"Math is Cool" Championships - 2002-03

9th - 12th Grade - November 1, 2002

School Name_____ Team #_____

Proctor Name_____ Room #_____

Final Score:

Full Name:_____

1st Score

Out of 40

Individual Contest - Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1			
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	Answer	1 or 0	1 or 0
21			
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"Math is Cool" Championships-2002-03

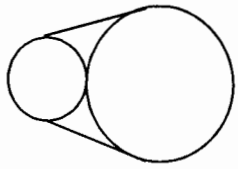
9^{th} - 12^{th} Grade - November 1, 2002
Individual Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the green cover sheet.

1	A triangle has angles of 45 degrees and 30 degrees. If the side length opposite the 45 degree angle is 10, what is the length of the side opposite the 30 degree angle?
2	What is the length of the diagonal of a rectangle that has an area of 32 and the length that is twice the width?
3	Find "a" so that the slope between the two points (3, 15) and (7, a) is 2.
4	A sample of 20 people is chosen randomly from 500 people. 15 out of 20 said they practiced math 7 nights a week. How many people would you expect practice math 7 nights a week out of the 500?
5	Yes or no: Is the converse to the following statement true. If I live in Washington, then I live in Seattle.
6	Solve for x: $8-2(3-(4x+3))+2 = 7(8-2(3x-4))$
7	Find "a" so that the mean of the following set of data {1,8,7,a} is 4?
8	The sum of three consecutive integers is 48. What is the smallest of the three integers?
9	If $x + y + z = 7$ and x, y , z are distinct positive integers, what is the largest value one of the integers can be?
10	How many of the following numbers are rational? $e \quad \frac{1}{2} \quad \frac{1}{9} \quad \pi \quad \sqrt{2} \quad \sqrt{9} \quad i \quad 2^3 \quad 9^3$
11	Solve for x: $25^{7x+1} = 125^{8x+3}$
12	Evaluate: $\left(\frac{64}{27}\right)^{\frac{-3}{2}}$
13	Find "b" so that $5x^2 + bx + 8$ has one real solution.

14	Find the sum of the integers that are not part of the solution of $x^2 - 15x + 56 > 0$.
15	Find the coefficient of the x^3y^5 term in the expanded and simplified form of $(2x - 3y)^8$.
16	Evaluate: $\sqrt{8 + \sqrt{8 + \sqrt{8+\ldots}}}$
17	Solve for x: $25^{x+1} + 6 \cdot 5^{x+1} - 7 = 0$
18	True/False: Given 7 points on a line segment of length 1, there must be at least two points separated by no more than a length of 1/6.
19	What are the coordinates of the point on the line 3x-y=-2 which is collinear with (-2,-15) and (3,15)?
20	An equilateral triangle is circumscribed by a circle such that the three vertices of the triangle lie on the circle. What is the area of the area trapped between the circle and the triangle if the area of the triangle is $\sqrt{3}$?
21	How many subsets does the set {A,B,C,D} have?
22	Evaluate: $\sum_{r=0}^{5} {}_5C_r$
23	A perfectly spherical onion with diameter 16 units is sliced so that the area of the cross-section is 49π units^2. How far from the center was the slice made?
24	A planet Silas K-13, is perfectly spherical. Tim stands on the roof of a skyscraper of height 1 mile and the horizon is 27 miles away from him. Find the radius of the planet. Ignore Tim's height.
25	Your younger brother has decided to open a lemonade stand. It is a hot day, and he has determined that if he charges p cents a cup, he will get $480-4p$ customers. What value of p will cause brother to make the most money?
26	What is the remainder when 2^{2002} is divided by 7?
27	If $f(x) = \frac{x+2}{x-1}$, what is the inverse function $f^{-1}(x)$?
28	If I randomly choose a four digit palindrome, what is the probability that it is divisible by 15?
29	What is the smallest possible sum of n and k. ${}_{12}C_2 + {}_{12}C_3 = {}_nC_k$

Challenge Questions

30	If for two distinct positive numbers x and y, $\frac{x}{y} = \frac{x^2 + y + x}{4x + y} = \frac{y - x^2}{y - x}$ find the numerical value of $\frac{x}{y}$.
31	If the line $y = mx + 1$ intersects the ellipse $x^2 + 4y^2 = 1$ exactly once, what is the value of m^2?
32	1, a, b, 10 is a sequence in which 1, a, b forms a geometric sequence while a, b, 10 forms an arithmetic sequence. Find a+b?
33	A string is wrapped tightly around two circular objects as shown. The centers of the two circular objects are 8 units apart. The radii of the two circular objects are 2 and 6. How many feet long does the string need to be to go around once? 
34	If a and b are positive real numbers and each of the equations $x^2 + 3ax + b = 0$ and $x^2 + bx + 3a = 0$ has real roots, then find the smallest value of a + b.
35	List the three complex cube roots of 8i in a + bi form.
36	In a rectangle ABCD, a point P exists inside of the rectangle such that the following measurements hold. AB = 34, AD = 20, AP = 16, and BP = 30. Find DP.
37	Write $\sqrt{29 - 12\sqrt{5}}$ in the form $a + b\sqrt{5}$.
38	In the following addition problem; A, B, C and D each represent a distinct digit. What is the largest possible value of A? $\begin{array}{r} ABC \\ +CD \\ \hline DBA \end{array}$
39	A triangle has sides 7, 9, and x with an area of $6\sqrt{5}$. What is x?
40	How many roots does the following equation have? $\sin x = \frac{x}{100}$

"Math is Cool" Championships - 2002-03

9th - 12th Grade - November 1, 2002

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

Individual Multiple Choice Contest-Score Sheet

1st Score

Out of 18

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer		
1			
2			
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"Math is Cool" Championships-2002-03

9^{th} - 12^{th} Grade - November 1, 2002
Individual Multiple Choice Contest

1	How many 3-digit numbers are multiples of 2, 3 or 5? a) 660 b) 661 c) 662 d) 663 e) none of the above
2	When Tom rides his bike to work, his course is mostly downhill, and so his average speed is 15mph faster than when he rides home after work (uphill). Tom's total commuting time each day is 20 minutes, which is 10 minutes longer than if his commute were downhill both ways. How far does Tom commute each day? (in miles) a) 4 b) 9/2 c) 15/4 d) 17/4 e) none of the above
3	Express $0.08\overline{3}$ as a fraction in lowest terms. a) 83/100 b) 1/99 c) 1/11 d) 1/12 e) none of the above
4	What is the largest integer for which $\frac{2}{x}$ is larger than $\frac{3}{19}$? a) 9 b)11 c)15 d) 18 e) none of the above
5	Mr. Sampson has 9 identical gumballs he plans to give to the 4 members of the Math Team who showed up for 3:00 AM Morning Practice. If each team member receives at least one gumball, in how many different ways can Mr. Sampson distribute the gumballs? a) 36 b)56 c)108 d) 126 e) none of the above
6	Suppose $2 \le f(x) \le (1-x)^2 + 2$ for all $x \ne 1$ and that $f(1)$ is undefined. What is $\lim_{x \to 1} f(x)$? (a) 2 (b) 5/2 (c) 3 (d) 4 (e) none of the above
7	$\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}} =$ (a) 6 (b) -6 (c) 0 (d) $+\infty$ (e) none of the above
8	If a, b, c are the three sides of a triangle and if the arithmetic mean of a, b, c is equal to the geometric mean of a, b, c, then the triangle must be. a) scalene b) equilateral c) isogonal d) obtuse e) not enough information
9	A gold ball 5 inches in diameter weighs 70 pounds. How many pounds does a golden ball 10 inches in diameter weigh? a) 140 b) 280 c) 560 d) 1120 e) none of the above

"Math is Cool" Championships - 2002-03

9th - 12th Grade - November 1, 2002

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

1st Score

Out of 10

Team Contest-Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1			
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"Math is Cool" Championships-2002-03

9^{th} - 12^{th} Grade - November 1, 2002

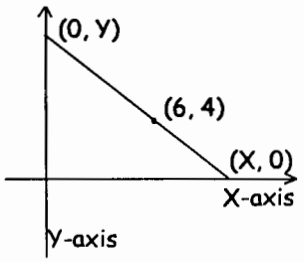
Team Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on colored cover sheet.

1	What is the shortest distance from the point (2,7) to the line 3x-4y=2
2	How many natural numbers less than 50,000 have exactly 35 positive integer factors?
3	The roots of $x^2 + bx + c = 0$ are the reciprocals of the roots of $x^2 + 2x + 3 = 0$. What is the sum of b and c?
4	A fish (tail, torso and head) has a tail which is $\frac{1}{3}$ of its total length. If its torso is $\frac{3}{2}$ the length of its tail and its head is 4 inches long? What is the total length of the fish?
5	1452 digits are used to consecutively number the pages of a book, starting with page 1. How many pages did the book have?
6	Find the smallest integer N such that $N > \left(\sqrt{2} + 9 - \sqrt{129 - 16\sqrt{2}}\right)^{-1}$.
7	What is the smaller angle between the hour and minute hands of a 12-hour analog clock at 6:14pm? (in degrees)
8	What is the remainder when $x^{17} + x^6 - 2x^2 + 1$ is divided by $(x-1)$?
9	For how many integer values of x will the sum $x^2 + (x+1)^2 + (x+2)^2$ be a perfect square?
10	A convex polygon has integer interior angle measures which form an arithmetic sequence with common difference 3. What is the largest possible measure of an angle in this polygon?

"Math is Cool" Championships-2002-03

9^{th} - 12^{th} Grade - November 1, 2002
Pressure Round

1	What is the sum of the integer solutions of the equation $[x/5] = 0$? Definition: $[x]$ is the greatest integer less than or equal to x.
2	The area of the triangle shown at the right formed by the x-axis, y-axis and the line segment passing through (6,4) is 48. Find the slope of the line segment. 
3	Urn A contains 5 green balls and 5 blue balls. Urn B contains 4 green balls and 6 blue balls. A ball is picked at random from each urn. The ball picked from urn A is placed in urn B. The ball picked from urn B is placed in urn A. One urn is selected at random and a ball is chosen at random from that urn. What is the probability that the ball chosen is green?
4	Factor $x^8 + x^4 + 1$ into factors at most of degree 2 with real coefficients.
5	For how many integers is $3 \leq \lvert x - 2 \rvert \leq 7$?

"Math is Cool" Championships-2002-03

9^{th} - 12^{th} Grade - November 1, 2002

Mental Math

Express all answers as reduced fractions in terms of radicals and π, where applicable, unless otherwise instructed.

Person 1		
1	What is the sum of 5 base 7 and 3 base 9 in base 9?	$8_{[9]}$
2	112 students attend a certain high school. 83 students are taking Calculus while 75 students are taking Physics. How many students are taking both Calculus and Physics? if everyone takes at least one of these classes.	46
3	What is the units digit of 7^{10}?	9
4	Solve for x: $64^x = 128$	7/6
Person 2		
1	I bought a horse and a cow. Together they cost $18.00. The cow cost twice as much as the horse. How much, in dollars, did the horse cost?	($)6
2	What is the distance between the x and y intercepts of the line: 3x+4y=12	5
3	An even function has a point (5,3) on its graph. State another point on its graph.	(-5,3)
4	Evaluate: $252^2 - 251^2$	503
Person 3		
1	A 45°- 45°-90° triangle has a hypotenuse of length 7. What is the area of the triangle?	49/4
2	Solve for x: $x^2 + 8x = 9$	1 & -9
3	Find the sum of the following infinite sequence: $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \ldots$	3
4	What is the domain of $f(x) = \sqrt{x - 3}$	$x \geq 3$ or $[3, +\infty)$
Person 4		
1	Solve for x: $5x + 3 = 7x + 6$	-3/2
2	What is the area of a square that has a diagonal of length 8?	32
3	What is the 11^{th} term in the sequence if the first term is 3, the second term is 6, the third term is 9 and the 4^{th} term is 12?	33
4	What is the slope of the line perpendicular to the line passing through (7, 8) and (3, 4)?	-1

"Math is Cool" Championships-2002-03

9^{th} - 12^{th} Grade - November 1, 2002

College Knowledge Bowl Questions #1

1	What is the diameter of a circle whose numeric value of the diameter is twice the numeric value of the area?	$2/\pi$
2	What is the sum of the first 16 terms of an arithmetic series with first term 8 and common difference -2?	-112
3	When seven coins are flipped, what is the probability there are exactly four tails?	$\frac{35}{128}$
4	Find the sum of the reciprocals of the roots of $5x^2 - 11x - 7 = 0$	-11/7
5	$5^6 = 15625$. What is 15625 in base 5?	$1000000_{[5]}$
6	What is the sum of an infinite geometric sequence with first term 54 and common ratio $\frac{2}{5}$?	90
7	What is the sum of the exterior angles of a pentagon in degrees?	360[°]
	Extra Question: Only use it if needed	
8	What's the least common multiple of 54, 30, and 135?	270

"Math is Cool" Championships-2002-03

9th - 12th Grade - November 1, 2002

College Knowledge Bowl Questions #2

1	How many non-degenerate triangles are there with integer side lengths between 3 and 8 inclusive?	49
2	When two cards are drawn, without replacement, from standard 52-card deck, what is the probability that two Jacks are drawn?	$\frac{1}{221}$
3	What is the area of an equilateral triangle with sides of length 8?	$16\sqrt{3}$
4	Alice has a box of 384 sugar cubes. Starting on Monday she eats one-fourth of the cubes remaining in the box each day. How many sugar cubes does she have left after Wednesday?	162
5	What is the probability that a sum of 10 occurs when 3 fair 6-sided dice are rolled?	$\frac{1}{8}$
6	When two cubed is added to five squared, the result is 14 more than what number?	19
7	How many diagonals can be drawn in a convex nonagon?	27
	Extra Question: Only use it if needed	
8	What is the radius of a circle with area 225π?	15

"Math is Cool" Championships-2002-03

9^{th} - 12^{th} Grade - November 1, 2002

College Knowledge Bowl Questions #3

1	What is the smallest natural number which is one more than a multiple of 3, but two less than a multiple of four?	10
2	The probability of event A occurring is $\frac{1}{4}$. The probability of event B occurring is $\frac{2}{3}$. The probability of both events occurring is $\frac{1}{7}$. What is the probability that neither occurs?	$\frac{19}{84}$
3	What is the length of the longer leg of a 30-60-90 triangle with hypotenuse 7?	$\frac{7\sqrt{3}}{2}$
4	What is the total surface area of a cylinder with base diameter 6 and height 1?	24π
5	Find 1/a + 1/b if a + b = 6 and ab=3.	2
6	Five students have a combined average of 74 points on a test, while another group of twelve students have a combined average of 91. Find the overall average.	86
7	Joe randomly picks a three digit number that does not begin with a zero. What is the probability that all three digits are distinct?	18/25
	Extra Question: Only use it if needed	
8	How many sides would a polygon have if the sum of the interior angles is 1260?	9(sides)

"Math is Cool" Championships -- 2001-02

9th - 12th grade - November 1, 2002

School Name_____ Team #_____

Proctor Name_____ Room #_____

Key

Full Name:_____

1st Score

Out of 40

Individual Contest - Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	$5\sqrt{2}$		
2	$4\sqrt{5}$		
3	23		
4	375[people]		
5	Yes		
6	51/25		
7	0		
8	15		
9	4		
10	5		
11	-7/10		
12	$(81\sqrt{3}) / 512$		
13	$\pm 4\sqrt{10}$		
14	15		
15	-108,864		
16	$(1 + \sqrt{33}) / 2$		
17	-1		
18	True		
19	(5/3,7)		
20	$(4\pi / 3) - \sqrt{3}$		

	Answer	1 or 0	1 or 0
21	16		
22	32		
23	$\sqrt{15}$		
24	364 [miles]		
25	60		
26	2		
27	$f^{-1}(x) = (x + 2) / (x - 1)$		
28	1/30		
29	16		
30	2/3		
31	3/4		
32	35/4 or 2		
33	$(28\pi / 3) + 8\sqrt{3}$		
34	16/3		
35	$0 - 2i, \sqrt{3} + i, -\sqrt{3} + i$		
36	$(4 / 17)\sqrt{1649}$		
37	$-3 + 2\sqrt{5}$		
38	7		
39	4, 2		
40	63		

"Math is Cool" Championships -- 2001-02

9th - 12th grade - November 1, 2002

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Individual Multiple Choice Contest-Score Sheet

1st Score

Out of 18

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer	-1, 0 or 2	-1, 0 or 2
1	A		
2	C		
3	D		
4	E ans 12		
5	B		
6	A		
7	B		
8	B		
9	C		

"Math is Cool" Championships -- 2001-02

9^{th} - 12^{th} grade - November 1, 2002

School Name_____ Team #_____

Proctor Name_____ Room #_____

Key

1^{st} Score

Out of 10

Team Contest-Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	$\frac{24}{5}$		
2	3		
3	1		
4	24 [in]		
5	520		
6	10		
7	103[°]		
8	1		
9	0 or none		
10	177[°]		

"Math is Cool" Championships -- 2001-02

9^{th} - 12^{th} grade - November 1, 2002

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Pressure Round - Score Sheet

	Answer		
1.	10		
2.	-2/3		
3.	9/20		
4.	$(x^2 + x + 1)(x^2 - x + 1)(x^2 + x\sqrt{3} + 1)(x^2 - x\sqrt{3} + 1)$		
5.	10		