"Math is Cool" Championships-2000-01

November 10, 2000

Individual Contest, Grade 8

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.

1. What is the area of a square with side length 8?

2. Solve for x: $3x + 8 = 29$

3. The sum of three consecutive odd integers is 21. What is the smallest of the integers?

4. What is the next number in the sequence 6,7,9, 12,16, ____

5. What is the sum, in degrees, of the interior angles of a regular pentagon?

6. The sum of Biff and Eho's ages is 28. What will the sum of their ages be 11 years from now?

7. What is the sum of the first 12 positive odd integers?

8. Evaluate: $\sqrt{13+\sqrt{4+\sqrt{25}}}$

9. What is the slope of the line passing through (-8, 7) and (-2, -3)?

10. What is the radius of a circle with area equal to 169π?

11. What is the volume of a sphere with a radius of 4?

12. Write $.\overline{7}$ as a ratio of two integers.

13. What is $\sqrt{2}$ to the nearest tenth?

14. What is the hypotenuse of a right triangle with legs of lengths 8 and 15?

15. Libbey drives to work at 10 m.p.h., then drives home at 15 m.p.h. along the exact same route. What was her average speed, in miles per hour, for the entire trip?

16. Yes or No: Are (1,12), (7,30), and (-2, 3) collinear?

17. Dan can read 2 pages in one minute. He reads 5 hours a day. At this rate, how many days will it take him to read the 1500 page annotated version of *War and Peace*?

18. What is the 54^{th} digit to the right of the decimal point in the decimal expansion of 5/9?

19. What is the area of a circle inscribed in a square with area 2?

20. What is the sum of the prime factors of 74?

21. In a horse race of seven horses, how many ways can 1^{st}, 2^{nd}, and 3^{rd} place be awarded?

22. If one diagonal of a rhombus is 8 units long, and the other is 6 units long, what is the perimeter of the rhombus?

23. An urn contains 5 red marbles, 17 green marbles, 9 purple marbles, and 11 orange marbles. On the first draw, what is the probability that the marble will be purple?

24. Find the missing number: $\frac{5+2}{7+3} = \frac{5}{7} + ?$

25. If $x+2=9$ and $x+5=7$, what does $x^2+7x+10$ equal?

26. What is the sum of 12_5 and 2_4 in base 6?

27. How many distinct isosceles triangles with a perimeter of 15 can be created?

28. What is the units digit of 7^{100}?

29. Find the negative value of x that makes the following equation true:

$$\frac{2}{X} = \frac{X}{32}$$

30. Factor completely: $x^2+9x+20$

Challenge Questions

31. The ratio of the perimeters of two similar polygons is 5:3. The area of the smaller polygon is 18. What is the area of the larger polygon?

32. Evaluate: $\frac{n!}{(n-1)!} + n$ when n=100.

33. How many ordered pairs of positive integers (x, y) satisfy $x^2 + y = 100$?

34. How many integer solutions are there to the equation: $|3x + 4| \leq 5$

35. What is the diagonal of a cube with surface area 216?

36. A fair six sided die is tossed 5 times. What is the probability a 2 or 5 would appear each time?

37. What is 34th digit to the right of the decimal point in the decimal expansion of 2/13?

38. How many distinct solutions does $|5 + |x|| = |x + 1|$ have?

39. What is the exact value of $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+...}}}}$

40. To the ancient Greeks and many artists of the Renaissance, a rectangle was most pleasing to the eye when the length *l* and width *w* satisfied the equation: $\frac{l}{w} = \frac{l+w}{l}$. A rectangle with this property is said to be a *golden rectangle* and the ratio $\frac{l}{w}$ is called the *golden ratio*. What is the exact numeric value of the golden ratio?

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Individual Multiple Choice Contest, Grade 8

During the summer, the Math Team Expedition Force (MTEF) climbed several mountains. Use the tables below to answer the questions. For purposes of this round, assume all hikes rise at a constant rate.

Hike Statistics			
Mountain	Maximum Elevation	Initial Elevation	Length of Hike in One Direction
Old Baldy	10,000 feet	5,000 feet	10 miles
Grizzly Peak	7,000 feet	4,000 feet	15 miles
Mt. Chillytoes	15,000 feet	8,000 feet	5 miles
Glacier Ridge	8,000 feet	3,000 feet	2 miles

Percentage of Hike Completed in One Direction				
MTEF Member	Old Baldy	Grizzly Peak	Mt. Chillytoes	Glacier Ridge
Sampson	90%	100%	90%	100%
Carl	40%	20%	70%	100%
Silas	30%	20%	70%	100%
Dillon	10%	60%	70%	100%
Sarah	20%	80%	100%	20%
Kirsten	100%	20%	70%	20%

1. Which hike had the least elevation gain?
   A. Old Baldy B. Grizzly Peak C. Mt. Chillytoes D. Glacier Ridge
2. Which hike had the greatest elevation gain?
   A. Old Baldy B. Grizzly Peak C. Mt. Chillytoes D. Glacier Ridge
3. Which hike was the steepest climb?
   A. Old Baldy B. Grizzly Peak C. Mt. Chillytoes D. Glacier Ridge
4. Who hiked the least number of miles over the summer?
   A. Sampson B. Carl C. Silas D. Dillon E. Sarah F. Kirsten
5. Who reached the highest elevation above sea level over the summer?
   A. Sampson B. Carl C. Silas D. Dillon E. Sarah F. Kirsten
6. Who gained the most elevation?
   A. Sampson B. Carl C. Silas D. Dillon E. Sarah F. Kirsten
7. How much elevation was gained by the whole MTEF?
   A. 70,000 B. 77,500 C. 80,000 D. 80,100 E. Answer not given
8. How many total miles hiked by the whole MTEF?
   A. 80.5 B. 150.6 C. 200 D. 212.6 E. Answer not given
9. What is the least number of members it would take to match or surpass the total elevation gained by Mr. Sampson?
   A. 1 B. 2 C. 3 D. 4 E. 5

"Math is Cool" Championships-2000-01

November 10, 2000

Team Contest, Grade 8

Express all answers as reduced fractions unless stated otherwise.
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Do not round any answers unless stated otherwise.

1. Last year a pair of pants cost \$30 and a shirt cost \$20. This year the cost of the pants increased by 4%, and the cost of the shirt increased by 8%. What is the percent increase of the combined cost of the pants and shirt?

2. Solve for x: $5^5 + 5^5 + 5^5 + 5^5 + 5^5 = 5^x$

3. Solve for x: If $5(2h+4y) = z$ then $15(6h+12y) = xz$

4. A brown paper bag is filled with glue sticks and scissors. 40% of the objects in the bag are glue sticks. If 80% of the scissors in the bag are right-handed scissors, what percentage of the total objects are left-handed scissors?

5. Some girls are standing in a circle. They are evenly spaced and numbered in order. The 7^{th} girl is directly opposite the 17^{th}. How many girls are in the circle?

6. The Lewis and Clark Math Team pays a water purifying company a fixed monthly rate to rent a water dispenser plus a charge for each jug of water used during the month. The May bill was \$22.00, but in June the bill was \$42.00 because they had used three times as many jugs of water. What is the fixed monthly rental charge?

7. Colin has a bag of jelly beans in his locker at school. Each day he eats 10% of what is in the bag. After eating from the bag three times he noticed there was 729 jelly beans left. How many jelly beans were in the bag originally?

8. At a math competition the ratio of boys to girls was 3:4. How many girls were present if 350 students were in attendance?

9. The length of the diagonal of a square is 5. What is the area of the square?

10. What is the name of the geometric shape that has twice as many sides as a square?

"Math is Cool" Championships-2000-01

November 10, 2000

Pressure Round, Grade 8

1. If an equal number of 2's, 3's and 7's are added together to get 96, what was the total number of 7's to get used?

2. What is the number halfway between 1/7 and 1/15?

3. Max can mow a lawn in 3 hours. Aaron can mow the same lawn in 6 hours. How long, in hours, would it take them to mow the lawn working together?

4. Dillon and Dylan go climbing, and between the two of them they have 120 feet of rope. Dylan's rope is three times half the length of Dillon's rope. How long, in feet, is Dillon's rope?

5. A trapezoid has a median of length 13 and height of 6. What is the area of the trapezoid?

"Math is Cool" Championships-2000-01

November 10, 2000

Mental Math, Grade 8

Express all answers as reduced fractions in terms of radicals and π, where applicable, unless otherwise instructed.

Person A

1. List the prime factors of 12.
2. Evaluate 3 to the fourth.
3. What is 15% of 300?
4. What is the area of a circle with diameter 12?

Person B

1. What is the first prime number greater than 50?
2. What is the sum of the first 5 positive integers?
3. What is the sum of two-thirds and five-sevenths?
4. What is the area of a rectangle with perimeter 24 with the length twice the width?

Person C

1. What is the greatest common factor of 12 and 32?
2. Evaluate 4! (Read as: Evaluate 4 factorial)
3. What is the length of a diagonal in a rectangle with sides of length 5 and 12?
4. What is the volume of a cube with side length 4?

Person D

1. What is the square root of 196?
2. What is the area of a triangle with sides of length 6, 8 and 10?
3. What is the reciprocal of the sum of one-half and one-fourth?
4. Every dimension of a pyramid is tripled. What is the ratio of the old volume to the new volume?

"Math is Cool" Championships-2000-01

November 10, 2000

Grade 8

College Knowledge Bowl Questions #1

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1. How many distinct ways can the letters in the word PASCAL be arranged?

Ans: 360

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2. Evaluate for x=6: x^2-6x+7

Ans: 7

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3. A turtle moves at 1 inch per second. If it takes him 3 minutes to cross the road, how many feet did he travel?

Ans: 15(feet)

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4. What is the diameter of a circle with area 121π?

Ans: 22

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5. Jenny is printing a 30 page report. The status bar says it will take 25 minutes and 15 seconds to finish the job. If it is 4:51:36,[read as four fifty-one and 36 seconds] at what time will the printer be done if the estimated time is correct. (Answer must include hour, minute, and second.)

Ans: 5:16:51

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6. Kyla has Ostriches and llamas on her farm. If there are 30 heads and 100 feet, how many Ostriches are there?

Ans: 10 (ostriches)

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7. A CD is 1 hr, 15 minutes long. The average length of a track is 5 minutes. How many tracks are there?

Ans: 15 (tracks)

Extra Question:
Adam, Bill, Carl, and Dean were buying tickets to a movie. In how many different ways could they line up?

Ans:24(ways)

"Math is Cool" Championships-2000-01

November 10, 2000

Grade 8

College Knowledge Bowl Questions #2

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1. What is the larger of the smallest two distinct primes whose sum will be a perfect square?

Ans: 7

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2. My Father and Father-in-Law turned 60 and 54 this year in October. In what year was my Father twice the age of my Father-in-Law?

Ans: 1952 or 52

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3. Krista, Josh, and Kristen took a math test. The average of their three scores was 97. Krista's score was 98 and Josh's score was 99. What was Kristen's score?

Ans: 94

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4. If there are pigs and chickens in a field, and there is a total of 22 heads and 64 feet, how many chickens are in the field?

Ans: 12(chickens)

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5. Each day a snail crawls up a wall 10 inches. Each night the snail slides down 3 inches. On which day will the snail reach the top of the wall if the wall is 12 feet 6 inches tall?

Ans: 21(days)

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6. What is the probability of drawing a king or a red card from a standard deck of 52 cards on the second draw knowing that the first card was a king?

Ans: 9/17

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7. What is the length of the hypotenuse of a right triangle with legs of length 15 and 36?

Answer: 39

Extra Question: The measure of the angles of a triangle are in ratio 4:5:6. What is the measure of the largest angle in degrees?

Answer: 72°

"Math is Cool" Championships-2000-01

November 10, 2000

Grade 8

College Knowledge Bowl Questions #3

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1. Solve for x: $3^x = 243$

Answer: 5

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2. If the difference between two numbers is 2, and their product is 35, what is the larger number?

Answer: 7

---

3. The Lewis and Clark drill team found they could arrange themselves in rows of 5, 9 and 15 with no one left over. What is the minimum number of students on the drill team?

Answer:45

---

4. Two trains are headed directly towards each other at 35 mph and 33 mph. How many miles apart are they 15 minutes before impact?

Answer: 17 miles or 17

---

5. How many diagonals can be drawn in a regular 12 sided figure?

Answer: 54

---

6. What is the sum of the interior angles of an octagon?

Answer: 1080

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7. What is the probability of rolling a sum of 4 on two six sided dice?

Answer: 1/12

Extra Question: Simplify: $\sqrt[4]{\frac{16}{81}}$ (Read: The fourth root of the quantity 16 divided by 81)

Answer: 2/3

Math is Cool" Championships -- 2000-01

8th Grade - November 10, 2000

School Name_____ Team #_____

Proctor Name_____ Room #_____

Key

Full Name:_____

1st Score

Out of 40

Individual Contest - Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer		
1	64		
2	7		
3	5		
4	21		
5	540°		
6	50(years)		
7	144		
8	4		
9	-5/3		
10	13		
11	$256\pi/3$		
12	7:9 or 7/9		
13	1.4		
14	17		
15	12 (mph)		
16	yes		
17	2.5(days) or 5/2		
18	5		
19	$\frac{1}{2}\pi$		
20	39		

	Answer		
21	210(ways)		
22	20		
23	3/14		
24	-1/70		
25	63		
26	$13_{(6)}$		
27	~~X~~ ∞		
28	1		
29	-8		
30	(x+4)(x+5)		
31	50		
32	200		
33	9		
34	4		
35	$6\sqrt{3}$		
36	1/243		
37	8		
38	0		
39	2		
40	$\frac{1+\sqrt{5}}{2}$ Or $\frac{2}{-1+\sqrt{5}}$		

Math is Cool" Championships -- 2000-01

8th Grade - November 10, 2000

School Name_____Team #_____

Proctor Name_____Room #_____

Key

1st Score

Out of 18

Individual Multiple Choice Contest-Score Sheet

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer		
1.	B		
2.	C		
3.	D		
4.	C		
5.	E		
6.	A		
7.	E		
8.	D		
9.	B		

Math is Cool" Championships -- 2000-01

8th Grade - November 10, 2000

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Team Contest-Score Sheet

1st Score

Out of 10

DO NOT WRITE IN SHADED REGIONS

	Answer		
1.	5.6(%)		
2.	6		
3.	9		
4.	12		
5.	20(girls)		
6.	($)12		
7.	1000(Jelly Beans)		
8.	200(girls)		
9.	25/2		
10.	Octagon		

Math is Cool" Championships -- 2000-01

8th Grade - November 10, 2000

School Name_____ Team #_____

Proctor Name_____ Room #_____

Key

Pressure Round - Score Sheet

	Answer		
1.	8(sevens)		
2.	11/105		
3.	2(hrs)		
4.	48(feet)		
5.	78		

Math is Cool" Championships -- 2000-01

8th Grade - November 10, 2000

School Name_____ Team #_____

Proctor Name_____ Room #_____

Key

Mental Math - Score Sheet

A. 1. 2, 3

2. 81

3. 45

4. 36π

B. 1. 53

2. 15

3. 29/21 or 1 8/21

4. 32

C. 1. 4

2. 24

3. 13

4. 64

D. 1. 14

2. 24

3. 4/3 or 1 1/3

4. 1:27