"Math is Cool" Masters-2001-02

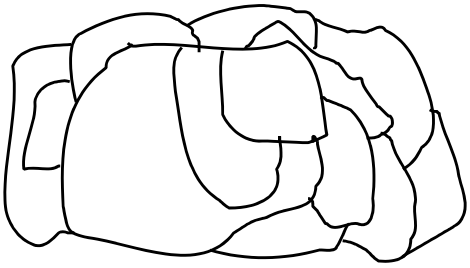
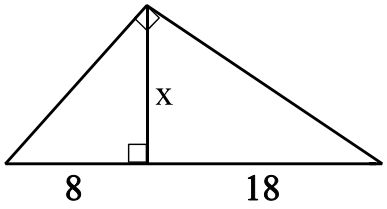
Sponsored by: LeMaster & Daniels

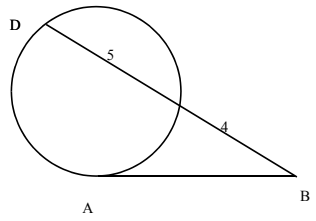
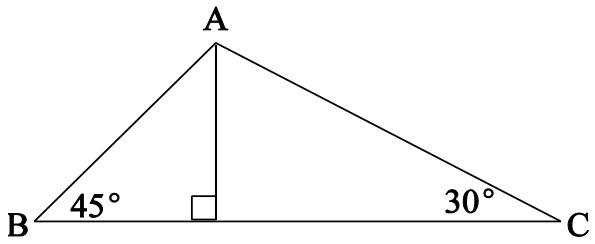
9^{th} & 10^{th} Grade - May 11, 2002

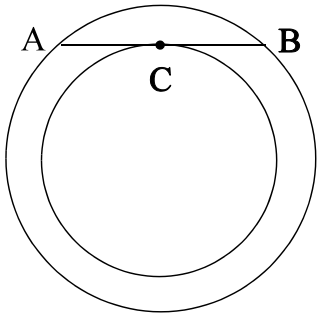
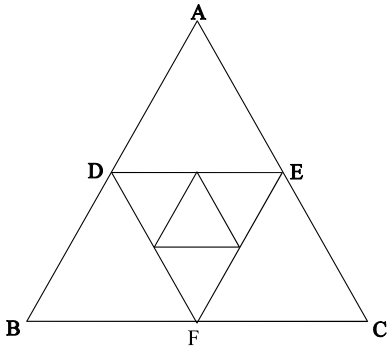
Individual Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

1	What is the greatest common divisor of 228 and 437?
2	What is the volume of a cone, in cm^3, with height 40 cm and radius 30 cm?
3	Find the area of the triangle formed by the line x + y = 20 and the x and y axes.
4	How many solutions does the following set of equations have: 3x - 2y = 10 4y - 6x = -2
5	Find the sum: 1+4+7+10+....+181.
6	What are the last three digits of 5^{2002}?
7	The sides of a right triangle form the diameters of semicircles. What is the ratio of the sum of the areas of the semicircles on the legs to the area of the semicircle on the hypotenuse?
8	A triangular number can be written as the sum of the first n whole numbers, where n is a positive integer. What is the second smallest triangular number that is also a square number?
9	What is the sum of the coordinates of the intersection of the altitudes of the triangle formed by the lines y=2, x=3 and y= -x+10?
10	What is the area of a triangle with sides 24, 45, and 51?
11	A bathtub with dimensions of 8ft, 2ft, and 30 inches drains at 5 ft^3/min. (1 foot = 12 inches) A faucet fills the bathtub at 3 ft^3/min. Assume the bathtub is completely full. How long, in minutes, will it take a full bathtub to become 1/4 full if the drain is left open and the faucet is running?
12	Give a Pythagorean triple that gives the same numerical value for the area and perimeter.

13	A "Best of the Hamburger 5" CD spins at 60 revolutions per second. If the song "Would You Like Fries With That?" lasts 4 minutes and 32 seconds and the song "Ketchup as Thick as Blood" lasts 6 minutes and 1 second, how many more revolutions does the CD make during "Ketchup as Thick as Blood" than during "Would You Like Fries With That?"
14	Which is greatest: 2^{35}, 5^{15}, or 6^{14}?
15	The country Malmania is divided into regions as shown. If each region is given a color, what is the minimum number of colors used so that no adjacent regions have the same color? 
16	Lily has twice as many flowers as Katie. Katie has 6 more flowers than James. James has 2 less than Silas. Lily has 3 times as many as Silas. How many do they have all together?
17	On an 8x12 grid of squares, a line is drawn from one corner to the opposite corner. How many squares does it pass through?
18	Find x: 
19	The inscribed circle of a regular hexagon has a radius of $\sqrt{3}$. Find the area of the hexagon.
20	Solve for x: $x^3 - 3x^2 - x + 3 = 0$
21	Find the volume of the solid given by the inequality $16x^2 + 4y^2 + 9z^2 - 144 \# 0$
22	A ball is shot from one corner of a 6 foot by 10 foot pool table at a 45° angle. How many times will it bounce off the sides before landing in a corner pocket?

23	Find the length of AB,where AB is tangent to the circle. 
24	How many ways can you divide 6 people into 3 groups of 2?
25	Find the area of ^{a}ABC, given AB = 20. 
26	Determine the vertical asymptote(s) of the function $f(x) = \frac{x^2 - 5x + 6}{x^2 - 7x + 12}$?
27	Find the distance between the center of one face of a cube and one of the opposite vertices, given that the side length equals 10.
28	ABCD is a regular tetrahedron. A plane cuts through the midpoints of AB, AC, CD, and BD. Given that AC = 4, find the area of the intersection of the plane and the tetrahedron.
29	Find the volume of a regular pyramid with a square base of area 100 units^2 and lateral faces each of area 65 units^2.
	# Challenge Quetions
30	How many divisors does 10! have?
31	A beetle sits on each square of a 9x9 checkerboard. Each beetle can crawl diagonally to a neighboring square, leaving some squares empty and others with multiple beetles. What is the smallest possible number of empty squares after each beetle has moved exactly once?

32	Find the area of the "ring" (region between two concentric circles) where AB = 10, C is the midpoint of AB, and AB is tangent to the inner circle. 
33	A 13 by 13 by 13 cube consists of 13^3 unit cubes. What is the maximum number of cubes that can be seen from a single viewpoint?
34	Let the area of ^{a}ABC = 30, and let D, E, and F be the midpoints of AB, AC, and BC, respectively. ^{a}DEF is called the medial triangle of ^{a}ABC. If you continue drawing medial triangles in this manner, what is the sum of all the areas of the triangles? 
35	If x, y, and z are chosen randomly on the interval (0,1) what is the probability that the mean of x, y, and z is less than $\frac{1}{4}$?
36	How many paths are there from (0,0) to (7,7) (by moving in 1 unit increments up or to the right), that do not go above the line y = x?
37	Find the four digit number abcd such that the product of 4 and abcd equals dcba.
38	How many integer solutions are there to $x^2 + y^2 + z^2 = x^2y^2$?
39	Write $\sqrt{29+12\sqrt{5}}$ in the form $a+b\sqrt{c}$ where a, b *and* c are integers.
40	Consider 64 objects, each with a different weight. What is the minimum number of pair wise weight comparisons needed to determine the heaviest and 2nd heaviest objects?

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9^{th}, 10^{th}, 11^{th} & 12^{th} Grade - May 11, 2002

Individual Multiple Choice Contest

1	A piece of wire 22 cm long is cut into two pieces in order to make two squares. If the sum of the areas of the two squares is 15.25 cm^2, what is the length of one side of the smaller square? A) 2.5 cm B) 3 cm C) 3.5 cm D) 4 cm E) Answer not given
2	Factor completely over the set of integers: $x^3 + x^2 - 4x - 4$ A) 2,-2,-1 B) (x-2)(x+2)(x+1) C) (x-4)(x+1)(x-1) D) 4,-1, 1 E) Answer not given
3	Consider a pentagon with 5 equal interior angles. It follows that: A) each interior angle is 72° B) the interior angles are equal to the exterior angles C) the pentagon is obtuse D) the pentagon is regular E) none of the above
4	What is the largest integer value that makes the following inequality true: $\frac{2x+3}{x-4} < 0$ A) 0 B) 1 C) 2 D)3 E) Answer not given
5	How many ways can 10 identical postcards be given to 4 people, if each person must receive at least 1 postcard? A) 24 B) 16 C) 56 D) 84 E) Answer not given
6	Find the coefficient of the x^6y^8 term in the expansion $(x - 2y^2)^{10}$ A) 3360 B) 1200 C) 1500 D) 2800 E) Answer not given
7	What is the vertex of the curve described by $y = 3x^2 - 2x + 4$? A) (1, 8) B) (3, 5) C) (1/3, 11/3) D) (-2 , 7) E) Answer not given
8	If $e^{g(x)} = \frac{x^x}{x^2-1}$, then $g(x) =$ A) $x\ln x - 2x$ B) $\frac{\ln x}{2}$ C) $(x-2)\ln x$ D) $\frac{x\ln x}{\ln(x^2-1)}$ E) $x\ln x - \ln(x^2-1)$
9	The domain of the function defined by $f(x) = \ln(x^2 - x - 6)$ is the set of all real numbers x such that: A) $x > 0$ B) $-2 \le x \le 3$ C) $x \le -2$ *or* $x \ge 3$ D) $-2 < x < 3$ E) x < -2 or x > 3

"Math is Cool" Masters-2001-02

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9th & 10th Grade - May 11, 2002

Team Contest

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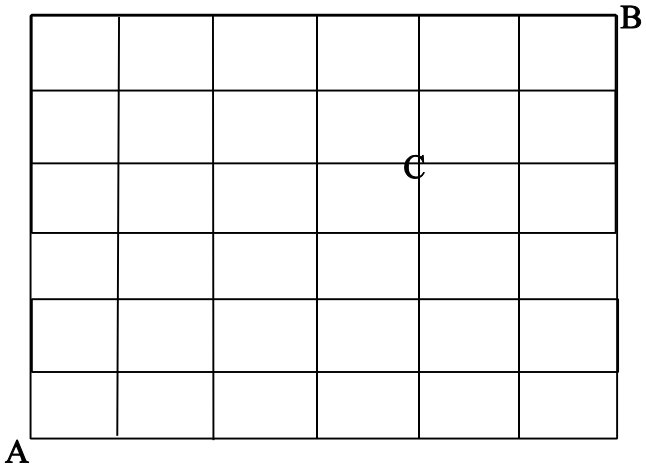
Record all answers on colored cover sheet.

1	Max is in a bike race. He biked the first half of the race plus $\frac{1}{2}$ a mile at 30 mph. He biked $\frac{1}{2}$ the remaining distance plus half a mile at 20 mph. He then biked the remaining 15 miles at 10 mph. How many minutes did he spend biking?
2	Round Lake is in the shape of a circle, with a radius of 2000 feet. A swimmer starts in the middle of the lake and swims at a speed of 25 feet per minute due north. A runner starts due south of the swimmer and runs around the edge of the lake. How fast, in feet per minute, must the runner travel in order to arrive at the north shore of the lake at exactly the same time as the swimmer?
3	 How many ways can A travel to B, given the restriction that A can only go up or right, and cannot go through c?

4	Mara types 84 words per minute. Given that each page contained 10 questions, each question contained exactly 12 words and she typed for 5 consecutive hours starting from page 1, how many pages did she type?
5	If $ab - ac - bc = -10$, $a^2 = 36$ and $b^2 + c^2 = 19$, what is $(a + b - c)^2$?
6	What is the sum of the coefficients of the expansion of $(a + b)^{12}$?
7	How many numbers less than 1,000 are the product of two square numbers greater than 1?
8	Solve for x: $x^4 - 13x^2 = -36$
9	An ant and a fly both start at one corner of a $4 \times 6 \times 8$ rectangular prism. The fly can fly straight to the opposite corner, but the ant must crawl along the faces of the box. What is the difference between the distance the ant travels and the distance the fly travels assuming the ant travels the shortest distance possible?
10	$\sum_{n=1}^{100} \frac{1 + ni}{i}$ Write answer in a +b*i* form.

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9^{th}, 10^{th}, 11^{th} & 12^{th} Grade - May 11, 2002

Pressure Round

1	The numbers 1 through 36 are arranged in a 6x6 square so that the sums of the numbers in each row, column and diagonal are the same. What is that special sum?
2	Each person in an office contributed the same amount of money per week to a charity fund, which received a total contribution of $20 per week from the office. Two new people came to work in the office today and everyone's contribution was reduced by 50¢ per week with the total contribution remaining the same. How many people work in the office now?
3	Solve for x: $\sqrt{3x+8}-\sqrt{6x+2}=1$
4	What is the area of an equilateral triangle with side length 8?
5	A total of 15 pennies are put in 4 nonempty piles so that each pile has a different number of pennies. What is the smallest number of pennies that could be put in the largest pile?

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9^{th}, 10^{th}, 11^{th} & 12^{th} Grade - May 11, 2002

Mental Math

Express all answers as reduced fractions and in terms of radicals and π, where applicable, unless otherwise instructed.

Person 1		
1	Find the sum of the following arithmetic series: 5 + 10 + 15 + ... + 100	1050
2	A square has a perimeter of 36. Connect the midpoints of the sides of the square to form a new square. If this process is repeated an infinite number of times, what is the sum of all the areas of the squares? Your sum should include the area of the original square.	162
3	What is the product of 11 base 2 and 101 base 2? Express your answer in base 3.	$120_{[3]}$
4	How many factors does 48 have?	10
Person 2		
1	Evaluate 6 factorial.	720
2	Eric the babysitter can clean the entire house in 3 hours. It takes the child 6 hours to make a mess of the house. If the house is a complete mess, how long will it take Eric to clean the house assuming the child is making a mess of the house at the same time?	6[hours]
3	How many seconds are in one day?	86,400 [seconds]
4	What is the units digit in 9 factorial?	0
Person 3		
1	What is the smallest prime number greater than 200?	211
2	What is the probability of drawing either an ace or a diamond from a standard deck of 52 cards?	4/13
3	What is the volume of a cone with height $\frac{1}{\pi}$ and base radius $\sqrt{\pi}$?	$\frac{\pi}{3}$
4	What is the sum of the first 9 odd numbers?	81
Person 4		
1	Round to the tenths place: $\sqrt{200}$.	14.1
2	What is the perimeter of an equilateral triangle with area $\sqrt{3}$?	6
3	What is the volume of a hemisphere with a radius 30?	18,000π
4	Which is greater: 3^{30} or 2^{40} ?	3^{30}

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College Knowledge Bowl Questions #1

1	What is the sum of the first 1000 whole numbers?	500500
2	If Drew has 10 coins, all of which are quarters or nickels, and he has $1.90, how many nickels does he have?	3
3	If f(x) is an one-to-one, odd function and f(3)=10, what is $f^{-1}(-10)$? **Read as:** If f of x is an one-to-one, odd function and f of 3 equals 10, what is f inverse of negative 10?	-3
4	For how many integer values of x is $x^2 \leq 3x$?	4
5	Evaluate: 8 factorial.	40320
6	Solve for x: $x^2 - 8x = -15$.	3,5 (need both answers and order does not matter)
7	What is the product of the least common multiple and greatest common factor of 9, 12 and 15?	540
	Extra Question: Only use it if needed	
8	If two six-sided dice are rolled, how many distinct results are possible for the product of the faces?	18

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College Knowledge Bowl Questions #2

1	What is the radius of a sphere with surface area 484π?	11
2	What is the sum of the geometric series 2, 1, $\frac{1}{2}$, 1/4...	4
3	What is the lateral surface area of a cone with base radius of 3 and height of 4?	15π
4	Find the sum of all natural numbers ≤ 1000 that have an odd number of positive divisors.	10416
5	Find the area of an ellipse with semi-axes of 3 and 6.	18π
6	What is the product of all the positive divisors of 30?	810,000
7	Solve for x: $\binom{2002}{999} = \binom{2001}{998} + \binom{2001}{x}$ ***Read as:*** Solve for x in the equation "2002 choose 999 is equal to the sum of 2001 choose 998 and 2001 choose x."	999
	Extra Question: Only use it if needed	
8	Find the shortest distance between the two parallel lines $x + 2y + 10 = 0$ and $2x + 4y + 60 = 0$	$4\sqrt{5}$

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College Knowledge Bowl Questions #3

1	Find the area of an equilateral triangle with side length equal to 7.	$\frac{49\sqrt{3}}{4}$
2	Determine the values of k such that $5x^2 + 4x + k = 0$ will have two non-real, complex solutions.	k> 4/5 Answers can also be in interval form
3	Two new high-tech commuter planes, designed and built by the Silas Aerodynamics Corporation, are 12,000 miles apart and are flying toward each other. Their speeds differ by 300 mph. If they pass each other in 10 hours, what is the speed of the slower plane?	450[mph]
4	What is the geometric mean of 2 and 512?	32
5	Evaluate: $221_5 + 432_5$ in base 5	$1203_{[5]}$
6	On a math team, the ratio of females to males is 7 to 3. When two males were not able to make it to the competition, the ratio of females to males for that competition was 3 to 1. How many males are on the team?	9
7	A trapezoid has a base of length 21 and a median of length 13. What is the length of the other base?	5
	Extra Question: Only use it if needed	
8	What is the surface area of a cylinder with height 18 and radius 2?	80π

"Math is Cool" Masters - 2001-02

9th & 10th Grade - May 11, 2002

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

Full Name:_____

1st Score

Out of 40

Individual Contest - Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

	Answer	1 or 0	1 or 0
21			
22			
23			
24			
25			
26			
27			
28			
29			
30			
31			
32			
33			
34			
35			
36			
37			
38			
39			
40			

"Math is Cool" Masters - 2001-02

9th & 10th Grade - May 11, 2002

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

Individual Multiple Choice Contest-Score Sheet

1st Score

Out of 18

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer		
1			
2			
3			
4			
5			
6			
7			
8			
9			

"Math is Cool" Masters - 2001-02

9th & 10th Grade - May 11, 2002

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

1st Score

Out of 10

Team Contest-Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

"Math is Cool" Masters - 2001-02

9th & 10th Grade - May 11, 2002

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

"Math is Cool" Masters -- 2001-02

9th & 10th grade - May 11, 2002

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Full Name:_____

1st Score

Out of 40

Individual Contest - Score Sheet
DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	19		
2	12,000π [cm^3]		
3	200 [$units^2$]		
4	0		
5	5551		
6	625		
7	1:1 or 1		
8	36		
9	5		
10	540		
11	15 [minutes]		
12	6,8,10 or 5,12,13 (Only one answer needed)		
13	5340 [rev]		
14	6^{14}		
15	4		
16	50 [flowers]		
17	16		
18	12		
19	$6\sqrt{3}$		
20	-1,1,3		

	Answer	1 or 0	1 or 0
21	96π [$units^3$]		
22	6 [times]		
23	6		
24	15		
25	$100+100\sqrt{3}$		
26	x = 4		
27	$5\sqrt{6}$		
28	4		
29	400 [$units^3$]		
30	270		
31	9 [squares]		
32	25π [$units^2$]		
33	469 [cubes]		
34	40		
35	9/128		
36	429 [paths]		
37	2178		
38	1		
39	$3+2\sqrt{5}$		
40	67		

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School Name_____Team #_____

Proctor Name_____Room #_____

Key

Individual Multiple Choice Contest-Score Sheet

1st Score

Out of 18

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer	-1, 0 or 2	-1, 0 or 2
1	A		
2	B		
3	E		
4	D		
5	D		
6	A		
7	C		
8	E		
9	E		

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School Name_____Team #_____

Proctor Name_____Room #_____

Key

1st Score

Out of 10

Team Contest-Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	202 (minutes)		
2	25π (ft/min)		
3	504 (ways)		
4	210 (pages)		
5	35		
6	4096		
7	19		
8	$\pm 2, \pm 3$		
9	$2\sqrt{41} - 2\sqrt{29}$		
10	5050 - 100i		

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9^{th} & 10^{th} grade - May 11, 2002

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Pressure Round - Score Sheet

	Answer		
1.	111		
2.	10 [people]		
3.	1/3		
4.	$16\sqrt{3}$		
5.	6		