"Math is Cool" Championships-2003-04

9th - 12th Grade - October 17, 2003
Individual Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
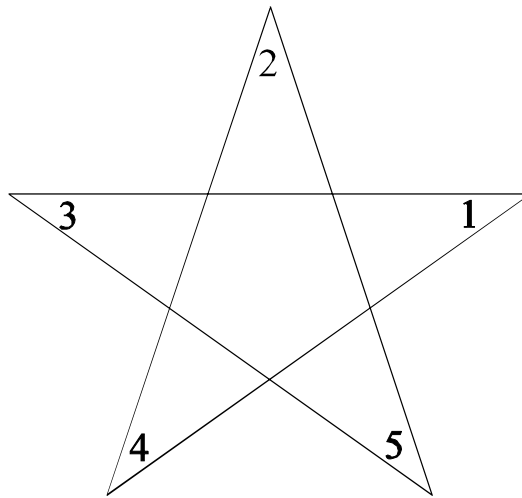Record all answers on the green cover sheet.

1	How many factors does 72 have?
2	A wallet contains 10 coins, all nickels or dimes, totaling 80 cents. How many dimes are there?
3	If the midpoint of the line segment between (-2, 5) and (8, a) is (3, 3), what is "a"?
4	Express as an ordered triple the mode, mean, and median of the following data set: {13, 21, 17, 17, 28, 20, 23, 30, 28, 47, 2, 12, 28}
5	There are goats and ducks in a field. If there are 34 heads and 92 feet, how many goats are present?
6	Evaluate $2^{10}/2^5$.
7	How many distinct arrangements of the letters in "MAMBA" are possible?
8	How many cubes three inches on a side are required to build a solid cube 1 foot on a side?
9	Two lines are eight units apart. A circle, tangent to both lines, has a square circumscribed around it. What is the area of that square?
10	What is the radius of a sphere whose volume is 64 times its surface area?
11	A circle with diameter $3\pi^3$ is externally tangent to a circle with area 26. How far apart are their centers?
12	For what value of x will the function f(x) = x^2 - 10x + 24 be at a minimum?
13	If y varies inversely with x and y = 3 when x = 4, find y when x = 18.
14	Convert 101110101110_2 to base 4.
15	Simplify the product of the square root of negative 3 and the square root of negative 12.

16	Evaluate and express as a common fraction: $16^{-1/4} - 8^{-2/3}$.
17	The base of a pyramid is a 27-sided polygon. Find the positive difference between the number of vertices and the number of edges of this pyramid.
18	If a679b is a 5-digit integer in which a and b stand for unknown digits (either the same or different), and this 5-digit integer is divisible by 72, determine a and b. Answer in the form (a,b).
19	Suppose you sum the digits of an integer x, then sum those digits, and so on until you reach a single-digit number. This single-digit number is the *digital root* of x. The year 2003 has a digital root of 5. How many years before 2003 was the most recent year with a digital root of 5?
20	How many distinct pairs of positive integers (a, b), where a $\leq$ b, can be formed such that the product of a and b is 360?
21	Name the figure represented by the following plane equation: $4x^2 + 5y^2 - 10x + 20y - 100 = 0$
22	If Christmas Day (Dec. 25) falls on a Monday in a certain leap year, on what day of the week was Valentine's Day (Feb. 14) of that year?
23	How many integers from 1001 through 2001 inclusive have exactly three of their digits the same?
24	Simplify (6 - 7i)/i where i^2 = -1.
25	Annie draws a circle, a triangle, and a square. If no side of the triangle is collinear with a side of the square, what is the largest possible number of points of intersection in the drawing?
26	Circle O is inscribed in square ABCD. Point P is the intersection of AO with the circle. What is the ratio of AP to OP?
27	Reduce the following fraction to lowest terms: the numerator is the quantity (1/x + 1/y) and the denominator is 1/(xy).
28	A parabola passes through the points (3, 3), (-3, 4), (-1, 1), and (2, a). Find a.
29	If $f(2x) = 4x^2 - 8x + 3$, determine $f(x)$.

Challenge Questions

30	Find the sum, in degrees, of the angles marked 1, 2, 3, 4, and 5 in the pentagram shown.
31	Kristina has a ½ chance of losing during each minute of playing a video game, while Jon has only a 1/3 chance of losing during the same length of time. What is the probability that Jon loses first and Kristina survives on to the following minute?
32	Rebecca takes two coins out of her purse, without replacement. The purse contains both nickels and dimes, and no other coins. The probability that both coins are nickels is ½. What is the smallest number of coins there could be in Rebecca's purse?
33	Evaluate: $\displaystyle\sum_{x=1}^{2003} \frac{1}{x^2 + 7x + 12}$
34	If $n(p) = ap^3 - cp + 8$ and $n(-13) = 5$, determine $n(13)$.
35	Find the largest positive real value of a satisfying the following equations: $123_a = 334_b$, $144_b = 49_{10}$
36	A ball in the shape of a single point, when dropped, will bounce to half its original height. Two such balls are dropped at one time, one from 10 meters above the ground, the other from 5 meters directly above the other ball. When the two balls collide, they bounce, effectively exchanging velocities. What is the total distance traveled by the two balls?
37	If $x^2 + y^2 = 12$ and $x + y = -4$, determine the value of $x^3 + y^3$.
38	How many natural numbers less than 100 are congruent to 2 mod 7?

39	How many positive four-digit numbers contain exactly three distinct digits?
40	Find the sum of all the real values of g which satisfy $$\left(2g^2 + 5g - 13\right)^{2g^2+7g+3} = 1$$

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Individual Multiple Choice Contest

Record answers on colored sheet

1	The base of a triangle is twice as long as a side of a square and the areas of the triangle and the square are the same. What is the ratio of the altitude of the triangle to a side of the square? (A) 1:4 (B) 1:2 (C) 1:1 (D) 2:1 (E) 4:1
2	The volume of a rectangular solid with side, front, and bottom faces of 12 cm^2, 8 cm^2, and 6 cm^2, respectively, is: (A) 576 cm^3 (B) 24 cm^3 (C) 9 cm^3 (D) 104 cm^3 (E) none of these
3	Successive discounts of 10% and 20% are equivalent to a single discount of: (A) 30% (B) 15% (C) 72% (D) 28% (E) none of these
4	A car travels from A to B at 30 miles per hour (mph) but returns the same distance at 40 miles per hour. The average speed for the round trip is closest to: (A) 33 mph (B) 34 mph (C) 35 mph (D) 36 mph (E) 37 mph
5	The simultaneous inequalities x<1/(4x) and x<0 are equivalent to which of the following? (A) 0<x<1/4 (B) 0<x^2<1/4 (C) x<-½ (D) 0<x<1/2 (E) none of these
6	Which of the following are true? (1) a(x - y) = ax - ay (2) a^{x-y} = a^x - a^y (3) (a^x)(a^y) = a^{xy} (4) (a^x)(a^y) = (a^2)(a^{x+y}) (5) a(xy) = ax•ay (A) only 1 and 5 (B) only 1 and 4 (C) only 1 and 3 (D) only 1 and 2 (E) only 1 is true
7	How many ounces of water are needed to reduce 9 ounces of shaving lotion containing 50% alcohol to a lotion containing 30% alcohol? (A) 3 (B) 4 (C) 5 (D) 6 (E) 7 (F) none of these
8	The number of distinct positive integral divisors of (30)^4, excluding 1 and (30)^4, is: (A) 100 (B) 125 (C) 123 (D) 30 (E) none of these
9	In a room containing n people, n > 3, at least one person has not shaken hands with everyone else in the room. What is the maximum number of people in the room that could have shaken hands with everyone else? (A) 0 (B) 1 (C) n - 1 (D) n (E) none of these

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9th - 12th Grade - October 17, 2003
Team Contest

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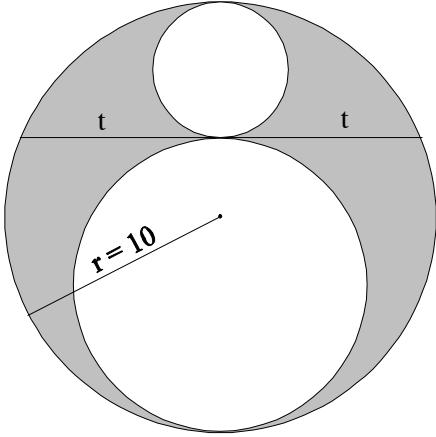
Leave answers in terms of π where applicable.

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Record all answers on colored cover sheet.

1	Andy and Bob arrange to meet for lunch, setting a date and location. However, being both vague and impatient, they agree that each will arrive independently at a random moment between 12 noon and 1 PM, and that each will stay only 5 minutes, leaving if the other isn't there during this interval. What is the probability that Andy and Bob actually will meet for lunch?
2	Put the letters of the following in order from least to greatest number of factors: (a) 140 (b) 140 •2 (c) 140 •3 (d) 140 •4 (e) 140 •5
3	In base 10, a number is divisible by 3 if and only if the sum of its digits is divisible by 3. What is the next largest base for which this is true?
4	The complete graph K_n is a set of n points in which each pair of points is connected with a curve called an edge. What is the largest value of n such that K_n can be drawn in the plane with no edge crossing another?
5	When you cut a certain rectangle in half, you obtain two rectangles that are both similar to the original rectangle. Find the ratio of the longer side length to the shorter side length.
6	To play the game Be Square, Joe draws numbered tiles from a box. If he can make a square number by adding 1 to the product of two of his tiles, he scores a point. Joe draws 4, 6, and a third tile with the integer n on it. If he can score 3 points with these 3 tiles, what is the smallest possible positive value for n?
7	A 6-digit integer has 1 as its leftmost digit. The 1 is removed from this position and moved to the right-hand end of the number. The new integer created is 3 times as big as the original number. What was the original 6-digit number?
8	Each of 10 scientists on a research project contributes one unique piece of data, different from that of any other researcher and at first unknown to anyone else. The scientists communicate by e-mail. When one scientist e-mails another, the first tells the second everything that the first scientist knows. What is the minimum number of e-mails needed so that all 10 scientists each have all 10 pieces of data?
9	How many prime numbers between 10 and 99 remain prime when their digits are reversed?
10	A circle is inscribed in a right triangle with sides 3, 4, and 5. Find the area of the circle.

"Math is Cool" Championships-2003-04

1	It is given that $x^2 - y^2 = z$, where z is a positive integer which has zero as its units digit, and both x and y are positive integers less than 10. Find the sum of all possible values of z.
2	The only coins currently minted in the US are the dollar, the half-dollar, the quarter, the dime, the nickel, and the penny. Libbey has $1.01, all in US coins. List all of the following numbers of coins Libbey could <u>not</u> have? 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
3	Factorize the expression: $a^4(b^2 - c^2) + b^4(c^2 - a^2) + c^4(a^2 - b^2)$
4	How many integer values are possible for b in the equation $\lvert 2b - 3 \rvert < 19$?
5	In the figure, the two smaller circles are tangent to each other and to the large circle and chord length of 2t. The radius of the large circle is r = 10. If t = 5, find the area of the shaded region.

"Math is Cool" Championships-2003-04

Mental Math

Express all answers as reduced fractions in terms of radicals and π, where applicable, unless otherwise instructed.

Person 1		
1	Solve for x in the equation 5x+3 = 9.	6/5
2	Express as a reduced fraction the product of 3 factorial and 4 factorial divided by 5 factorial.	6/5
3	Find the reflection of the point (4, 3) through the point (2, 0).	(0, -3)
4	The sum of two numbers is 28 and the difference is 22. What is the smallest of the two numbers?	3

Person 2		
1	If two angles in a triangle are 72 degrees and 31 degrees, what is the measure of the other angle?	77
2	What is the smallest composite number whose prime factorization contains a number other than 2, 3, 5, or 7?	22
3	A circle of radius r is inscribed in a square. Find the ratio of the area of the circle to that of the square.	π/4
4	What are the roots of x squared minus 49 equals zero?	7, -7 need both answers

Person 3		
1	What is the area of a triangle with sides of length 6, 8, and 10?	24
2	Of 100 students surveyed, 80 were taking a math class and 75 were taking a science class. If 70 students were taking both math and science, how many were taking neither?	15
3	Three fair cubical dice are rolled. Find the probability that all three numbers rolled are different.	5/9
4	What is the area of an equilateral triangle with side length 8?	$16\sqrt{3}$

Person 4		
1	Solve for x: 9x+3x=24.	2
2	Find the smallest integer greater than 1 that is a perfect square, a perfect cube, and a perfect fourth power.	4096
3	The sum of the squares of two consecutive positive integers is 85. Find the smaller of the two integers.	6
4	The side lengths of two similar regular octagons are 1 and 3. What is the ratio of the smaller area to the larger area?	1:9

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College Knowledge Bowl Questions #1

1	A rock band, The Wrath of Kahn, has just released a new CD. The length of the CD in hours is the sum of the reciprocals of the first three prime numbers. What is the length of the CD in minutes?	62
2	What is the smallest positive integer N such that the product of one-hundred thirty-five and N is a perfect square?	15
3	If F of X equals X squared, plus four X, plus one, find F of the quantity X plus 2.	$x^2 + 8x + 1 3$
4	Jar A contains one black marble and one red marble. Jar B contains one black marble and two red marbles. Jar C contains one black marble and three red marbles. If a jar is picked at random and then one marble is randomly taken from it, what is the probability it is black?	13/36
5	Find the area of an equilateral triangle with altitudes of length 12.	$48\sqrt{3}$
6	Find the sum of the series one plus two minus three plus four plus five minus six, continuing that pattern until the final two terms are plus 38 minus 39..	234
7	In slope-intercept form, give the equation of the line passing through the point three comma negative two and parallel to the line five X minus four Y equals eight.	$y = \dfrac{5}{4}x - \dfrac{23}{4}$
	Extra Question: Only use it if needed	
8	Factor completely: X cubed plus two X squared minus X minus two	(x-1)(x+1)(x+2)

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9th - 12th Grade - October 17, 2003

College Knowledge Bowl Questions #2

1	In a bag are 6 red and 5 white marbles. If 3 are selected at random, what is the probability that they will all be of the same color?	2/11
2	What is the maximum area, in square feet, that can be enclosed by 12 feet of rope?	$36/\pi$ ft^2
3	Solve the following system of equations and express your answer as an ordered pair: 2x - 3y = -7 ; 3x + y = -5.	(-2, 1)
4	Twice the complement of angle A is 40° less than the supplement of angle A. Find angle A, in degrees.	40
5	A polygon with n sides has 2n diagonals. What is n?	7
6	A 6-foot person standing 15 feet from a streetlight casts an 8-foot shadow. What is the height of the streetlight, in feet?	17 1/4 or 69/4 or 17.25 [ft]
7	For what value of x does 1 + 2 + 3 + … + x = 120?	15
	Extra Question: Only use it if needed	
8	If f(x) = x^3 - 1 and g(x) = x + 1, find f(g(3)).	63

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College Knowledge Bowl Questions #3

1	What is the least integer value of n such that $3^n + 2^n$ is a 3-digit number?	5
2	If a stack of 8 quarters is ½ inch high, what is the dollar value of a stack of quarters one foot high?	($)48
3	Solve for x: $4 + |2x + 3| < 5$.	$-2 < x < -1$
4	My number is a four-digit integer. The product of the digits is equal to 6 factorial. What is the largest my number could be?	9852
5	In how many distinct ways can 5 different keys be arranged on a keychain? Keys appear the same regardless of orientation.	12
6	Give the y coordinate of the vertex of the parabola $y = x^2 + 2x - 8$.	-9
7	The two diagonals of a rhombus are 18 and 24. What is its perimeter?	60
	Extra Question: Only use it if needed	
8	Solve for x: $1/x^2 > 1/4$.	$-2 < x < 2$

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9th - 12th grade - October 17, 2003

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Full Name:_____

1st Score

Individual Contest - Score Sheet
DO NOT WRITE IN SHADED REGIONS

Out of 40

	Answer	1 or 0	1 or 0		Answer	1 or 0	1 or 0
1	12			21	ellipse		
2	6[dimes]			22	Monday		
3	1			23	36		
4	(28, 22, 21)			24	-7 - 6i		
5	12[goats]			25	20		
6	32			26	$\sqrt{2}$ -1:1		
7	30 [arrangements]			27	x + y		
8	64[cubes]			28	3/2 or 1.5		
9	64			29	f(x) = x^2 - 4x + 3		
10	192			30	180[°]		
11	$(3/2)\pi^3 + \sqrt{26\pi}\,/\,\pi$			31	1/4		
12	5			32	4		
13	2/3			33	2003/8028		
14	232232[4]			34	11		
15	-6			35	$2\sqrt{23} - 1$		
16	1/4			36	75 [meters]		
17	26			37	-40		
18	(3,2)			38	14		
19	9			39	3888		
20	12			40	-9/2		

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School Name_____Team #_____

Proctor Name_____Room #_____

Individual Multiple Choice Contest-Score Sheet

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

1st Score

Out of 18

DO NOT WRITE IN SHADED REGIONS

	Answer	-1, 0 or 2	-1, 0 or 2
1	C		
2	B		
3	D		
4	B		
5	C		
6	E only 1 is true		
7	D		
8	C		
9	E		

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Team Contest-Score Sheet

DO NOT WRITE IN SHADED REGIONS

1st Score

Out of 10

	Answer	1 or 0	1 or 0
1	23/144		
2	a,b,e,d,c		
3	13		
4	4		
5	$\sqrt{2}$		
6	20		
7	142,857		
8	18		
9	9		
10	π		

Pressure Round - Score Sheet

	Answer		
1.	200		
2.	1		
3.	(a+b)(b+c)(c+a)(b-a)(c-b)(a-c)		
4.	18		
5.	$25\pi/2$		