"Math Is Cool" Championships-1996-7

May 9, 1997

Individual Contest, High School

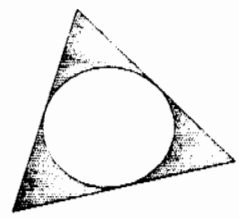
Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.

1. Two spheres have a volume of 1.25369 units each. One of the spheres is inflated so that the ratio of the new radius to the old radius is $\sqrt[3]{(\sqrt{5})^2} : \sqrt[3]{2}$. What is the ratio of new volume to old volume of the spheres?

2. Given the following sequence:
1, -2, 0, 1, -2, 0, 2, -3, 0, 3, -4, 0, 5, -6, 0, 8, -9, 0, 13, -14, 0, 21,-22,0,34,-35,0...
Find the sum of the next two numbers in the sequence.

3. You start with a square of area 10, divide the square into nine smaller congruent squares. The eight smaller squares along the edge also divide into nine smaller congruent squares while the middle one is colored black. Do this over and over again an infinite number of times. What is the sum of the area of the black squares?

4. Find the area of the shaded region given that the perimeter of the equilateral triangle is 12 and it circumscribes the circle.



5. Evaluate: $\sec(\arcsin(\frac{2}{4}))$

6. Solve for n: $n+2=2\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}}$

7. Solve for x over the set of real numbers: $343^x+49^x+7^x=3$

8. Find: $a_4 + a_5 + a_6 + a_7$

   Given: $a_n = (-1)^{n-1}(n-4)$

9. Roy rode his scooter from home to the beach and returned along the same path. He traveled at a constant rate of 30 m.ph. to the beach. Roy's average speed for the whole trip was 24 m.p.h. How fast did he travel on the way back?

10. A six inch pizza costs $6.00. How much does a 12 inch pizza, of equal thickness, cost assuming the six inch pizza and the 12 inch pizza cost the same per square inch?

11. A bag of marbles contains 5 red, 3 green and 11 purple marbles. If two marbles are drawn without replacement, what is the probability that both are red?

12. What is the sum of the roots of:
    $11X^5 + 2X^4 - 5X^3 + 7X^2 + X - 1 = 0$

13. If x is positive and $2X^3 + 3X^2 + 2X + 1 = 2321$
    then $X^3 + 9X^2 + 9X + 7 = ?$

14. Which is greater?

$$\sqrt{\frac{1}{2}} \quad \text{or} \quad \sqrt[3]{\frac{1}{3}}$$

15. Evaluate: $g(x)f(x)h(x)$

    Given:
    $g(x) = x^2 - 25$

$$h(x) = \frac{x^3 - 125}{x^2 - 10x + 25}$$

$$f(x) = (x + 5)^{-1}$$

16. Jenny has 3 pennies, 3 nickels, and 3 dimes. How many different sums of money can Jenny make using one or more of these coins?

17. The sum of two consecutive integers is 1997. What is the smallest of the two integers?

18. Find $f^{-1}(x)$ if $f(x) = x^3$

19. Simplify:(all exponents must be positive)

$$\frac{\frac{x^2}{y^2}\cdot(1-x)}{2x-(x-x)^2}\cdot\frac{-z^0(a-2)^0}{2y^2}$$

20. Simplify: $[8(x + 5)-x(8 + 5) - 5(1 -x)] - [8(x - 5) - x(8 + 5) - 5(1 - x)]$

21. Factor: $(x - a)(x^2 + xa + a^2) - 3xa(x - a)$

22. Three cards are drawn, without replacement, from an ordinary deck. Find the probability that exactly 2 cards are red.

23. There are 16 coins in a bank. If the coins are all nickels and dimes and they total \$1.05, how many nickels are there?

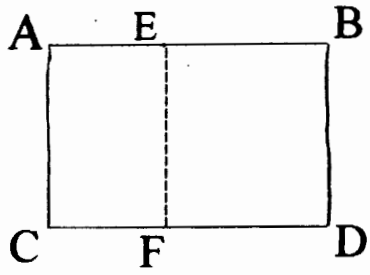
24. Find the area of a circle with equation: $(x-\sqrt{3})^2+(y+7)^2=\frac{9}{\pi}$

25. Find the smallest positive integer which, when divided by 5 has a remainder of 2 and when divided by 9 has a remainder of 1.

26. Let the ratio of $\frac{\overline{AB}}{\overline{BD}} = \frac{\sqrt{5}+1}{2}$ = golden ratio

and $\frac{\overline{EB}}{\overline{BD}} = 1$.

Find the ratio of $\frac{\overline{BD}}{\overline{AE}}$.



(Please note that ABCD and EBDF are rectangles)

27. It is given that x varies directly as y and inversely as the square of z, and that x = 10 when y = 4 and z = 14. What is x when y = 16 and z = 1?

28. Find the units digit of 19^{97}.

29. If a + b = 0, but a ≠ 0, what is the value of $\frac{a^{1997}}{b^{1997}}$.

30. Nineteen soldiers, numbered 1 through 19, stood in a circle in clockwise numerical order, all facing the center. They began to count out loud in clockwise order: the first soldier called out the number 1, the second called out 2; and each soldier then called out the number 1 more than the number called to his right. What was the number of the soldier who called out the number 1997?

"Math Is Cool" Championships-1996-7

Individual Multiple Choice Contest, High School

May 9th, 1997

1. Express the absolute value of the difference between $0.\overline{23}$ and .23 as a common fraction?

A) 23/9999 B) 23/99 C)23/9900 D) answer not given E) undefined

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2. The radius of the first circle is 1 inch, that of the second is $\frac{1}{2}$ inch, that of the third is 1/4 inch and so on indefinitely. What is the sum of the areas of the circles?

A) 2 inch^2 B) 3 inch^2 C) 4 inch^2 D) answer not given E) infinite

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3. Ten math team members, five girls and five boys, decide to ride a merry-go-round with exactly ten seats arranging themselves at random. If the boys and girls are seated randomly, what is the probability that boys and girls will be seated alternately?

A) 1/1024 B)1/120 C)1/60 D) answer not given E) undefined

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4. If $3^x = 5$, what is the value of 3^{2x+3}?

A)$\log_3 5$ B) 1/120 C) 675 D) answer not given E) undefined

5. Find the length of the side of a rhombus which has area 12 and diagonals with lengths $2x$ and $3x$.

A) 18/5 B) $\sqrt{13}$ C) 3 D) answer not given E) undefined

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6. Evaluate: $\log_6(\log_2(\log_3(\log_1 3)))$

A) 0 B) 6 C) 8 D) answer not given E) undefined

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7. A square table is positioned on an un-level surface so that all four legs of equal length are sitting firmly on the surface. What is the least number of degrees θ ($\theta > 0$) that you can rotate the table and the table will still be stable?

A) 45° B) 90° C) 360° D) answer not given E) undefined

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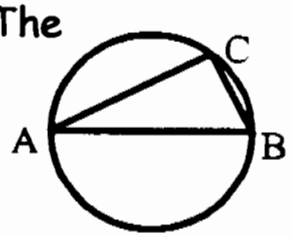
8. A and B lie on a diameter of the circle. The length of the diameter of the circle is 10 and the measure of $\triangle CAB$ is 30^0. Find the length of line segment CB.



A) 5 B) 6 C) 7 D) answer not given E) undefined

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9. What is the base 7 number 3456_7 in base 10?

A) 3456 B) 3455 C) 1266 D) answer not given E)undefined

"Math Is Cool" Championships-1996-7

Team Contest, High School

May 9th, 1997

Express all answers as reduced fractions in terms of radicals.

1. A crew of 100 workers can build a bridge in 400 days. After 25% of the bridge is built, plans are changed; the company wants the bridge built in 200 days, not 400 days. How many more workers must be hired?

2. How many factors are there for $3^{18} - 2^{18}$?

3. When four numbers are added three at a time, the sums are 71, 77, 85, and 91. What is the sum of all four numbers?

4. Morten paddled his canoe upstream for 1 hour and downstream for 2. The rate of the current was 3 mph. When he stopped, Morten looked at his Global Positioning Unit and calculated that he was down stream 14 miles from where he had started. How many hours will it take him to paddle back to his starting point?

5. A knight of the round table has a furious joust fight with the great Norwegian. Both jousters ride at a constant speed toward each other at 5 ft/sec from a distance of 300 feet apart. As they run toward each other a killer bee flies from one jouster to the other at a speed of 15 ft/sec. The killer bee flies between the two jousters in this manner until they crash into each other. How far did the killer bee fly before it was crushed between the jousters?

6. In an eat-off, the lumberjack, the highschool senior and the Norwegian compete furiously. The probability that the lumberjack wins is 1/8, the highschool senior 1/4 and the Norwegian 5/8. On a day when the lumberjack is ill in bed. What is the probability the Norwegian will win?

7. In the year 1997 Jina and Mr. Sampson's ages are both prime. Their ages are 17 and 31 respectively. 1997 is also prime. This event occurs again in 2003 when their ages are 23 and 37 respectively. Assuming they both live long enough, when next will their ages and the year all be prime?

8. A piece of metal of uniform density and uniform thickness has its vertices at (1,2), (5,5), (5,10) and (1,7). Find the metal's center of mass.

9. A survey of 60 math team members at Lewis and Clark High School produced the following results.

   19 of the students read *Calculus Anonymous;*
   18 read *The Math Street Journal;*
   50 read *Euler's Inferno;*
   13 read *Calculus Anonymous* and *The Math Street Journal;*
   11 read *The Math Street Journal* and *Euler's Inferno;*
   13 read *Calculus Anonymous* and *Euler's Inferno;*
   9 read all three

   How many students read none of the publications?

10. What are all values x of for which $(x^2-5x+5)^{x^2-9x+20}=1$?

Practice relay
Person#1

What is the sum of the interior angles of a triangle?

Practice relay
Person#2

What is the quotient of TNYWG and the closest whole number to π?

Practice relay
Person#3
Solve:

$5x + TNYWG = 135$

Practice relay
Person#4

Find the sum of the reduced numerator and denominator of TNYWG/20

Relay #1
Person#1
Let X = the sum of the interior angles of the quadrilateral ABCD
Let Y = the sum of the exterior angles of the 17-gon EFGHIJKLMNOPQRSTW

Find X - Y

Relay#1
Person#2
What is $(TNYWG + 5)^2 + 5^{-1} + 174.8$?

Relay#1
Person#3
During week 1 a dress sells for $TNYWG.
During week 2 the price is discounted 10%.
During week 3 the price is increased 10%.

What is the price during week 3?

Relay#1
Person#4
$g(x) = x - 200$
$f(x) = 2x + 3$

Find $f(f(g(TNYWG + 1)))$.

Relay#2
Person#1
Find the sum of 6 and all solutions to $|2x + 1| = 15$

Relay#2
Person#2
Convex polygon W has TNYWG sides. How many distinct diagonals does polygon W have?

Relay#2
Person#3
The shorter leg in a 30^0-60^0-90^0 triangle ABC is TNYWG units. If AB is the longer leg and $X = (AB)\sqrt{3} - 3$, what is X^2?

Relay#2
Person#4
TNYWG is a number written in base 6. What is it in base 10?

"Math Is Cool" Championships-1996-7

May 9th, 1997

Mental Math, High School

Express all answers as reduced fractions in terms of radicals and π, where applicable, unless otherwise instructed.

Person #1

1. What is the units digit of the product of the first 99 primes?

2. In a right triangle, the lengths of the legs are 333 and 444. What is the length of the hypotenuse?

3. What is the first two digit positive integer which is exactly twice the sum of its digits?

4. What is the value of x which satisfies the equation $\sqrt{x}+\sqrt{x}=\sqrt{64}$?

Person #2

1. What is the units digit of 5^{25}?

2. How many integer values x satisfy the inequality $25 \le x \le 144$?

3. What is the smallest positive integer n that when divided into 83 leaves a remainder of 8

4. Evaluate: 2^{11}

Person#3

1. The sum of two positive real numbers equals the sum of the reciprocals of the same two real numbers. What is the product of these two real numbers?

2. The product of 20 positive integers is 100. What is the largest sum of these 20 positive integers?

3. I have twice as many nickels as dimes. If the value of the nickels is $5.00, what is the value of my dimes?

4. Heather has a bag of bagels. In the bag there are six blueberry, 12 plain, seven cinnamon-raisin, and one onion bagel. What is the probability she draws a blueberry bagel from the bag in one draw?

Person#4

1. The sum of two numbers is 44, and the difference is 12. What is the smaller of the two numbers?

2 On the Galapagos Islands, Darwin saw tortoises and finches. If there were 25 heads and 90 feet how many tortoises were there?

3. What is the probability of rolling a sum of seven with two six-sided dice?

4. Robert has 2 pairs of red gloves, 3 pairs of blue gloves, and 9 pairs of yellow gloves in a drawer. How many gloves must he draw out in the morning with out looking to ensure that he gets a matching pair?

"Math Is Cool" Championships -- 1996-7
High School
College Knowledge Bowl Questions #1

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1. List any quadrant 4 solutions between 0 and 2π in radians to the equation $\cos x = -\frac{\sqrt{3}}{2}$

Answer: No solutions

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2. Factor completely $x^3 - 4x^2 + x + 6$

Answer: $(x + 1)(x - 3)(x - 2)$

---

3. Find the reciprocal of $4 + i$ in the form $a + bi$ where a and b are integers and a/b is in lowest terms .

Answer: $\frac{4}{17} - \frac{1}{17}i$

---

4. Angles x and y are measured in radians. They are different, each is between 0 and 2π and each terminates in quadrant 2. If the measure of each is an integer, find their sum.

Answer: 5

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5. Simplify: $i^{291} + i^{263} + i^{-3}$

Answer: $-i$

---

6. Solve for x: $8^{2x} - 5(8^x) + 4 = 0$

Answer: 2/3 and 0 (order doesn't matter)

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7. Find the vertical asymptotes of $f(x) = \frac{x^2 - 16}{x^2 - 6x + 8}$

Answer: x=2 (x=4 is not an answer, it is a removable discontinuity)

Extra: Only Use if Needed

How many units of phase shift and in which direction, left or right, does the following curve have in relation to a basic sine curve?

$Y = 6 + 2\sin(4x - 48)$

Answer: 12 units right

"Math Is Cool" Championships-1996-7
High School
College Knowledge Bowl Questions #2

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1. Find the vertex of $f(x) = 5x^2 - 3x + 2$

Answer: $(\frac{3}{10},\frac{31}{20})$ or $(\frac{3}{10},1\frac{11}{20})$ (Order matters)

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2. What is the logarithm of $(27)(\sqrt[4]{9})(\sqrt[3]{9})$ base 3?

Answer: $\frac{25}{6}$ or $4\frac{1}{6}$

---

3. How many points with integer coordinates are exactly 5 units away from (0,0)?

Answer: 12

---

4. We are given the line $3x + 5y = 15$ and a point on this line equidistant from the coordinate axes. In which quadrants can such a point exist?

Answer: Quadrants 1 and 2

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5. Find the discriminant of $2x^2 + 5x + 3$.

Answer: 1

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6. What are the coordinates of the point on segment AB that is two-thirds of the way from A to B, where A has coordinates (2,4) and B has coordinates (-1,1)?

Answer: (0,2)

---

7. Find the distance from (1,2) to the center of the circle $x^2 + y^2 - 2x + 6y = 20$.

Answer: 5

Extra Question if needed
Where does the line $3x + 5y = 8$ intersect $-2x + 7y = 5$?

Answer: (1,1)

"Math Is Cool" Championships-1996-7

High School

College Knowledge Bowl Questions #3

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1. Solve the system for x:

$$\begin{aligned} 2x + 3y - 2z &= 2 \\ y - 5z &= 13 \\ 6z &= -18 \end{aligned}$$

Answer: 1

---

2. What is the negative value of k that makes $x^2 + kx + 64$ a perfect square trinomial?

Answer: -16

---

3. On a coed soccer team, the ratio of females to males is 6 to 5. When two males were not able to make it to a game, the ratio of females to males for that game was 3 to 2. How many males are on the team?

Answer: 10

---

4. Find the equation of the line parallel to $5x + 3y = 10$ and passing through (-1, 17) in the form $Ax + By = C$, where A, B, and C are integers and A is positive.

Answer: $5x + 3y = 46$

---

5. Four students tried to find the sum of the first 21 primes. Their answers were 709, 711,712, and 713. Only one answer was correct. What is the sum of the first 21 primes?

Answer: 712

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6. How many diagonals can be drawn in a convex polygon that has ten sides?

Answer: 35

---

7. A circle is inscribed in a equilateral triangle What is the ratio of the area triangle to the area of the circle?

Answer: $\frac{3\sqrt{3}}{\pi}$

Extra question if needed:

In terms of pi, what is the length of an arc that is subtended by angle of 60^0 with a radius of 12?

Answer: 4π

"Math Is Cool" Championships -- 1996-7

May 9, 1997

Individual Contest - Score Sheet

Score:
KEY

Full Name:_____ School:_____

DO NOT WRITE IN SHADED REGIONS

	Answer		
1.	5/2 or 5:2		
2.	-1		
3.	10		
4.	$4\sqrt{3}-\frac{4}{3}\pi$		
5.	$\frac{2}{\sqrt{3}} else 2\frac{\sqrt{3}}{3}$		
6.	2		
7.	0		
8.	2		
9.	20		
10.	24 or $24		
11.	10/171		
12.	-2/11		
13.	1997		
14.	$\sqrt{\frac{1}{2}}$		
15.	$X^2 + 5x + 25$		

	Answer		
16.	39		
17.	998		
18.	$\sqrt[3]{x}$		
19.	$\frac{x^2-x}{4y^4}$		
20.	80		
21.	$(x - a)^3$		
22.	13/34		
23.	11		
24.	9		
25.	37		
26.	$\frac{\sqrt{5}-1}{2}$		
27.	7840		
28.	9		
29.	-1		
30.	2		

"Math Is Cool" Championships -- 1996-7

May 9, 1997

Individual Multiple Choice Contest - Score Sheet

Score:
KEY

School:_____ Team #:_____

DO NOT WRITE IN SHADED REGIONS

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

	Answer		
1.	C		
2.	D		
3.	D		
4.	C		
5.	B		
6.	E		
7.	B		
8.	A		
9.	C		

"Math Is Cool" Championships -- 1996-7

May 9, 1997

Team Contest - Score Sheet

Score:
KEY

School:_____ Team #:_____

DO NOT WRITE IN SHADED REGIONS

	Answer		
1.	200		
2.	32		
3.	108		
4.	7		
5.	450		
6.	5/7		
7.	2027		
8.	(3,6)		
9.	1		
10.	1,2,3,4,5 (order doesn't matter)		

"Math Is Cool" Championships -- 1996-7

May 9, 1997

Mental Math - Score Sheet

School:_____ Team #:_____

A. 1. 0

2. 555

3. 18

4. 16

B. 1. 5

2. 120

3. 15

4. 2048

C. 1. 1

2. 119

3. 5 or $5

4. 3/13 or 3 out of 13

D. 1. 16

2. 20

3. 1/6 or 1 out of 6

4. 4

"Math Is Cool" Championships -- 1996-7

May 9, 1997

Relay Contest - Score Sheet

Practice relay

● 7

Answer for relay #1

5

Answer for relay #2

64