"Math is Cool" Masters-1999-00

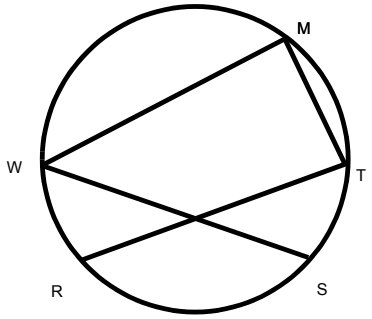
9^{th} -10^{th} Grade -April 29, 2000

Individual Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the green cover sheet.

1. Simplify: $\dfrac{y+\dfrac{2y}{y+2}}{1+\dfrac{4}{y^2-4}}$

2. In the circle, minor arc RS = 40E,
then $m\text{p}W + m\text{p}T =$

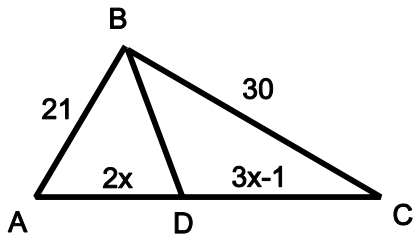


3. A, B, C and D are four different weights. When they are placed on a balance scale, the following observations are made:
A and B exactly balance C and D
A and C together out weigh B and D together
C is lighter than D
Arrange the weights in order from heaviest to lightest.

4. If a>b>0 then

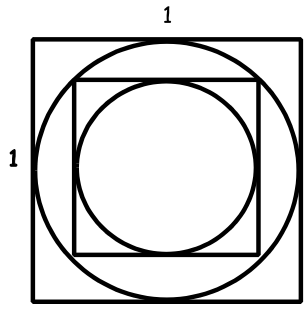
a) $\dfrac{1}{a}=\dfrac{1}{b}$ b) $\dfrac{1}{b}>\dfrac{1}{a}$ c) $\dfrac{1}{b}<\dfrac{1}{a}$ d) $\dfrac{1}{b}<0$ e) $\dfrac{1}{a}<0$

5. In Î ABCD, BD bisects pB. Then X=



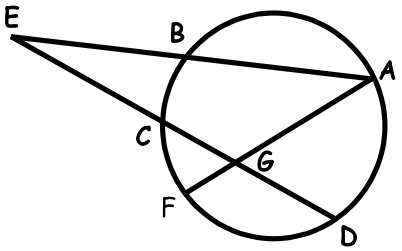
6. A line through (2,3) has x-intercept twice its y-intercept. Its equation is:

7. All are circles or squares. What is the area of the smaller circle?



8. If m+1 varies inversely as 2n-1 and if n=13 when m=3, find n when m=24.

9. BE=5, AB=7, CE=4 and AG=6.
Then GF=

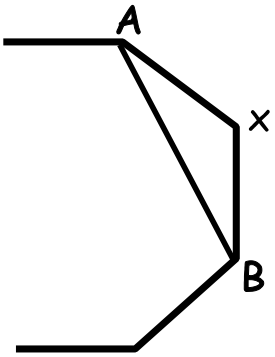


10. In $\hat{I}ABC$, if a=2b and mpB=30E, then mp$A$=

11. If logX = 3log3 + $(\frac{1}{2})$log5 - 2log7 then x=

12. If 5x-7y=3 and -2x+4y=0, then x+y=

13. Ax and Bx are two adjacent sides of a regular polygon. If the measure of angle ABx equals 1/3 the measure of angle AxB, how many sides does the regular polygon have?



14. One root of x^2-kx+18=0 is twice the other. Then k=

15. A boy ran up a hill at $1\frac{1}{2}$ mph and came down the hill at $4\frac{1}{2}$ mph. The trip took him 6 hours. How far, in miles, is it to the top of the hill?

16. In a city, 85% of the taxi cabs are green and 15% are blue. A taxi cab has an accident, and a witness who is accurate 80% of the time identifies the cab as blue. What is the probability that it was a blue rather than green cab involved in the accident?

17. A number that is 11 less than twice the square root of 49 is:

18. If (6/5)x=10 then 3x=

19. N is a 3-digit number whose units digit is 3. What is the probability that N is divisible by 3?

20. Three kinds of nuts are mixed in a can. How many nuts must you take to be sure you have at least n nuts of one kind?

21. Suppose a, b and c are integers between 0 and 9. Then the number represented by the infinite repeating decimal 0.abcabcabcabc... is a rational number. Express it as a fraction.

22. The yearly changes in population census of a town for four consecutive years are, respectively, 25% increase, 25% increase, 25% decrease, 25% decrease. To the nearest percent, what is the net change over the four years?

23. Simplify: $$\cfrac{1}{1+\cfrac{2}{1+\cfrac{2}{1+\cfrac{2}{1+....}}}}$$

24. How many diagonals does a regular 2000-sided polygon have?

25. The ordered pair (2,-3) is the common solution of the system 2x+3y=A and 5x-2y=B. What is A+B?

26. A cyclist rides 30 km at an average speed of 9 km/hr. At what rate must she cover the next 10 km in order to bring her overall average speed up to 10 km/hr?

27. Consider the linear function f(x)=ax+b. If the x-intercept is (-32,0) and the y-intercept is (0,24), determine a+b.

28. Gregg can mow 600 square yards of grass in 1 ½ hours. At this rate, how many minutes would it take him to mow 600 square feet?

29. Find the max of : x^2+5x+7 on [-3,5]

30. You have 5 liters of 10% HCI and 3 liters of 30% HCI. You want 1 liter of 16% HCI. How much 30% solution should be used?

31. What is the geometric mean of: 1, 3, 3, 7, 8

32. The distance between A and B is 300 miles. A car traveled from A to B at 60 mph. How fast must a car travel, in miles per hour, from B to A to average 40 mph for the round trip?

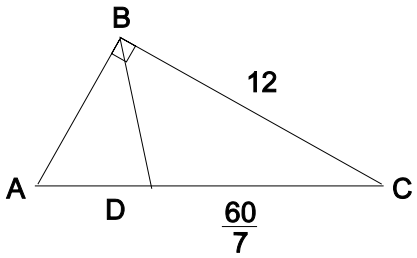
33. I am twice as old as my brother. In 3 years my sister will be twice as old as my brother. How much older am I than my sister?

34. $\angle ABC = 90^\circ$ $\angle ABD = 45^\circ$

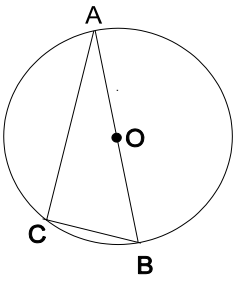
$\overline{DC} = \frac{60}{7}$

$\overline{BC} = 12$

$\overline{AB} = ?$



35. Let $\overline{AB}$ be a diameter of circle O with radius 5. If $\overline{AC}$ = 4, what is the length of $\overline{BC}$?

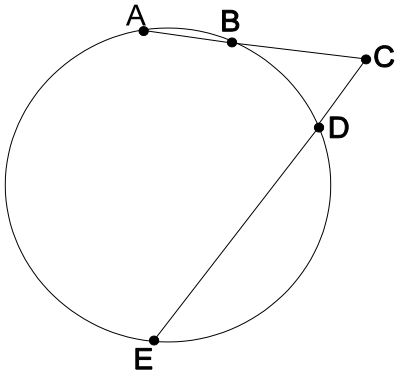


36. There are 10 blobs in a glob, 100 globs in a drop and 25 pseudodrops in a drop. How many blobs are in 50 pseudodrops?

37. Simplify $\frac{6^9}{12^{10}}$

38. What is the area of a rhombus with diagonals of 19 and 20?

39. Define $a \mathbin{r} b$ as $\frac{a+b}{a^2+b^2}$. Evaluate $2 \mathbin{r} (3 \mathbin{r} 4)$



40. $\overline{AB} = \overline{CD} = x$

$\overline{BC} = 1, \overline{DE} = 2$

What is x?

"Math is Cool" Masters-1999-00

9^{th} -10^{th} Grade -April 29, 2000

Individual Multiple Choice Contest

1. If $i^2=-1$ and $x=1+i$, then $\frac{1}{x^2}$ is equal to:

   a) $\frac{-1}{2}i$ b) $\frac{1}{2}i$ c) 2i d) -2i e) none of these

---

2. What is the shortest distance between the lines y=x and y=x+2?

   a) $\sqrt{2}$ b) $\sqrt{3}$ c) 2 d) 3 e) none of these

---

3. Town X and Town Y are separated by 400 miles. A car going from X to Y travels at 50 mph. How fast must the car travel from Y to X to obtain an average speed of 100 mph?

   a) 100 mph b) 150 mph c) 300 mph d) 450 mph e) not possible

---

4. If log2 + log15.6 = log7.8 - logx, then x=

   a) 1 b) 1/4 c) 31.2 d) 7.8 e) 5.8

---

5. Let q be a binary operator, defined for positive integers, such that $aqb=a^2+b^2$ Which of the following are true?

   I. q is commutative

   II. q is associative

   III. q is distributive over standard addition

   a) I only b) II only c) I and II only d) II and III only e) I, II and III

---

6. x is a non-zero real number. $x^{-\frac{1}{3}}=$

   a) $\frac{-x}{3}$ b) $\sqrt[3]{x}$ c) $\sqrt[3]{-x}$ d) $\sqrt[3]{\frac{1}{x}}$ e) $\frac{1}{x^3}$

---

7. The circumference of a circle is equal to its areas. The radius of the circle is?

   a) $2/\pi$ b) 2 c) 1 d) $\frac{1}{2}$ e) $1/\pi$

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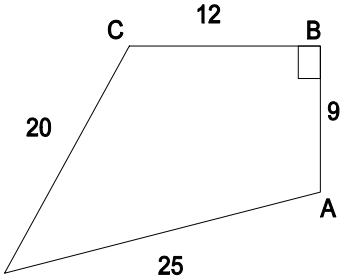
8. The area of convex quadrilateral ABCD is?

   a) 204 b) $241\frac{1}{2}$ c) 304 d) $166\frac{1}{2}$ e) 370



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9. If a, b and x are non-zero numbers so that a…b and $1/a+1/x=1/b$ then x is equal to?

   a) b-a b) a-b c) $\frac{a\cdot b}{b-a}$ d) $\frac{b-a}{a\cdot b}$ e) $\frac{a\cdot b}{a-b}$

"Math is Cool" Masters-1999-00

9^{th} -10^{th} Grade -April 29, 2000

Team Contest

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1. Solve for x. x^2-1999x+2000=0

2. A work crew of 3 people requires 3 weeks and 2 days to do a certain job. How long would it take a work crew of 4 people to do the same job if each person on both crews works at the same rate as each of the others? Note: each week contains 6 work days.

3. For what integers value of x is the following inequality satisfied?
$$\frac{x^2+10x+24}{x^2+6x+8} \le 0$$

4. 3 coins are picked at random from an unlimited supply of pennies, nickels and dimes. Given that the total value of coins chosen exceeds 15¢, what is the probability at least 2 dimes were chosen?

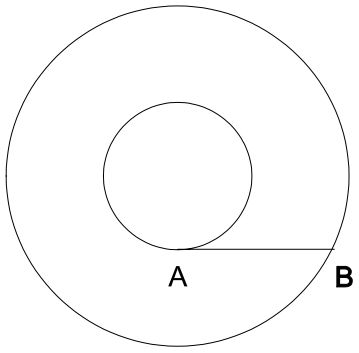
5. How many ways are there to divide 10 identical pennies among Robert, Bill, Eric, Greg and Tim if each person must receive at least one penny and Robert must receive an even number of pennies.

6. Diophantus spent 1/6th of his life in childhood, 1/12th in youth and 1/7th more as a bachelor. Five years after his marriage was born a son who dies 4 years before his father, at ½ his father's final age. How old was Diopheutus when he died?

7. If 2x-3y-z=0 and x+3y-14z=0, z…0, the numerical value of $\frac{x^2+3xy}{y^2+z^2}$ is ?

8. What is the area of a square which has a diagonal one unit longer than one of the sides?

9. Consider two concentric circles with $\overline{AB}$ tangent to the inner circle. Line segment AB is one unit long. What is the area of the annular region between the two circles?



10. What is the value of $\frac{2^{2000}-2^{1999}}{2^{2000}+2^{1999}}$?

"Math is Cool" Masters-1999-00

9^{th} -10^{th} Grade -April 29, 2000

Pressure Round Contest

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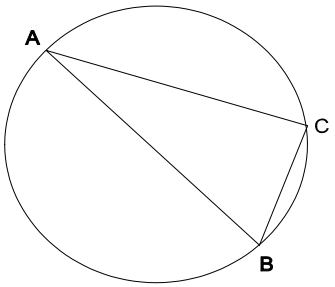
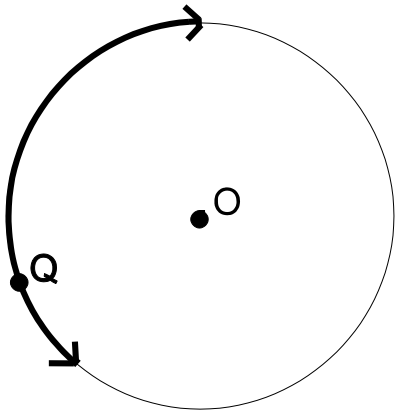
1. Given a three dimensional tic-tac-toe type game consisting of four levels of 4-by-4 squares, how many ways could you get 4 in a row?

2. In the diagram, AB is the diameter of the circle. If AB=4 and the area of triangle ABC is 4, calculate the perimeter of the triangle ABC.



3. Any number of the form 1000a+100b+10a+b where a and b are integers is always divisible by what prime?

4. Simultaneously, two particles start at point Q, on circle O, and move in opposite directions around the circle. If the speed of one particle is 4 times that of the other, then the particles will next meet at point P on the circle. If pPOQ is acute, what is mpPOQ, in degrees?



5. Augustus De Morgan, a famous mathematician who lived solely in the 19^{th} century made the following statement. "I was x years old in the year x^{2}". When was he born?

"Math is Cool" Masters-1999-00

9^{th} -12^{th} Grade -April 29, 2000

1MM	What is the fourth term in the geometric progression that begins: 1, 2, . . .	8
1MM	What is the area of a rectangle with diagonal of 5 and a long side of 4?	12
1MM	What is the sum of the first 5 primes?	28
1MM	2 coins are flipped simultaneously. What is the probability of 1 head and 1 tail appearing?	½
1MM	Find the x-value where x^2+x+1 reaches a minimum?	-1/2
MM	Two runners are running around a 1 mile track. One runner runs a 6-minute mile and the other runs an 8-minute mile. They start at the same place. How many minutes go by until the slower runner is lapped by the faster runner?	24 minutes
MM	Robert's prize fish weighs 45 pounds plus 1/6 of its weight. How many pounds does the fish weigh?	54 pounds
MM	Factor completely: $X^4+3x^3+2x^2$	$x^2(x+1)(x+2)$
MM	How many times larger than 12^{72} is 12^{144}?	e) 12^{72}
MM	What is the units digit of 13^{44}?	9
MM	In what base does 26x4=116?	9
MM	Six people are in a circle and begin counting in turn. (i.e., the first person says one, the second two, ... and they continue around). Which person (1, 2, 3, 4, 5 or 6) will be the one to say 2000?	2
MM	What is the slant height of a right circular cone with volume 30π and radius 3?	$\sqrt{34}$
MM	Factor completely: $x^2+30x+225$	$(x+15)^2$
MM	How many distinct prime factors does 2000 have?	2
MM	Bill has 6 red socks and 4 blue socks. Bill first draws a red sock. How many more socks must Bill draw to guarantee he has a pair of red socks?	5

CB	What is the median of the mode, median and mean of the following list of test scores? 60, 70, 85, 60, 95	70
CB	Factor completely: $x^3+9x^2+26x+24$	(x+2)(x+3)(x+4)
CB	Simplify $i^{2000}+i^{-2000}+i^{1999}-1^{-1999}$ where $i^2=-1$	2-2i or 2(1-i)
CB	The population of Holleyville increased 25% two years ago and then decreased 25% last year. The population is now 4500 people. What was the population before the two changes?	
CB	Sean and Roy are playing a game. Sean rolls two fair four-sided dice. Roy rolls a single fair ten-sided die. The person with a higher total sum wins. What's the probability that Roy wins?	½
CB	What is $\sin\frac{19\pi}{6}$?	$\frac{-\sqrt{3}}{2}$
CB	What value(s) of k will allow the following to be factored as the square of a linear polynomial? $4x^2+kx+49$	±28
CB	How many ways can you rearrange the letters in the word Bassoon?	1260
CB	What is the sum of the factors of 48?	124
CB	Where do the following two functions intersect? $f(x)=x^2+2$ $g(x)=-2x^2+8$	$\left(\sqrt{2},4\right)$ and $\left(-\sqrt{2},4\right)$
CB	$f(x)=x+3$, $g(x)=x^2-9$, $h(x)=3-2x$ What is $h(f(g(x^2)))$?	$15-2x^4$
CB	What is the vertex of the parabola $f(x)=2x^2-3x+1$?	$\left(\frac{3}{4},\frac{-1}{8}\right)$
CB	Three water pipes are used to fill a swimming pool. The first pipe alone can fill the pool in 8 hours, the second in 12 hours, and the third in 24 hours. If all three pipes are opened at the same time, how long will it take to fill the pool?	
CB	Express $0.31\overline{7}$ as a simple fraction	143/450

CB	Ryan, Mark and Nadav are running around a track. Ryan runs 12 laps per hour, Mark runs 6 laps per hour and Nadav runs 3 laps per 2 hours. If they all start together, how long will it take them to all return to the beginning at the same time?	2/3 hour or 40 minutes
CB	The slope of line L is m= $-\sqrt{3}$. What is the inclination of line L in degrees?	120E
CB	A certain cube has a side length of 2. If we triple its volume to form a new cube, what is its new side length?	$2\sqrt{3}$
CB	Simplify: $\frac{8!}{2\cdot4\cdot6\cdot8\cdot10\cdot12\cdot14\cdot16}$	1/256
CB	What is the remainder when 6! Is divided by 7?	6
CB	What is the area of a square circumscribed about a circle with circumference 1?	$\frac{1}{\pi^2}$
CB	Bill and Robert are playing a game. In an Urn there are 6 marbles, one black and five white ones. Bill draws a marble and then passes the Urn to Robert who draws a marble and passes the Urn to Bill. After all the marbles are drawn the person with the black marble wins. What is the probability that Bill wins?	1/2
CB	Two fair six-sided dice are rolled. What is the probability a sum of 7 is rolled given that at least one 3 is rolled?	2/11
CB	Which is larger 3^{180} or 2^{300} ?	2^{300}
CB	Bill has a right, circular cylinder of volume 10. If Bill doubles the radius of the cylinder, what is the new volume of the cylinder?	40

"Math is Cool" Masters -- 1999-00

9th & 10th grade - April 29, 2000

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Full Name:_____

1st Score

Out of 40

Individual Contest - Score Sheet
DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	$(y^2+2y-8)/y$	or factored	
2	160E		
3	A>D>C>B or ADCB		
4	b		
5	7		
6	x+2y=8		
7	$\frac{\pi}{8}$		
8	5/2		
9	4		
10	90E		
11	$\frac{27\sqrt{5}}{49}$		
12	3		
13	5		
14	±9		
15	6 3/4 miles		
16	12/29 . .41		
17	3		
18	25		
19	1/3		
20	3n-2		

	Answer	1 or 0	1 or 0
21	$\frac{abc}{999}$		
22	-12%		
23	$\frac{1}{2}$		
24	1,997,000		
25	11		
26	15		
27	24 3/4		
28	10		
29	57		
30	3/10 or .3 (Liters)		
31	$\sqrt[5]{504}$		
32	30 mph		
33	-3		
34	16		
35	$\sqrt{84}$ or $2\sqrt{21}$		
36	2000		
37	1/6144		
38	190		
39	1425/2549		
40	$\left(-1+\sqrt{5}\right)/2$		

"Math is Cool" Masters -- 1999-00

9th & 10th grade - April 29, 2000

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Team Multiple Choice Contest-Score Sheet

1st Score

Out of 18

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points. Answers need to be recorded as letters.

DO NOT WRITE IN SHADED REGIONS

Answer		-1, 0 or 2	-1, 0 or 2
1	A		
2	A		
3	E		
4	B		
5	A		
6	D		
7	B		
8	A		
9	E		

"Math is Cool" Masters -- 1999-00

9th & 10th grade - April 29, 2000

School Name_____Team #_____

Proctor Name_____Room #_____

Key

1st Score

Out of 10

Team Contest-Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	$\left(1999 \pm \sqrt{398001}\right)/2$		
2	2 weeks 3 days or 15 days		
3	-6, -5, -3		
4	7/16		
5	40		
6	84		
7	7		
8	$3+2\sqrt{2}$		
9	π		
10	1/3		

"Math is Cool" Masters -- 1999-00

9th & 10th grade - April 29, 2000

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Mental Math - Score Sheet

1	8
2	12
3	28
4	1/2
1	-1/2
2	24(minutes)
3	54(pounds)
4	$x^2(x+1)(x+2)$
1	12^{72}
2	1
3	9
4	2
1	$\sqrt{109}$
2	$(x+15)^2$
3	2
4	5

"Math is Cool" Masters -- 1999-00

9th & 10th grade - April 29, 2000

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Pressure Round - Score Sheet

	Answer		
1	76 ways (24 on diagonals)		
2	✳√+△✳		
3	101		
4	72E		
5	1806		