"Math is Cool" Masters-2000-01

Sponsored by: Genie

9^{th} & 10^{th} Grade - May 12, 2001

Individual Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

1. The point (3,2) is reflected across the line y = x. What is the coordinates of the new point?

2. What is the greatest common divisor of 14, 49 and 343?

3. What is the sum, in degrees, of the exterior angles of a regular polygon?

4. Solve for x: -3(5x + 4) - 2(2x - 9) = 12

5. What is the slope of the line passing through the points (3,8) and (11,2)?

6. What is the perimeter of a rectangle with sides of length 7 and 8?

7. What is the length of the hypotenuse of a right triangle with sides of length 10 and 24?

8. Are the following set of points collinear? { (2,3), (5, 11), (8, 17)}

9. What real number cannot be divided by itself?

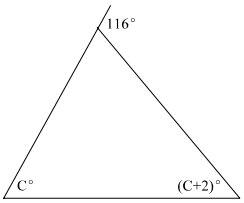
10. How many integers make the following statement true? $-18 \leq x \leq 200$

11. A triangular number is any number of the form (1/2) (n^2 + n) where n is a counting number. What is the second smallest triangular number that is also a perfect square?

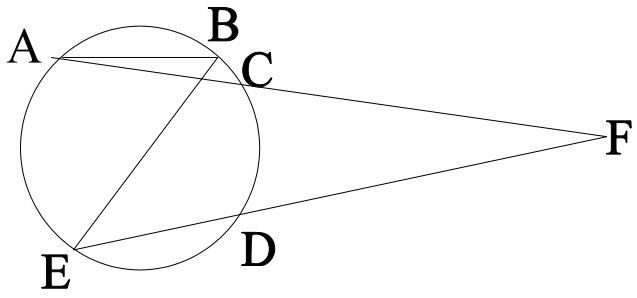
12. Factor completely: $x^3 + 8$

13. Two parallel lines are cut by a transversal. The measure of two alternate interior angles are $(2x + 61)^\circ$ and $(6x - 51)^\circ$. Find the value of x.

14. What is the value of c in the diagram?



15. In the figure below, <ABE = 40°, < AFE = 30°. What is the measure of minor arc CD?

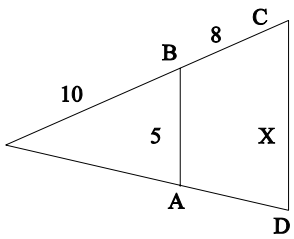


16. What is the LCM of $45x^2y$, $66xy$, 90?

17. z is jointly proportional to x^2 and y^3. If z = 192 when x = 4 and y = 2, find z when x = 2 and y = 4.

18. The length of one leg of a right triangle is twice the length of the second leg. The hypotenuse is 15 meters. Find the length of the shorter leg.

19. Find the value of X in the diagram. Given: $\overline{AB} \| \overline{CD}$



20. What is the vertex of $y = 3x^2 + 4x + 5$?

21. Solve for x: $(27^{-x})((1/3)^{-2}) = (1/9)^5$

22. Solve for X: $23_4 + 13_5 = X_3$

23. Evaluate: $\sum_{k=1}^{3} \frac{1}{2k}$

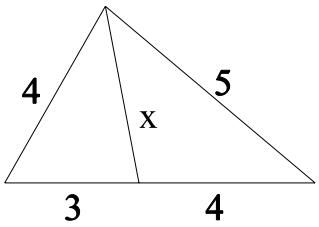
24. Solve for x: $\frac{x-2}{x-3} + \frac{x-3}{x-2} = \frac{2x^2}{x^2-5x+6}$

25. What is the sum of the reciprocals of the 5 solutions to: $5x^5 + 4x^4 - 3x^3 + 2x^2 + x - 1 = 2000$

26. Noah's favorite flower shop sells 5 different types of flowers. How many different ways are there to buy 5 flowers if there are no restrictions on how many of each flower may be purchased?

27. If a, b, c, d, e are positive integers and ab is even, bc is even, cd is odd, de is even. Of a, b, c, d, e, which must be even?

28. What is the value of x^2 in the diagram?



29. How many positive integers < 2001 have an odd number of distinct factors?

30. *n* points are chosen arbitrarily on the circumference of a circle. Each pair of points is then connected with a straight line. What is the maximum number of regions into which the interior of the circle can be divided if *n* = 6?

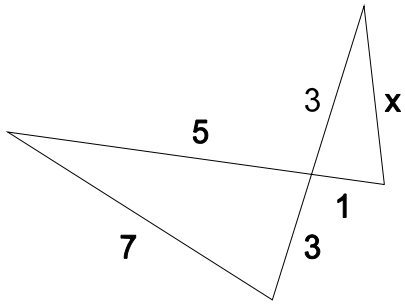
Challenge Questions

31. Silas, Powei, Eric and Ester are all planning on competing at the Spring Mu Alpha Theta math competition. (For purposes of this questions they are the only competitors.) The probabilities of each student taking first place on the Linear Algebra are 7/32, 7/32, 1/4 and 5/16 respectively. A week before the competition it was determined that Silas and Eric would be unable to attend due to the fact they would compete at State Knowledge Bowl on the same day. What is the probability now, that Ester will take first place on the Linear Algebra Exam?

32. Sean and Roy are playing a game. Sean rolls two fair four-sided dice. Roy rolls a single fair ten-sided die once. If Roy rolls higher than the sum of Sean's two dice, Roy wins. Otherwise, Sean wins. What's the probability that Roy wins?

33. A bug is placed on a 2 dimensional grid. Every second, the bug moves 1 unit up, down, left or right, with each direction having an equal chance of being chosen. If the bug is allowed to move for 6 seconds (6 moves), what is the probability the bug ends up where it started?

34. What is the value of x in the diagram?



35. How many ways can 6! be written as x$\binom{}{}$y where x and y are integers and x<y?

36. If 5 numbers are chosen randomly from the interval (0,1), what is the probability the median of the 5 numbers is less than $\frac{1}{4}$?

37. A cylinder has radius 10 and height 8. If x is subtracted from the radius, the absolute change in volume of the cylinder is y. If x is subtracted from the height instead of the radius, the absolute change in volume is also y. What is x?

38. $\sum_{i=1}^{10} \binom{10}{i}$

39. Find b and c so the non-zero numbers $-c+\sqrt{bc}$ and $-c-\sqrt{bc}$ are the roots of $x^2+bx-c=0$

40. Spencer has an urn with 10 balls in it – some green and the rest blue. Jina has an urn containing exactly 3 green balls and 7 blue balls. If Spencer randomly chooses a ball from his urn and places it in Jina's urn, and Jina then draws a ball from her urn, the probability that Jina's ball is green is 31/110. How many green balls are initially in Spencer's urn?

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9th & 10th Grade - May 12, 2001

Individual Multiple Choice Contest

1. If $2^x = 7$, what is the value of 2^{2x}?

A)$\log_2 7$ B) 1/49 C) 49 D) answer not given

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2. How many points with integer coordinates are exactly 10 units away from (0,0)?

A) 10 B) 11 C) 12 D) answer not given

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3. Four students tried to find the sum of the first 9 primes. Their answers were 99, 100, 97 , and 95. Only one answer was correct. What is the sum of the first 9 primes?

A) 99 B) 100 C) 97 D) 95

---

4. If $h(x) = \dfrac{x^2+1}{x^2}$ where $x > 1$, then $h^{-1}(x) =$

(a) $\dfrac{1}{\sqrt{x-1}}$ (b) $\sqrt{\dfrac{x}{1+2x}}$ (c) $\dfrac{-1}{\sqrt{x}}$ (d) $\dfrac{1}{\sqrt{x-1}+1}$ (e) $\dfrac{1}{-\sqrt{x-1}}$

---

5. If $f(x) = \sqrt{1-x^2}$, which of the following is NOT true?

(a) Domain of $f = [-1,1]$ (b) $(f(x))^2 + x^2 = 1$ (c) Range of f *is* $[0,1]$

(d) $f(x) = f(-x)$ (e) The line $y = 1$ intersects the graph of f at two points.

6. If $f(x) = \dfrac{\sqrt{x+2}}{x+2}$ *and* $g(x) = \dfrac{1}{x} - 2,$ then $f[g(x)] =$

(a) $\dfrac{\sqrt{\dfrac{1}{x} - 2}}{\dfrac{1}{x} - 2}$ (b) $\sqrt{\dfrac{1-2x}{x}}$ (c) $\dfrac{\sqrt{\dfrac{1}{x-2} + 2}}{\dfrac{1}{x-2} + 2}$ (d) $\sqrt{x}$ (e) $\dfrac{\sqrt{x}}{x}$

---

7. If $f(x) = \log_b x,$ then $f(bx) =$

(a) $bf(x)$ (b) $f(b)f(x)$ (c) $1 + f(x)$ (d) $x\ f(b)$ (e) $f(x)$

---

8. The vertical asymptote and horizontal asymptote for $f(x) = \dfrac{\sqrt{x}}{x+4}$ are

(a) $x = -4, y = 0$ (b) no vertical asymptote, $y = 0$ (c) no vertical or horizontal asymptote
(d) $x = -4,$ no horizontal asymptote (e) $x = -4, y = 1$

---

9. Which of the following are logically true, assuming x and y are real numbers?

I. If $xy > 0,$ *then* $\dfrac{x}{y} > 0$

II. If $xy \le 0,$ *then* $\dfrac{y}{x} < 0$

III. If $x + y > 0$ *then* $\dfrac{x}{y} > 0$

a) I only b) II only c) I and II only
d) I and III only e) I, II and III

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9th & 10th Grade - May 12, 2001

Team Contest

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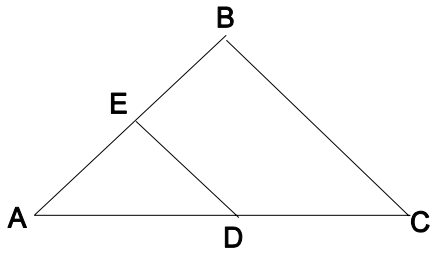
1. $\overline{BC} \parallel \overline{ED}$

   $\overline{AE} = 4, \overline{DE} = 5$

   $\overline{BC} = 8, \overline{AC} = 12$

   What is $\overline{CD}$?



2. Factor completely over the rational numbers: $x^5 - x^4 + x^3 - x^2 + x - 1$

3. Gregg has 2, fair, 6-sided dice. If Gregg rolls the dice until he gets a sum of 7 or 11, what is the probability Gregg rolls a 7?

4. Solve for x: $\sqrt{2x+3} - \sqrt{x+5} = 1$

5. How many integer values make the following equation true? $\left|\frac{3x}{2} - 4\right| \le 5$

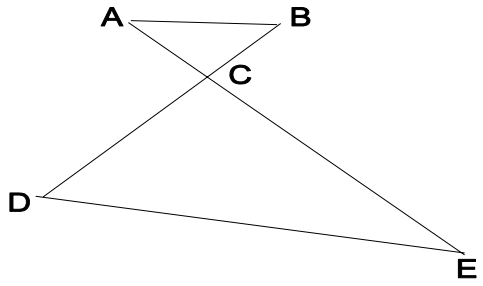
6. Simplify: $\dfrac{\frac{2}{x} + \frac{3}{4y}}{\frac{1}{2x} - \frac{5}{3y}}$

7. $\overline{AB} \parallel \overline{DE}$

   $\overline{AC} = 1, \overline{CD} = 2$

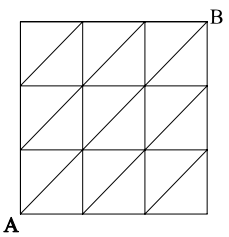
   $\overline{CE} = 3, \overline{DE} = 4$

   What is $\overline{AB}$?



8. Sam's miracle test for anosmia correctly diagnoses the disorder 90% of the time in rats. Of rats tested that do not have anosmia, 10% of the tests result in false positives. It is also known that anosmia affects 10% of the rats in Sam's test population. If a rat from Sam's test population tests negative for anosmia, what is the probability the rat actually has the disorder?

9. How many paths are there from point A to B assuming each move in the path must be North, East, or Northeast (North means up as on a map)?



10. How many positive real values of x satisfy: $\left(x + \sqrt{2} + \sqrt{3} + \sqrt{6}\right)^4 = 1500x$

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9th & 10th Grade - May 12, 2001

Pressure Round

1. Find the distance between the two points (4, 3) and (11, 2)?

2. At a certain casino your odds of winning a game are 1:9. The cost of the game is 50¢ each time you play. If you win the jack pot is $2.00. If you play the game 150 times how much money would you expect to lose?

3. Find the inverse function of $f(x) = \frac{2x+1}{3x-1}$.

4. Evaluate the partial, geometric series: $\frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{1024}$

5. If f(x+y) = f(x) + f(y) for all values of x and y, how many possible values are there for f(0)?

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9^{th}, 10^{th} , 11^{th} & 12^{th} Grade - May 12, 2001

Mental Math

Express all answers as reduced fractions in terms of radicals and π, where applicable, unless otherwise instructed.

	Person #1	
1	Evaluate: 2346 divided by 3	782
2	The sum of two numbers is 32, and the difference is 2. What is the smaller of the two numbers?	15
3	Solve for x: $8^x = 16$	4/3 or 1 1/3
4	The ratio of the volume of two spheres is 8:27. What is the ratio, respectively, of the circumferences of the great circles?	2:3
	Person #2	
1	What is the tens digit of the product of 210 and 427?	7
2	What is 7 written in base 2 ?	$111_{(2)}$
3	Biff can mow a lawn in 4 hours by himself. Eho can mow the same lawn in 2 hours by himself. How many minutes would it take them to mow the lawn together?	80 (minutes)
4	The odds of winning a certain game are 9 to 7. What is the probability of winning the game?	9/16
	Person #3	
1	Evaluate: 2^{12}	4096
2	I bought a chicken and an egg for $31.00. I paid 30 dollars more for the chicken than the egg. How much did I pay for the egg?	50 (cents) or .50
3	What is the volume of a square pyramid with a height of 9 and a base area of 20?	$60(\text{units}^3)$
4	If Sean scores an 80 on 2 exams and a 90 on 2 exams, what is his average score?	85
	Person #4	
1	The product of 30 positive integers is 15. What is the largest sum of these 30 positive integers?	44
2	Express in standard a+bi form, a and b are real: $i^5 + i^9 + i^4$	$1 + 2i$
3	How many ways can you arrange 5 different books on a shelf?	120
4	Kyla has ostriches and llamas on her farm. If there are 30 heads and 100 feet, how many ostriches are there?	10 (ostriches)

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9^{th}, 10^{th}, 11^{th} & 12^{th} Grade - May 12, 2001

College Knowledge Bowl Questions #1

1	Solve the system of equations for x: $3x + 2y = 17$ $x - y = -1$	3
2	What is the equation of the line, in standard form, passing through (8,5) and parallel $2x + 3y = \pi$?	2x+3y=31
3	How many real solutions does the equation $5x^2 + 3x + 4 = 0$ have?	None or 0
4	How many integers make the following statement true? $\lvert 3x + 2 \rvert \leq 8$	6(integers)
5	Two sides of a triangle measure 4.6 and 12.2 centimeters. The perimeter is 28.4 centimeters. Find the measure of the third side in centimeters.	11.6(cm)
6	A cord of wood is 128 cubic feet of wood. A stack of wood is 4 feet wide and 2 feet high. How long, in feet, must the pile be if it contains one chord of wood?	16(ft)
7	The surface area of an unopened cylindrical can is 32π square inches. The radius of the can is 2 inches. Find the height of the can in inches.	6(inches)
	Extra Question: Only use it if needed	
	The measure of three angles in a triangle are $(x+20)^\circ$, x° , and $(210 - 3x)^\circ$. Find the value of x.	50

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9^{th}, 10^{th}, 11^{th} & 12^{th} Grade - May 12, 2001

College Knowledge Bowl Questions #2

1	Biff earned $12,000 last year tutoring students in mathematics. He invested part at 8% simple interest and the rest at 9% simple interest. He made a total of $1050 in interest. How much did he invest, in dollars, at 8% interest?	($)3000
2	Ester can bike to work in 3/4 of an hour. When she takes the bus, the trip takes 1/4 of an hour. If the bus travels 20 miles per hour faster than Ester rides her bike, how far is it to her workplace in miles?	7.5(miles)
3	Robert is twice as old as his brother, Bill. In five years, Bill will be the same age as his brother was ten years ago. What is Robert's present age in years?	30(years)
4	Factor completely: $8r^2 + 6rs - 12rs - 9s^2$	(2r-3s)(4r+3s)
5	What is the area of a triangle with sides length 8,10 and 12?	$15\sqrt{7}$ (units^2)
6	A building has a floor area of 140 square meters. The building has the shape of a rectangle with length 4 meters more than the width. Find the width of the building in meters.	10(meters)
7	Solve for x: $3^{2x} = 81^{x+4}$	-8
	Extra Question: Only use it if needed	
	Solve the following equation for y: 3(2y-1) + y = 5y + 3 ***Read as:*** 3 times the quantity 2 y minus 1 end quantity plus y equals 5 y plus 3.	3

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9^{th}, 10^{th}, 11^{th} & 12^{th} Grade - May 12, 2001

College Knowledge Bowl Questions #3

1	The slope of the line containing the points (2, 3) and (x,11) is 4. What is the value of x?	4
2	What is the area of the region bounded by the following lines? x = 0, y = 0 and x + y = 10	50(units^2)
3	The difference of the two numbers is 6. Twice the smaller is 4 more than the larger. Find the larger of the two numbers.	16
4	If tickets for the show cost $2.00 for adults and $1.50 for children, how many children tickets were sold if a total of 300 tickets were sold for $525?	150(tickets)
5	What is the area of the region that represents the solution set of $x^2 + y^2 \# 49$?	49π
6	The ratio of the length of the sides of two similar rectangles is 3:4. The area of smaller rectangle is 18, what is the area of the larger rectangle?	32(units^2)
7	What is the negative value of k that makes $x^2 + kx + 121$ a perfect square trinomial?	-22
	Extra Question: Only use it if needed	
	What is the volume of a cone with base area of 20 and a vertical height of 6?	40(units^2)

"Math is Cool" Masters -- 2000-01

9th and 10th Grade - May 12, 2001

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Name:_____

1st Score

Out of 40

Individual Contest - Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	(2,3)		
2	7		
3	360(°)		
4	-6/19		
5	-3/4		
6	30		
7	26		
8	No		
9	0		
10	219		
11	36		
12	$(x+2)(x^2-2x+4)$		
13	28		
14	57		
15	20°		
16	$990x^2y$		
17	384		
18	$3\sqrt{5}$		
19	9		
20	(-2/3,11/3)		

	Answer	1 or 0	1 or 0
21	4		
22	201		
23	11/12		
24	13/10		
25	1/2001		
26	126		
27	b and e		
28	55/7 or 7 6/7		
29	44		
30	31		
31	10/17		
32	1/2		
33	25/256		
34	$\sqrt{13}$		
35	30		
36	53/512		
37	15/2 or 7.5		
38	1023		
39	b=2, c=1		
40	1		

"Math is Cool" Masters -- 2000-01

9th and 10th Grade - May 12, 2001

School Name_____Team #_____

Proctor Name_____Room #_____

Key

1st Score

Out of 18

Individual Multiple Choice Contest-Score Sheet

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	ANSWER	-1, 0 or 2	-1, 0 or 2
1	C		
2	C		
3	B		
4	A		
5	E		
6	D		
7	C		
8	B		
9	A		

"Math is Cool" Masters -- 2000-01

9th and 10th Grade - May 12, 2001

School Name_____Team #_____

Proctor Name_____Room #_____

Key

1st Score

Out of 10

Team Contest-Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	4.5 or 9/2		
2	$(x-1)(x^2+x+1)(x^2-x+1)$		
3	$\frac{3}{4}$		
4	11		
5	7		
6	$\frac{3(8y+3x)}{2(3y-10x)}$ Or multiplied out		
7	4/3 or 1 1/3		
8	$\frac{1}{82}$		
9	63(paths)		
10	0		

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School Name_____Team #_____

Proctor Name_____Room #_____

Key

Pressure Round - Score Sheet

	Answer		
1	$5\sqrt{2}$		
2	($)45		
3	$f^{-1}(x) = \frac{x+1}{3x-2}$		
4	$\frac{1023}{1024}$		
5	1		