"Math is Cool" Championships-2000-01

February 10, 2001

Mental Math, High School

Person 1		
1	Evaluate: 82 x 88?	7216
2	What is 8! divided by 5!?	336
3	Is the sum of the first n prime numbers, where n is odd, always odd, even or neither?	Even
4	Solve for x: $9^x = 27$	3/2
Person2		
1	How many integer values x satisfy the inequality $19 \leq x \leq 55$?	37
2	The odds of winning a certain game is 3 to 4. What is the probability of winning the game?	3/7
3	The surface area of a cube is increased from 9 units^2 to 25 units^2. What is the ratio of the new volume to the old volume of the cube?	125:27
4	What are all the possible remainders when a perfect square, greater than 1, is divided by 4?	0,1
Person 3		
1	What is the value of x which satisfies the equation $9x + 8 = 16$?	8/9
2	What is the units digit of 3^7?	7
3	What is the smallest positive integer n that when divided into 75, leaves a remainder of 2?	73
4	I have twice as many nickels as dimes. If the value of the nickels is $8.00, what is the value of my dimes, in dollars?	($)8.00
Person 4		
1	What is 7 written in base 3?	$21_{(3)}$
2	Biff can mow a lawn in 1 hour by himself. Eho can mow the same lawn in 2 hours by himself. How many minutes would it take them to mow the lawn together?	40(minutes)
3	In a jar is 7 slips of paper with numbers 2 through 8 written on them. One slip of paper is drawn and then replaced. Then another slip of paper is drawn. What is the probability that the sum of the numbers drawn is 8?	5/49
4	What is the geometric mean between 25 and 16?	20

"Math is Cool" Championships-2000-01

February 10, 2001

Pressure Round Contest, High School

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.

1. If the circumference of a circle is increased by 20, how much would the radius increase?

2. Evaluate: $\sum_{n=1}^{100} \frac{1}{n^2 + 5n + 6}$

3. If there are 220 ways to make a committee of 3 people from the employees of Sampson's Doughnut Shop, how many employees are there?

4. $f(x) = x^4 + x^3 + x^2 + x - 1$.
If it is given that $f(x)$ has at least 1 complex non-real root, how many negative real roots does it have?

5. Solve for x over the reals:
$9^x+9^2=10\cdot3^{x+1}$

"Math is Cool" Championships-2000-01

February 10, 2001

Individual Multiple Choice Contest, High School

1. On what domain(s) does the function $f(x) = x^3 - 3x$ have an inverse that is a function?

a) $x \geq \frac{1}{2}$ b) $x \leq \frac{1}{2}$ c) x > 2 or x < -2 d) all of the above e) none of the above

---

2. If the circumference of a cylinder is doubled, then the volume:

a) is halved b) remains the same c) doubled d) quadrupled e) none of the above

---

3. If a function defined on the entire real line is invertible, i.e. has an inverse that is a function, it must be

a) even b) odd c) monotonic d) None of the above

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4. If circle A has an irrational radius, then

a) the area of A must be irrational b) the area of A must be rational
c) the circumference of A must be irrational d) the diameter of A can be rational e) none of the above

---

5. If $1/(2 + i)$ is one root of a rational polynomial, what must be another root of the polynomial? Express your answer in the form $a + bi$ where a and b are reals.

a) $0.4 + 0.2i$ b) $4 + 2i$ c) $.8 + .4i$ d) $-0.4 + 0.2i$

---

6. Which of the following lines always divides a triangle into two parts of equal area?

a) a median b) an angle bisector c) a perpendicular bisector d) all of the above e) none of the above

---

7. The horizontal asymptotes of $f(x) = \frac{1 - |x|}{x}$ are given by:

a) $y = 1$ b) $y = -1$ c) $x = 0,\ x = 1,\ x = -1$ d) $y = 0$ e) $y = 1,\ y = -1$

---

8. If $\frac{f(x_1)}{f(x_2)} = f\left(\frac{x_1}{x_2}\right)$ for all real numbers x_1 and x_2 (except those for which $x_2 \neq 0$ and $f(x_2) \neq 0$, which of the following could define f ?

a) $f(x) = \frac{1}{x}$ b) $f(x) = x^2 + 3$ c) $f(x) = x + 1$ d) $f(x) = \ln x$ e) $f(x) = e^x$

---

9. $\frac{\ln(x^3 e^x)}{x} =$

a) $\frac{3(\ln x + e^x)}{x}$ b) $\ln(x^3 e^x - x)$ c) $\ln x^2 + 1$ d) $\frac{3\ln x + x}{x}$ e) $\frac{3\ln x}{x}$

"Math is Cool" Championships-1999-00

February 12, 2000

Team Contest, High School 9-10

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.

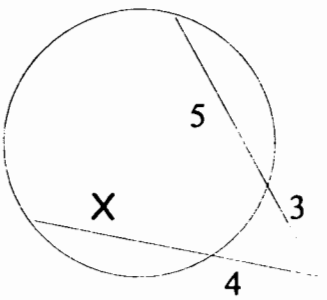
1. The areas of three sides of a rectangular box are 10, 12 and 15, respectively. What is the volume of the box?

2. In an arithmetic progression the first term is 8 and the eleventh term is 168. Find the fifteenth term.

3. How many ways can a committee of 3 be chosen from Ryan, Josh, Rhonda, Katie, Jude, Ingrid and Beth if Josh and Katie cannot both be on the committee?

4. Simplify $8^{\log_2 \pi}$

5. Find the value of x in the diagram.



6. Evaluate: $\sum_{n=1}^{10} (n^2 + 2n + 2)$

7. Mark knows the time is between 3p.m. and 4 p.m. If Mark notices that on a standard clock face, the hour and minute hands make a 180° angle, what time is it, to the nearest second?

8. If $x^2 + 3xy + y^2 = 4$, solve for x in terms of y.

9. What is the equation of the parabola that has a directrix of y = 0 and has its focus at (0,2)?

10. Given a quadrilateral ABCD inscribed in a circle with side AB extended beyond B to point E, if M∠BAD=92°, M∠ADC=69°. Find the measure of ∠EBC in degrees.

"Math is Cool" Championships-2000-01

February 10, 2001

Individual Contest, High School

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.

1. What is the slope of the line passing through (4, 8) and (-2, -7)?

2. How many positive integers less than 100 have rational square roots?

3. What three consecutive positive integers may not be the side lengths of an acute triangle?

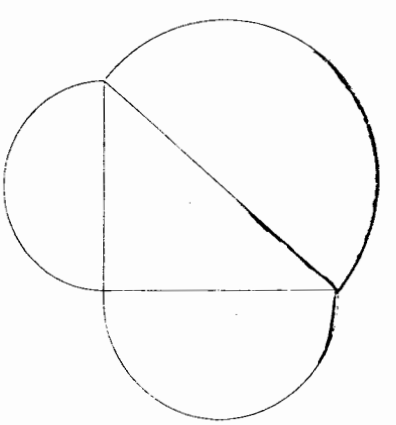
4. What is the maximum number of sides, of rational length, an isosceles right triangle can have?

5. Factor completely over the set of integers: $54x^3 - 2y^3$

6. Reduce to lowest terms: $\dfrac{x^3 - 2x^2 + 5x - 10}{x^3 - 8}$

7. x is an integer between 100 and 150. When x is multiplied by a positive integer larger than 20, the result is always divisible by 31. What is x?

8. A right triangle has a hypotenuse of 5 and a short leg of 3 with semicircles on each side. What is the ratio of the area of the small semicircle to the large semicircle?



9. The members of a Math Team were going to fly commercially to a competition at a total cost of $2420 which was to be divided equally among the members. At the last minute, two of the members decided to fly their own private planes. The cost to the remaining members increased $11 each. How many members flew commercially?

10. A contractor hires two bulldozers to clear the trees from a 20-acre tract of land. One works twice as fast as the other. It takes them three days to clear the tract working together. How many days would it take the slower dozer to clear the land working by itself?

11. Bill and Robert want to slice a circular pizza using straight line cuts. What is the maximum number of separate pieces that can be obtained with 4 cuts?

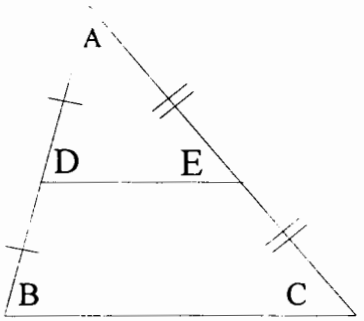
12. How many sides does a regular polygon have if the sum of the exterior angles of the polygon is exactly 1/3 the sum of the interior angles?

13. If the area of triangle ABC is 20, what is the area of triangle ADE?

14. In a right triangle with hypotenuse 14, what is the length of the median to the hypotenuse?

15. In a given rhombus, one diagonal is 3 times as long as the other. If the side length of the rhombus is 10, what is the length of the longer diagonal?

16. What is the largest area (in square feet) that can be enclosed by 100 feet of fence?



17. Josh is sitting at a switch board containing 100 switches, numbered 1 through 100. The switches can be in either an on or off position. Initially, all the switches are off. First, Josh flips every switch. Then, he flips all switches that are multiples of 2 (i.e., 2, 4, 6, . . . 100). He then proceeds to flip the multiples of 3, 4, 5, 6 . . .100. After he flips the multiples of 100, he stops. How many switches are in the "on" position when he stops?

18. Solve for x: $3^{4x-2} = 27^{2x-1}$.

19. What real value(s) of x satisfy: $\frac{3^{(x^2)}}{(3^x)^2} = 3$

20. If $3^y=7$ and $7^x=3$, what is $x \cdot y$?

21. Solve for x: $x^{\log x} = 10^4$

22. What is the coefficient of the x^{24} term in the expanded and simplified form of $(2x^2+x^3)^{10}$

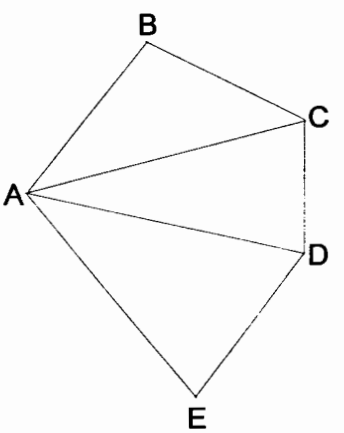
23. Mr. Sampson is distributing 10 brownies to Beth, Tasha and Jenny. How many ways can he do this if no one may get more than 5 brownies and everyone has at least one?

24. How many ways can the letters in the word MATH be rearranged (permuted) such that no letter maintains its original position, e.g. M cannot be the first letter?

25. How many integer values are not a solution to the equation $\frac{x^2-9}{x^2}>0$

26. Given a regular 20-sided polyhedron, what is the number of vertices minus the number of edges?

27. $\angle ABC=100°$
$\angle BCD=110°$
$\angle CDE=145°$
$\angle DEA=80°$
$\angle CAD=35°$
What is the measure of $\angle BAC=$ in degrees?



28. What is the ten's digit of the following sum?
$\sum_{n=1}^{20}(2n)!$

29. What is the shortest distance between the line $y=\frac{x}{2}-4$ and the origin?

30. A number is considered a "quad" if it can be represented by four repeated digits in some base. For example, 1111 and 2222 are both quads written in base 10, and 255 is a quad because it is 3333 in base 4. What is the largest quad less than 2001_{10}?

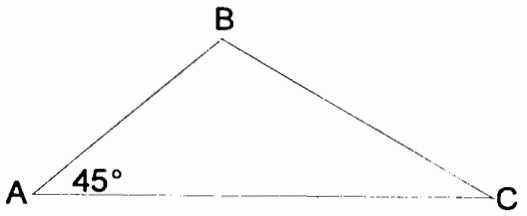
31. If 5 numbers are chosen randomly from the interval (0,1), what is the probability the median of the 5 numbers is less than 0.5?

32. Given:

$\overline{AC} = 1$

$\overline{BC} = \frac{\sqrt{6}}{3}$

$\overline{AB} < \overline{AC}$



$\angle BAC = 45°$

What is the measure of $\angle ABC$= in degrees?

33. If Robert rolls a fair, 12-sided die and Bill rolls three, fair, 4-sided dice, what is the probability that Robert's roll equals the sum of Bill's rolls?

34. In what base system are there exactly 990,000 (base 10) different 3 digit numbers?

35. Solve for all real values of x that satisfy $\log_{27} x \cdot \log_{16} x = \log_{27} 16$

36. For what positive, real value(s) of x and y is the following true?

$$\frac{5 + x + y^2}{3} = \left(5xy^2\right)^{\frac{1}{3}}$$

37. If an isosceles triangle has two angles of 75° and two sides of length 1, what is the length of the other side?

38. Nadav's calculator always displays numbers to the nearest thousandth's place. If Nadav uses the calculator to generate a number on the interval [0,1], what is the percent chance that the calculator outputs 1.000 after rounding?

39. A binary word consists of only zeroes and ones. How many 6 digit binary words exist that do not contain 3 consecutive zeroes?

40. The Fibonacci sequence is defined by the recurrence relation $a_n = a_{(n-1)} + a_{(n-2)}$ with $a_1 = 1$, $a_2 = 1$. It is also known that a closed formula for a_n is:

$$a_n = \frac{\sqrt{5}}{5}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

It is also known that a_n is the closest integer to the expression $x \cdot y^n$ for some real numbers x and y. Determine x and y.

"Math is Cool" Championships-2000-01

February 10 & 16, 2001

High School

College Knowledge Bowl Questions #1		
1	Find the distance between the points (-1, -3) and (4, 9).	13
2	If two 6-sided die are rolled, what is the probability that their sum is a prime number?	~~1/3~~ 5/12
3	Find the area of an equilateral triangle with side length equal to 10.	$25\sqrt{3}$
4	Determine the values of k so that $kx^2 + 4x - 3 = 0$ will have two non-real, complex solutions.	k< -4/3
5	Two planes, which are 2475 miles apart fly toward each other. Their speeds differ by 75 mph. If they pass each other in 3 hours, what is the speed of the slower plane?	375(mph)
6	How many ways can 6 charms be arranged on a bracelet without a clasp?	60
7	Evaluate: $2 - 1 + \frac{1}{2} - \frac{1}{4} + \ldots$	4/3
Number 8 is an extra question. Only use it if needed.		
8	What is the smallest, whole number base, such that a number written in that base is divisible by 3 if and only if the sum of the digits of the number is also divisible by 3 in the same base?	4

"Math is Cool" Championships-2000-01

February 10 & 16, 2001

High School

College Knowledge Bowl Questions #2

1	If the surface area of cube A is twice as large as the surface area of cube B, what is the ratio of the volume of cube A to cube B?	$2\sqrt{2}:1$ or $\frac{2\sqrt{2}}{1}$
2	What is the domain of $x^2 + 2x + y^2 = 3$?	$-3 \le x \le 1$
3	How many diagonals can be drawn in a convex 11-sided polygon?	44
4	Evaluate: $643_7 + 132_7$ in base 7	$1105_{(7)}$
5	Given the alphabet where $a = 5^0$, $b = 5^1$, $c = 5^2$, ... Solve for n in the equation: math = 5^n	~~42~~ 38
6	Simplify: $i^{291} + i^{263} + i^{-3}$	$-i$
7	What are the coordinates of the point on segment AB that is three-fourths of the way from A to B, where A has coordinates (1,5) and B has coordinates (5,13)?	(4,11)

Number 8 is an extra question. Only use it if needed.

8	Angles x and y are measured in radians. They are between 0 and 2π and terminate in quadrant 3. If the measure of each is an integer, find their sum.	8

"Math is Cool" Championships-2000-01

February 10 & 16, 2001

High School

College Knowledge Bowl Questions #3		
1	What is the negative value of k that makes $x^2 + kx + 36$ a perfect square trinomial?	-12
2	On a coed soccer team, the ratio of females to males is 5 to 4. When three females were not able to make it to a game, the ratio of females to males for that game was 1 to 1. How many males are on the team?	12
3	Find the equation of the line parallel to $2x + 5y = 10$ and passing through (-3, 2) in the form $Ax + By = C$, where A, B, and C are integers and A is positive.	$2x + 5y = 4$
4	Find the distance from (5,0) to the center of the circle $x^2 + y^2 - 4x + 8y = 20$.	5
5	How many positive integers less than 1000 are not divisible by 2, 3 or 4?	333
6	In base 6, what is the greatest common factor of all even-digit palindromes. Express your answer in base 6.	$11_{(6)}$
6	Evaluate $\binom{9}{3}$	84
7	A trapezoid has a base of length 12 and a median of length 16. What is the length of the other base?	20
Number 8 is an extra question. Only use it if needed.		
8	How many diagonals can be drawn in a convex polygon that has ten sides?	35

"Math is Cool" Championships -- 2000-01

High School - February 10, 2001

School Name_____ Team #_____

Proctor Name_____ Room #_____

Key

Full Name:_____

1st Score

Out of 40

Individual Contest - Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer		
1	5/2		
2	9		
3	1, 2, 3		
4	2(sides)		
5	$2(3x-y)(9x^2+3xy+y^2)$		
6	$\frac{x^2+5}{x^2+2x+4}$		
7	124		
8	9:25		
9	~~22~~(members) 20		
10	9(days)		
11	11(pieces)		
12	8(sides)		
13	5		
14	7		
15	$6\sqrt{10}$		
16	2500/π (sq ft)		
17	10(switches)		
18	1/2		
19	$1+\sqrt{2},\ 1-\sqrt{2}$		
20	1		

	Answer		
21	100 or 1/100		
22	13440		
23	18(ways)		
24	9(ways)		
25	7		
26	-18		
27	30(°)		
28	6		
29	$8\sqrt{5}/5$		
30	2000		
31	½ or .5		
32	120(°)		
33	1/12		
34	(base)100		
35	16, 1/16		
36	$x=5, y=\sqrt{5}$		
37	$\frac{\sqrt{6}-\sqrt{2}}{2}$ or $\frac{2}{\sqrt{6}+\sqrt{2}}$ or $\sqrt{2-\sqrt{3}}$		
38	0.05(%)		
39	44(words)		
40	$x=\sqrt{5}/5,\ y=(1+\sqrt{5})/2$		

"Math is Cool" Championships -- 2000-01

High School - February 10, 2001

School Name_____ Team #_____

Proctor Name_____ Room #_____

Key

Individual Multiple Choice Contest-Score Sheet

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

1st Score

Out of 18

DO NOT WRITE IN SHADED REGIONS

	Answer		
1	c		
2	d		
3	c		
4	e		
5	a		
6	a		
7	e		
8	a		
9	d		

"Math is Cool" Championships -- 2000-01

High School - February 10, 2001

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Team Contest-Score Sheet

1st Score

Out of 10

DO NOT WRITE IN SHADED REGIONS

	Answer		
1	$30\sqrt{2}$		
2	232		
3	30(ways)		
4	π^3		
5	2		
6	515		
7	3:49:05		
8	$\frac{-3y \pm \sqrt{5y^2+16}}{2}$		
9	$y = \frac{x^2}{4} + 1$		
10	69(°)		

"Math is Cool" Championships -- 2000-01

High School - February 10, 2001

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Pressure Round - Score Sheet

	Answer		
1	$10/\pi$		
2	100/309		
3	12		
4	1		
5	1,3 (order doesn't matter)		