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"Math is Cool" Championships-1997-8

February 27, 1998

Individual Contest, High School

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Leave answers in terms of simplified radicals where applicable.
Do not round any answers unless stated otherwise.

1. What is the units digit of 19^{98}?

2. Mr. Sampson is going to the Zoo. He travels at the speed limit (it's constant) on the way to the zoo and speeds 9 km/hr above the speed limit on his way home. He tells you that his average velocity was 40 km/hr. What is the speed limit?

3. What positive number is equal to 1 plus its own reciprocal?

4. A bag of marbles contains 6 black and 4 blue marbles. One marble is drawn and put back in the bag, then a second marble is drawn. What is the probability that both marbles are blue?

5. You have two cubes of equal volume. If you inflate one cube so that its surface area is 4 times that of the other, what is the ratio of the volume of the bigger cube to the smaller one?

6. Evaluate

$$\sin^{-1}(\sin(\frac{2\pi}{3}))$$

7. Solve for x over the reals $8^x - 3 \cdot 4^x - 2^{5+x} = 8^2$

8. Given: $g(x) = x+5$
$f(y) = y^2 + y + 1$
$h(z) = 1$

Find $f(g(x)) + h(g(x))$ as a polynomial of decreasing powers of x.

9. Chef Sampson is cooking up pancakes to feed the math team. According to his recipe 1 serving requires 8 fluid ounces of water. If he makes 18 servings how many gallons of water does he need?

10. Bill and Robert are playing a game. Bill has a fair 4-sided die and Robert has a fair coin. Bill goes first. If he rolls a one he wins otherwise it is Robert's turn. If Robert flips a head he wins otherwise it is Bill's turn again. The game continues until someone wins. What is the probability that Bill wins?

11. There are 8 red marbles and 1 blue marble in a bag. If all nine marbles are drawn out, one at a time, what is the probability that the blue marble will be drawn out on the 5th or 7th draw?

12. What is the area of an equilateral triangle inscribed inside of a unit circle?

13. What is the period of

$$f(x) = 12\pi \sin(4x+17) + \sqrt{2}\sin(x-\pi) - \sqrt{e}\cos(\frac{2}{3}x+1) \quad ?$$

14. Given: $f(x) = 2x^5+7x^4-x^3+8x^2+2x-8$
If the roots are a,b,c,d,e.

Evaluate: $\dfrac{abcd+acde+abde+abce+bcde}{a+b+c+d+e}$

15. Mike travels into the future and glimpses at some of the results of the "Math is Cool 1998" individual exam. The scores he saw were:
18, 27, 24, 20, 21, 17, 19, 25, 19, 24, 19

What is the mean of the mode and the median?

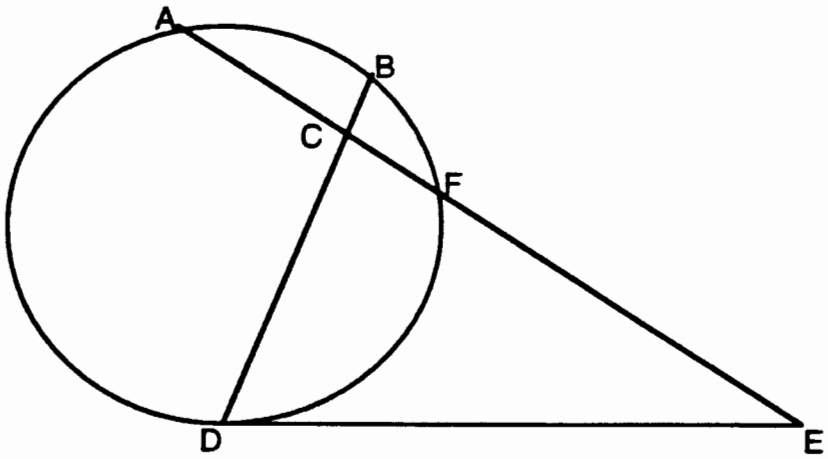
16. The probability team "A" beats team "B" = .25
The probability team "A" beats team "C" = .50
The probability team "A" doesn't beat team "D" = .2

If team "A" plays every other team once, what is the probability that team "A" wins exactly 1 game.

17. Which is smaller:

$$\sqrt[3]{\frac{1}{5}} \quad \text{or} \quad \sqrt[6]{\frac{1}{24}}$$

18. Given that $\overline{DE}$ is tangent to the circle at D and $\overline{AC}=6, \overline{BC}=2, \overline{CD}=15, \overline{CE}=9$



What is the length of $\overline{DE}$?

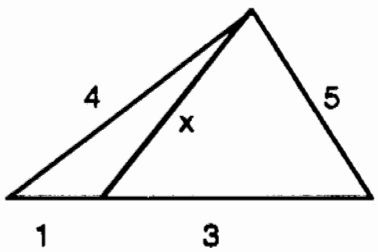
19. Simplify

$$3+x+4[(x-5)+9(2x+6)]-5[-3+5(8x+9)]$$

20. You have 2 homogenous mixtures. One is 20% water and 80% wine while the other is 40% water and 60% wine. You would like to mix these 2 solutions together to get a 5 ounce drink that is 25% water and 75% wine. How much of the 20% water and 80% wine solution do you need?

21. The arithmetic mean of 2 numbers is 13. Their geometric mean is 12. What is their harmonic mean?

22. Find x



23. What are the solutions of

$$\frac{\cos\theta(3-3\sin\theta-2\cos^2\theta)}{\sec\theta-\tan^2\theta\cos\theta}=0$$

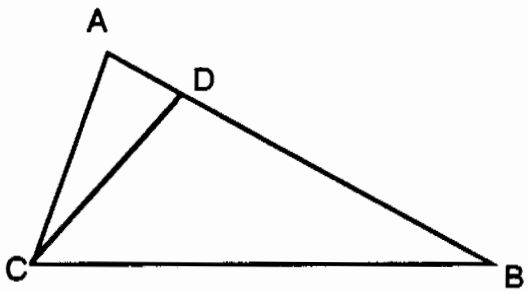
on the interval [0, 2π) ?

24. It is given that x varies directly with y and inversely with the square root of z. If y = 3 and z = 16, then x = 6. If x = 12 and y = 4, what is z?

25. How many digits does the integer part of $(\sqrt{125})^{27}(\sqrt{2})^{81}$ have?

26. Sampson, Tealah and Julie are going trick or treating. If ten identical pieces of candy are divided among the 3 of them, how many ways can this be done?

27. Given $\angle ACB=\angle CDB, \overline{CD}=2.4, \overline{AC}=3, \overline{AB}=10$



Find the perimeter of ΔADC

28. Spencer has a clock with an hour and a minute hand. The current time is between 2 and 3 such that the two hands on his clock line up exactly. How many hours will it take for the hands to line up again?

29. If $x\oplus y=\frac{x-y}{x+y}$ and $x\otimes y=\frac{x+y}{xy}$

Evaluate: $(1\oplus 9)\otimes(9\oplus 8)$

30. There exists a right rectangular prism (a right parallelepiped) with dimensions 21 by 36 by 49, composed of 37,044 unit cubes. If a line is drawn from one corner to the corner furthest away from it, through how many of the unit cubes will the line pass?

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"Math is Cool" Championships-1997-8

February 27, 1998

Individual Multiple Choice Contest, High School

1. What is the logical converse of the statement:
If one is a mathematician, then one has a winning personality.

A) If one is a mathematician, then one has a winning personality.
B) If one has a winning personality, then one is a mathematician.
C) If one is not a mathematician, then one does not have a winning personality.
D) If one does not have a winning personality, then one is not a mathematician.
E) All of these statements are false

---

2. Express $.19\overline{98}$ as a common fraction:

A) $\frac{1998}{9900}$ B) $\frac{222}{1111}$ C) $\frac{1979}{9900}$ D) undefined E) Answer is not given

---

3. Given: $\log(x) = .12$ and $\log(y) = .34$
Evaluate: $\log(x^2y^{-1})$

A) .7058 B) .5800 C) -.1000 D) -.3256 E) undefined

---

4) ΔABC is an isosceles triangle. If $\overline{AB} = 3$ and $\overline{BC} = 7$ what is the length of $\overline{AC}$?

A) 3 B) 5 C) 7 D) 8 E) Not enough information given

---

5. In which base is the following true?

$$3564 + 415 = 4201$$

A) base 6 B) base 7 C) base 8 D) base 9 E) base 13

---

6. For what values of x is this true: $\log(x+5) + \log(\frac{x+15}{x-2}) = \log(\frac{(x+5)(x+15)}{x-2})$

A) $(2,\infty)$ B) $(-15,-5)\cup(2,\infty)$ C) $(-\infty,-5)\cup(2,\infty)$ D) Always True E) Never True

7. Gregg is playing a number game. Each turn of this game consists of multiplying an input number by 4 and then adding 3 to it to produce an output number. If Gregg starts with the number 2, how many turns does it take to produce a perfect square if he uses the output of each turn as the input for the next turn?

A) 3 B) 5 C) 9 D) greater than 10 E) never

---

8. If you roll a fair 4-sided and a fair 8-sided die what is the probability that the sum is 7?

A) $\frac{1}{16}$ B) $\frac{1}{11}$ C) $\frac{5}{32}$ D) $\frac{1}{6}$ E) Answer is not given

---

9. Given: $f(x) = \frac{(x-3)(x+2)^2}{(x^2-x-6)}$

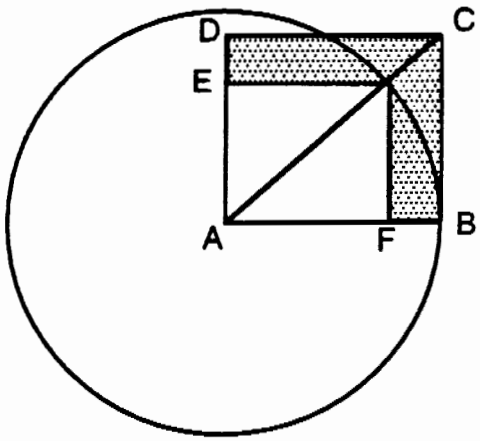
Evaluate f(3):

A) 0 B) 5 C) ∞ D) undefined E) Answer not given

6. Mr. Sampson goes for a walk and notices Robert digging a hole.
"How deep is the hole?" asks Mr. Sampson.
Robert, always one for games, responds, "Guess. I am 6 feet 1 inch tall. When I am finished digging the hole it will be twice as deep as it is now and my head will be twice as far below the ground as it is now above the ground."
Help out Mr. Sampson; how deep is the hole now? Express your answer as a decimal number of inches.

7. Nadav has a digital clock which displays military time. He notices that at 01:21, he can remove the colon to get 121, a perfect square. Nadav also notices that if he converts 01:21 to minutes, he gets 81, which is also a perfect square. When is the next time that both these facts will be true (i.e. both values will be perfect squares)?

8. In unit circle A, point G is the intersection of $\overline{AC}$ and circle A. If $\angle BAC = 30°$, $\overline{BC}$ is tangent to the circle and $ABCD$, $AEGF$ are both rectangles, what is the area of the shaded region?



9. Jude, Josh, Robert, Bill and Mark each own a hat. If one hat is randomly given to each person, what is the probability that no one gets their correct hat?

10. Evaluate

$$77 + \prod_{k=1}^{76} k - \sum_{i=1}^{75} i(i!)$$

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"Math is Cool" Championships-1997-8

February 27, 1998

Pressure Round, High School

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Leave answers in terms of simplified radicals where applicable.
Do not round any answers unless stated otherwise.

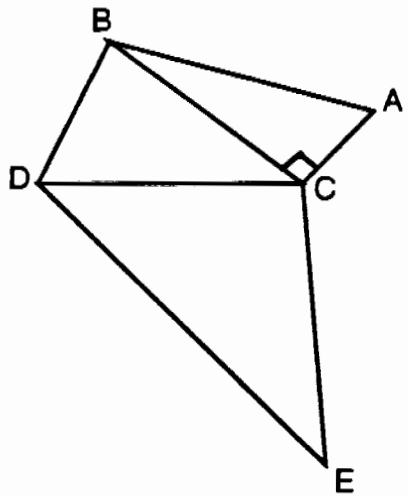
1. An amicable pair is a pair of numbers (a,b) such that the sum of the factors of "a" (excluding "a") equals "b," and the sum of the factors of "b" (excluding "b") equals "a." Which two numbers in the given list form an amicable pair.
70, 134, 136, 220, 260, 284, 302, 328

2. You have a right circular cone with a diameter of 24 meters and a slant height of 13 meters. If the cone is empty and oriented with its point pointed directly down and with water flowing in from the top at a constant rate of 2 cubic meters/ second, how long will it take until the height of the water level is half the height of the cone?

3. How many groups of 4 letters can be made from the letters in "Mississippi"? (Note: The order of the 4 letters does not matter i.e. {m,i,s,s} is the same as {s,m,i,s})

4) Let $i = \sqrt{(-1)}$. Express $(\sqrt{12} + 2i)^5$ in the form $a + bi$ where a and b are real numbers.

5) Given $\overline{AB} = 2\sqrt{6}$, $\overline{AC} = \sqrt{6}$, $\overline{BD} = 4$, $\angle CBD = 45°$, $\angle CDE = 45°$, $\angle DCE = 75°$



What is $\overline{CE}$?

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"Math is Cool" Championships-1997-8

February 27, 1998

Mental Math, High School

Express all answers as reduced fractions in terms of radicals and π, where applicable.

Person #1

1. Evaluate $6 \cdot 3 - 4 + 2$

2. What is the tenth smallest prime?

3. George has a total of twenty nickels and dimes. If the value of these coins is $1.90, how many nickels does he have?

4. How many integer values for x satisfy this inequality $45 < x < 107$

Person #2

1. Evaluate $19 \cdot 98$

2. How many positive integers are less than 15 and relatively prime to 15.

3. The sum of 2 numbers is 34, the difference between them is 19. What is the average of the two numbers?

4. In a right triangle, the length of the hypotenuse is 26 and one of the legs is 24. What is the length of the other leg?

Person #3

1. What is 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64?

2. Ryan has a bag of marbles. In the bag, there are 5 red, 3 blue and 2 orange marbles. If Ryan draws a single marble, what is the probability that the marble is orange?

3. What is the largest counting number less than 10,000 and divisible by 11?

4. What is the product of all integers whose absolute value is less than 100?

Person #4

1. To the nearest tenth, what is $\sqrt{200}$?

2. Six people are seated around a round table, how many ways can they arrange themselves?

3. What is the smallest integer, greater than 30, with an odd number of factors?

4. In a given triangle, the length of one side is 8, the length of another side is 6, and the angle between these sides is $\frac{3\pi}{4}$. What is the area of this triangle?

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"Math is Cool" Championships-1997-8

February 27, 1998

College Knowledge Bowl #1, High School

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1. If $\tan(x) = \frac{\sqrt{3}}{3}$ and x is measured in radians between π and 2π. What is x?

Answer: $\frac{7\pi}{6}$

---

2. What is the sum, in degrees, of 4 of the 5 external angles in a regular pentagon?
Answer: 288 degrees

---

3. Find all zeros of: $f(x) = \frac{x^2+7x+12}{x^2+6x+8}$
Answer: -3 (-4 is not an answer since it is a removable discontinuity)

---

4. Find the vertex of: $f(x) = 5x^2 - 10x + 15$
Answer: (1,10) order matters

---

5. Evaluate: $e^{10\ln(2)}$
Answer: 1024

---

6. If f(x) is an odd function and g(x) is an even function, is f(g(x)) necessarily even, odd, or neither?
Answer: Even

---

7. Find the area of the triangle with sides 5, 7, and 8.
Answer: $10\sqrt{3}$

Extra Question if needed
Find "k" such that the following trinomial is a perfect square:

$$kx^2 + 32x + 64$$

Answer: 4 or k=4

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"Math is Cool" Championships-1997-8

February 27, 1998

College Knowledge Bowl #2, High School

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1. Find the discriminant of: cx^2+ax+b

Answer: a^2-4bc

---

2. Factor completely: $x^3-20x-16$

Answer: $(x + 4)(x^2 - 4x - 4)$

---

3. How many prime numbers are there less than 100?

Answer: 25

---

4. If $x+y=4$ and $x^2y+y^2x=-2$, what is x^3+y^3?

Answer:70

---

5. Find the reciprocal of $2+3i$ in the form $a+bi$ where "a" and "b" are reduced fractions.

Answer: $\frac{2}{13}-\frac{3}{13}i$

---

6. Evaluate the infinite series

$1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots$

Answer: $\frac{3}{2}$ or $1\frac{1}{2}$

---

7. Find the equation of the line, in slope-intercept form, of the line that passes through the point (4,4) and is perpendicular to the line $2x+3y=\pi$.

Answer: $y=\frac{3}{2}x-2$

Extra Question if needed

Calculate the sum from i equals 1 to 100 of $8i^0$.

Answer: 800

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"Math is Cool" Championships-1997-8

February 27, 1998

College Knowledge Bowl #3, High School

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1. Solve the system for y:

$$5x + 3y + 2z = 7$$
$$-2x + 10y + 13z = 30$$
$$14x + 17y + z = -16$$

Answer: y=-2

---

2. How many sides does a regular polygon with 65 diagonals have?
Answer: 13

---

3. In a best of 5 tournament between the Aardvarks and the Banshees the chance of the Aardvarks winning any particular game is $\frac{1}{2}$. What is the probability, as a fraction, that all 5 games of the tournament are needed?

Answer: $\frac{3}{32}$

---

4. Given circle "a" of radius 6, what is the length in terms of π of the arc subtended by an angle of 45 degrees?

Answer: $\frac{3\pi}{2}$

---

5. Evaluate: $i^{-413} + i^{206} + i^{2008}$
Ans: $-i$

---

6. What is the logarithm of $(343)(\sqrt[3]{7})(\sqrt[3]{49})^{-1}$ base 7?

Answer: $\frac{8}{3}$ or $2\frac{2}{3}$

---

7. Given that points (5,3) and (-1,5) are on a circle, find the equation, in slope-intercept form, for the line which contains all possible centers for the circle.
Answer: y = 3x - 2

Extra Question if needed
98 students were given a survey about mathematics. If 35 students liked trigonometry, 71 students liked geometry and 19 students liked neither subject, how many students liked both trigonometry and geometry?
Answer: 27

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"Math is Cool" Championships - 1997-98

February 27, 1998

Individual Test, High School

Score:

Full Name:_____ School:_____

DO NOT WRITE IN SHADED REGIONS

	Answer		
1	1		
2	36 km/hr		
3	$\frac{(1+\sqrt{5})}{2}$		
4	.16 or $\frac{4}{25}$		
5	8:1		
6	$\frac{\pi}{3}$		
7	3		
8	$x^2 + 11x + 32$		
9	$\frac{9}{8}$ or $1\frac{1}{8}$ gallons		
10	$\frac{2}{5}$ or .4		
11	$\frac{2}{9}$ or $.\overline{2}$		
12	$\frac{3\sqrt{3}}{4}$		
13	6π		
14	$\frac{-2}{7}$		
15	19.5		

	Answer		
16	$\frac{2}{5}$ or .4		
17	$\sqrt[3]{\frac{1}{5}}$ or $\frac{\sqrt[3]{25}}{5}$		
18	$2\sqrt{15}$		
19	-123x - 11		
20	3.75 ounces		
21	$\frac{144}{13}$ or $11\frac{1}{13}$		
22	$\frac{\sqrt{61}}{2}$		
23	$\frac{\pi}{6}, \frac{5\pi}{6}$ Order doesn't matter		
24	$\frac{64}{9}$ or $7\frac{1}{9}$		
25	41		
26	66		
27	9		
28	$\frac{12}{11}$ or $1\frac{1}{11}$ hours		
29	$\frac{63}{4}$ or $15\frac{3}{4}$		
30	90		

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"Math is Cool" Championships - 1997-98

February 27, 1998

Individual Multiple Choice Test, High School

Score:

School:_____

Full Name:_____

DO NOT WRITE IN SHADED REGIONS

Please note: correct responses are worth 2 points, incorrect responses are worth -1 point, and no response is worth 0 points.

	Answer		
1	B		
2	C		
3	C		
4	C		
5	C		
6	A		
7	E		
8	E		
9	D		

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"Math is Cool" Championships - 1997-98

February 27, 1998

Team Test, High School

Score:

School:_____

Team:_____

DO NOT WRITE IN SHADED REGIONS

	Answer		
1	$\frac{24}{25}$		
2	$\frac{1}{2}n^2+\frac{1}{2}n+1$ or $\frac{n^2+n+2}{2}$		
3	6		
4	$999(1+\sqrt{5})$ or $999+999\sqrt{5}$		
5	$\frac{1}{7}$		
6	54.75 inches		
7	20:25		
8	$\frac{\sqrt{3}}{12}$		
9	$\frac{11}{30}$		
10	78		

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"Math is Cool" Championships - 1997-98

February 27, 1998

Pressure Round, High School

Score:

School:_____

Team:_____

DO NOT WRITE IN SHADED REGIONS

	Answer		
1	220, 284 (Order doesn't matter)		
2	15π seconds		
3	21		
4	$-512\sqrt{3}+512i$		
5	$\frac{2\sqrt{15}}{3}$		

"Math Is Cool" Championships - 1997-8

Score:

Mental Math - Score Sheet

School:_____ Team#:_____

A. 1. 16

2. 29

3. 2

4. 61

B. 1. 1862

2. 8

3. 17

4. 10

C. 1. 204

2. 1/5 or .2

3. 9999

4. 0

D. 1. 14.1

2. 120

3. 36

4. $12\sqrt{2}$