101
2	17
3	12
4	60
1	8
2	48
3	12
4	2,3
1	49
2	0
3	8
4	16

"Math is Cool" Masters -- 1999-00

Grades 7-8 - April 129,2000

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Pressure Round - Score Sheet

	Answer		
1	8		
2	129		
3	2341		
4	26		
5	$x^4+4x^3+8x^2+8x+4$		