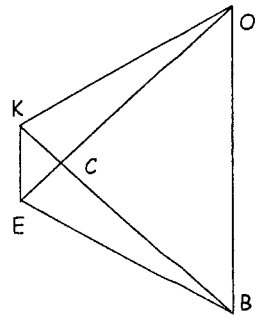
"Math is Cool" Championships-2001-02

February 15, 2002

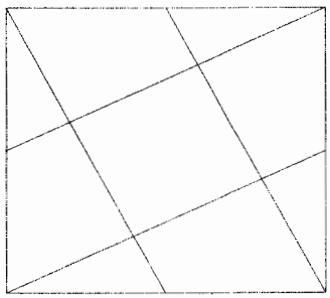
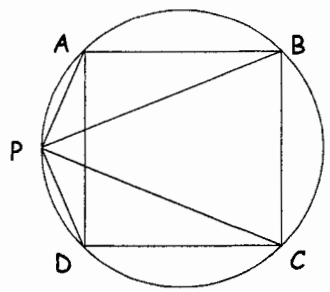
Individual Contest, High School

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.

1	What is the area of the shape with vertices (0, 0), (2, 7), (6, 7) and (10, 0)?
2	What is the units digit of the product of the first 20 prime numbers?
3	What is the slope of the line perpendicular to the line passing through (2, -3) and (-2, 7)?
4	The members of a Math Team were going to float down the St. Joe River at a total cost of $300, which was to be divided equally among the members. At the last minute, two of the members decided not to go. The cost to the remaining members increased $5 each. How many members ended up going on the trip?
5	What is the distance between the points (1, 2, 3) and (-3, 3, 1)?
6	What is the perimeter of a semicircle with area 2π?
7	Reduce to lowest terms: $\frac{x^2+x-6}{x+3}$, $x \neq -3$
8	How many integer values are not a solution to the inequality $x^2 - x - 6 > 0$?
9	If a regular polygon has 35 diagonals, how many sides does it have?
10	Factor completely over the set of integers: $x^3 + 27$
11	A natural number n has exactly 4 positive factors. Find the smallest possible value of n.
12	Let n be the smallest positive number such that the sum of the first n prime numbers is a perfect square. What is n?
13	What is the units digit of 2^{2002}?
14	A hamburger launched from a slingshot follows the parabolic path $y= -x^2 + 100x +24$, where x is the time in seconds from the launching and y is the height in feet. How high above the ground was the hamburger when it was initially launched?

15	What is the vertex of the parabola $y = x^2 - 2x + 1$?
16	A turtle is on a one mile journey. Each day he travels 1/4 of the remaining distance. On which day will he pass the 3/4 of a mile point?
17	What is the center of the circle with the equation $x^2 - 4x + y^2 + 8y = 1{,}000{,}000$?
18	What is the longest diagonal of a cube with volume 216?
19	Solve for x: $5^{x-2} = 25^{3x-5}$
20	Solve for x: $\log(x) + \log(2x) = \log(18)$
21	What is the shortest distance between the line $y = x + 7$ and the origin?
22	What is the ratio of the area of a circle to the area of an inscribed equilateral triangle?
23	How many ways can the letters in the word "POWEI" be rearranged such that no letter maintains its original position, e.g., P cannot be the first letter?
24	How many zeros does 100 factorial end in?
25	An equilateral triangle and an regular hexagon have the same perimeter. What is the ratio of the triangle's area to the hexagon's area?
26	Two cards are placed in a hat. One has two black sides and the other has a black side and an orange side. If Joel draws a card and sees that one side is black, what is the probability the other side is black?
27	In a nineteen game series, where both teams have an equal probability of winning, find the ratio of the probability of the series ending in 18 games versus 19 games.
28	KOBE is a trapezoid with KE parallel to OB, and KE:OB=1:4. Given OE=20, what is CE? 
29	How many ways can 8 people be divided into 2, non-empty, groups if there are no other restrictions on the sizes of the groups?
30	How many positive numbers less than 150 have an odd number of factors?

Challenge Questions

31	What is the volume of a tetrahedron with sides of length 4?
32	Evaluate: $\sum_{n=0}^{n=2002} i^{(n^2)} + i^n$
33	How many regular polygons can tesselate the plane?
34	The area of the larger square is 1. What is the area of the smaller square inside created by connecting the midpoints and vertices of the larger square as shown? 
35	Find the geometric mean of the positive factors of 100.
36	What is the radius of a sphere inscribed in a tetrahedron with vertices (0,0,0), (4,0,0), (0,-4,0) and (0,0,2)?
37	Square ABCD is inscribed in a circle, with AB = $5\sqrt{2}$. If P is on the minor arc AD, find $(AP)^2 + (BP)^2 + (CP)^2 + (DP)^2$. 
38	Let d(n) be the sum of the digits of n, where n is a positive integer. How many solutions does the following equation have? n + d(n) + d(d(n)) = 2002
39	In how many distinct orders can 5 people finish a race if ties are allowed?
40	How many ways are there to color the 6 faces of a cube using 6 different colors? Each face has exactly one color.

"Math is Cool" Championships-2001-02

February 15, 2002

Individual Multiple Choice Contest, High School

1	Solve for x: $3(5(2x + 3) - 2(4x + 7) + 3(4x - 5)) = 6(2(x - 3) + 3(5x - 1))$ (a) 4 (b) 5 (c) 6 (d) 7 (e) answer not given
2	If $y = \frac{x^2}{z}$ and $x \neq 0$, then $\frac{1}{x^2} =$ (a) yz (b) $\frac{y}{z}$ (c) $\frac{z}{y}$ (d) $y - \frac{1}{z}$ (e) $\frac{1}{yz}$
3	Through how many degrees does the minute hand of a clock turn from 3:10 a.m. to 3:25 a.m. of the same day? (a) 15° (b) 30° (c) 45° (d) 60° (e) 90°
4	If $a \times b \times c = 72$, where a, b, and c are integers and $a > b > c > 1$, what is the greatest possible value of a? (a) 12 (b) 18 (c) 24 (d) 36 (e) 72
5	What is the area of a right triangle whose perimeter is 36 and whose sides are x, x + 3 and x + 6? (a) 27 (b) 54 (c) 81 (d) 108 (e) 135
6	The largest possible domain of $f(x) = \sqrt{4 - x^2}$ is (a) $-2 \leq x \leq 2$ (b) $-2 \leq x \text{ or } x \geq 2$ (c) $-2 < x \text{ or } x > 2$ (d) $-2 < x < 2$ (e) $x \geq 2$
7	If f(x) is an even function and g(x) is an odd function, then the product of f(g(x)) and g(f(x)) must be: (a) even (b) odd (c) neither even nor odd (d) invertible (e) none of the above
8	If $f(x + c) = f(x) \cdot f(c)$ for all real numbers x and c and $f(0) \neq 1$, then $f(0) =$ (a) 1 (b) 0 (c) 0 and 1 (d) -1 (e) $\sqrt{2}$
9	What is the smallest positive integer k>2 such that given k people, there must exist either 3 people who all know each other, or 3 people who all don't know each other. Assume that for every pair of people, either both people know each other, or neither person knows the other. (a) 4 (b) 5 (c) 6 (d) 7 (e) No k exists

"Math is Cool" Championships-2001-02

February 15, 2002

Team Contest, High School

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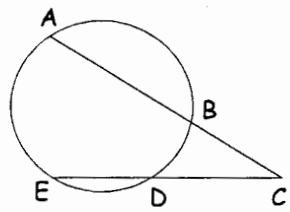
1	Find the mean of the median and mode of the following data set. {16, 4, 10, 9, 7, 7, 4, 4}
2	If Sampson drives from Spokane to Coeur d'Alene at 50 mph and returns at 100 mph along the same route, what is his average speed in mph?
3	If a 3-digit number is chosen at random, what is the probability that the hundreds digit is greater than the units digit?
4	If 8 students can write 600 questions in 3 days, how many questions can 20 students write in 5 days?
5	If a is directly proportional to b and inversely proportional to the square of c, and a = 10 when b = 50 and c = 4, what is a when b = 10 and c = 16?
6	A bag contains 16 gold marbles, 12 silver marbles, and 12 bronze marbles. If two marbles are drawn with replacement, what is the probability that exactly one is gold?
7	AB=6, BC = 4, ED = 3. What is CD? 
8	Find the sum of all the diagonals of a regular hexagon with side length 6.
9	Simplify: $\frac{2002^3 - 29^3 - 1973^3}{2002 \times 1973 \times 29}$
10	Find the midpoint of (4, 7) and (-8, 11).

"Math is Cool" Championships-2001-02

February 15, 2002

Pressure Round Contest, High School

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1	Find the value of x for which the points (-1,-14), (-9,1), (6,9) and (x,-6) form the vertices of a square.
2	What is the sum of the first 10 perfect squares?
3	How many solutions does the equation $a^2 + b^2 = c^2$ have if a, b, c are non-zero integers and a^2, b^2, c^2 are all less than 250?
4	What is the maximum number of times that a parabola can intersect a circle?
5	The triple (1,3,q) has the property that the product of any pair of members from the triple is one less than a perfect square. What is the smallest possible positive integer value of q, other than 1?

"Math is Cool" Championships-2001-02

February 15, 2002

Mental Math, High School

Person 1		
1	What is the sum of the first 8 positive odd numbers?	64
2	Express 11 in base 2.	$1011_{(2)}$
3	What is the number halfway between 1/3 and $\frac{1}{2}$?	5/12
4	What is the probability of rolling a sum of 5 with two six-sided dice?	1/9
Person2		
1	For how many integer values of x is the quantity (x - 3) squared less than or equal to 0?	1
2	Evaluate: 89 x 81.	7209
3	What is the volume of a sphere with radius 3?	36π
4*	What is the slope of the line described by the equation 2y - 6x = 10?	3
Person 3		
1	What is the area of the region described by the equation $x^2 + y^2 \leq 2002$?	2002π
2	What is the area of a triangle with sides of length 6, 8 and 10?	24
3	Evaluate: $i^{20} + i^0 + i^2$	1
4	What is the probability of drawing two orange marbles without replacement from a bag containing 10 orange marbles and 6 black marbles?	3/8
Person 4		
1	The radius of a cone is doubled and its height is tripled. What is the ratio of the old volume to the new volume?	1/12
2	Solve for x: 3x + 8 = -2x - 13	-21/5
3	How many factors greater than 1 of 30 are relatively prime to 3?	3
4	If a cube with side length 3 is painted and then divided into 27 unit cubes, how many of the resulting cubes will have exactly 2 faces painted?	12

"Math is Cool" Championships-2001-02

February 15, 2002

High School

College Knowledge Bowl Questions #1		
1	The ratio of cement to sand in a 90 pound bag of dry mix is 1 to 4. Find the number of pounds of sand in the bag.	72
2	What is the larger of two consecutive integers whose product is 702?	27
3	The sum of the first 14 terms of an arithmetic sequence is 2002 and the difference is 2. Find the first term.	130
4	Solve for the positive root of the equation $35x^2 + 11x - 6 = 0$.	2/7
5	How many ways can a six question true-false exam be answered? Assume that no questions are omitted.	64
6	Three points that are not on a line determine three lines. How many lines are determined by seven points, no three of which are on a line?	21
7	A florist wants to arrange a dozen flowers consisting of two varieties: carnations and roses. Carnations cost 75¢ each and roses cost $1.50 each. How many roses should the florist use so that the arrangement will cost $12.00?	4
Number 8 is an extra question. Only use it if needed.		
8	Jude can run 8 miles per hour, and Jina can run 11 miles per hour. If Jina gives Jude a 10 minute head start, how long, in hours, will it take her to catch Jude?	4/9 [hours]

"Math is Cool" Championships-2001-02

February 15, 2002

High School

College Knowledge Bowl Questions #2

1	A student has taken 3 quizzes, and obtained scores of 70 65 and 60. The quizzes account for two-thirds of her grade, with the rest of her grade being determined by the final exam. What grade must the student earn on the final in order to average 70?	80
2	A car has a highway fuel-economy rating of 50 miles per gallon. A trip that consists of 200 miles of city driving and 1200 miles of highway driving used 29 gallons of gasoline. What is the city fuel economy rating, in miles per gallon, of the car?	40 [mpg]
3	A forest fire is burning in a circular pattern. If the radius of the circle increases by 50 feet, the area of the fire increases by $27{,}500\pi$ square feet. What was the radius, in feet, of the burning area before the increase?	250 [feet]
4	A quantity F varies directly as a quantity G. If F = 5 when G = 3, find the value of F when G = 5.	25/3 *or* 8⅓
5	Solve for x in the following equation when y is 2. $5xy - 3x + 4y = 7y^2 - 4xy + 2$	22/15 *or* 1 7/15
6	What is the area of a triangle with sides of length 5, 7, 8?	$10\sqrt{3}$
7	What is the smaller of two integers whose sum is 11 and whose product is as large as possible?	5
	Number *8* is an extra question. Only use it if needed.	
8	How many integer values of x satisfy the inequality $12x^2 + 12x < 0$.	0

"Math is Cool" Championships-2001-02

February 15, 2002

High School

College Knowledge Bowl Questions #3

1	A certain professor grades his tests in such a way that for every correct answer, the student gets 1 point, but for every incorrect answer the student loses two points. If a student gets a score of 55 on a 100-question test using this method of grading, how many questions did he answer correctly? Assume the student attempted every question.	85 [correct]
2	A hose can fill a swimming pool 10 times faster than the drain can empty the pool. If the hose can fill the pool in 14 hours while the drain is open, how long, to the nearest hour, does it take to fill the pool when the drain is closed?	13 [hours]
3	If the major axis of an ellipse has length 5 and the minor axis has length 4, what is the distance between the foci?	3
4	Trisha has 5 blue marbles, 5 red marbles and 5 white marbles. If Trisha wants to choose 4 marbles, how many ways can she do this?	15
5	Find the positive value of k for which the graph of the equation $y= x^2 + kx + 3k + 7$ is tangent to the x-axis.	14
6	What is the sum of the roots of the equation $x^6 - 13x^5 + 6x^4 - 7x^3 - 29x^2 + x + 16 = 0$	13
7	Find the largest value of x such that the distance between the points (x, -3) and (2,3) is equal to 10.	10
	Number 8 is an extra question. Only use it if needed.	
8	Find the sum of every third integer, starting with 17 and ending with 92.	1417

"Math is Cool" Championships -- 2001-02

High School - February 15, 2002

School Name_____ Team #_____

Proctor Name_____ Room #_____

Key

Full Name:_____

1st Score

Out of 40

Individual Contest - Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer		
1	49		
2	0		
3	2/5		
4	10 [members]		
5	$\sqrt{21}$		
6	$4+2\pi$		
7	$x-2$		
8	6		
9	10 [sides]		
10	$(x+3)(x^2-3x+9)$		
11	6		
12	9		
13	4		
14	24 [feet]		
15	(1,0) *or* x=1, y=0		
16	5 [days]		
17	(2,-4) *or* x=2, y=-4		
18	$6\sqrt{3}$		
19	8/5		
20	3		

	Answer		
21	$7\sqrt{2}/2$		
22	$4\pi / 3\sqrt{3}$ This is a ratio		
23	44		
24	24 [zeros]		
25	2/3		
26	2/3		
27	1:1		
28	4		
29	127		
30	12		
31	$16\sqrt{2}/3$		
32	1002 + 1002i		
33	3		
34	1/5		
35	10		
36	$2\sqrt{6}-4$		
37	200		
38	0		
39	541		
40	30 [ways]		

"Math is Cool" Championships -- 2001-02

High School - February 15, 2002

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Individual Multiple Choice Contest-Score Sheet

1st Score

Out of 18

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer		
1	E		
2	E		
3	E		
4	A		
5	B		
6	A		
7	A		
8	B		
9	C		

"Math is Cool" Championships -- 2001-02

High School - February 15, 2002

School Name_____ Team #_____

Proctor Name_____ Room #_____

Key

Team Contest-Score Sheet

1st Score

Out of 10

DO NOT WRITE IN SHADED REGIONS

	Answer		
1	11/2 *or* 5 1/2		
2	200/3 *or* 66 2/3 [mph]		
3	1/2		
4	2500 [questions]		
5	1/8		
6	12/25		
7	5		
8	$36\sqrt{3} + 36$		
9	3		
10	(-2, 9)		

"Math is Cool" Championships -- 2001-02

High School - February 15, 2002

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Pressure Round - Score Sheet

	Answer		
1	14		
2	385		
3	64		
4	4		
5	8		