"Math Is Cool" Championships 98/99

March 5, 1999
Individual Contest, High School

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1. Two squares each have a perimeter of 8 inches. What is the sum of their areas?

2. Suppose $f(x) = 3x + 5$. What is x if $f(x) = 62$?

3. The area of $\triangle ABC$ is 4 sq. units. $\triangle XYZ$ is similar to $\triangle ABC$ and $\dfrac{\overline{XY}}{\overline{AB}} = 3$. What is the area of $\triangle XYZ$?

4. The equation $2_3 \cdot X_3 + 2_3 = 121_3$ is a true statement about base 3 numerals. What is X_3?

5. Two dice are rolled. What is the probability that the sum of the resulting faces is either 3 or 4?

6. Vertex angle C of the isosceles triangle ABC is $48°$. What is the measure of base angle B if $\overline{AC} = \overline{BC}$?

7. $(\log_2 8)^2 + \log_2 8^2 = ?$

8.
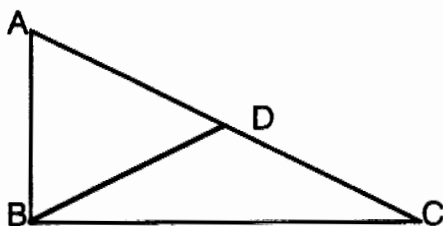
$\overline{AB} \perp \overline{BC}$, $\overline{BD} = \overline{DC}$. If $\angle ADB = 70°$, what is the measure of $\angle ABD$?

9. What is the units digit of 19^{99}?

10. 75 Holly-Mason HS students are reviewing their new schedules. 27 are taking physics, 18 are taking drama and 20 are taking French. Six are taking all three subjects; two are taking French and physics but not drama; one is taking drama and physics but not French; no one is taking French and drama but not physics. How many people are taking none of these subjects?

11. Luther visits REI on Monday and almost buys a pair of jeans for $40. On Wednesday, jeans go up in price by 15%. On Friday, Luther decides to buy the jeans because all merchandise has been marked down by 25%. How much money did Luther save buying Friday instead of Monday?

12.
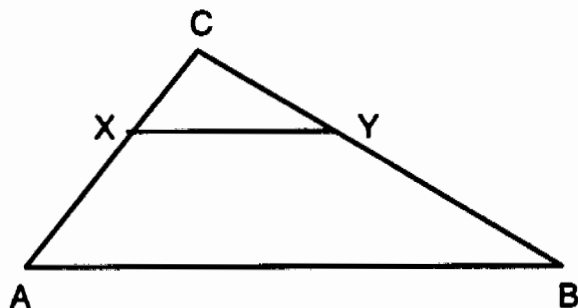
$\overline{AB} \parallel \overline{XY}$, $\overline{XC} = 3$, $\overline{XA} = 6$, $\overline{YC} = 4$, $\angle ACB = 90°$. What is $\overline{AB}$?

13. Consider the following collection of data:
1,1,2,2,2,8,10,14.

What is the mean of the mean, median and mode?

14. Simplify $\sqrt{x^2 + y^2 - 2xy}$ if $y > 0$ and $x < 0$.

15. Every Holly-Mason student must have an identification code consisting of 2 letters followed by a digit (e.g. AX5). If the letters must be different and the digit must be odd, how many codes are possible?

16.
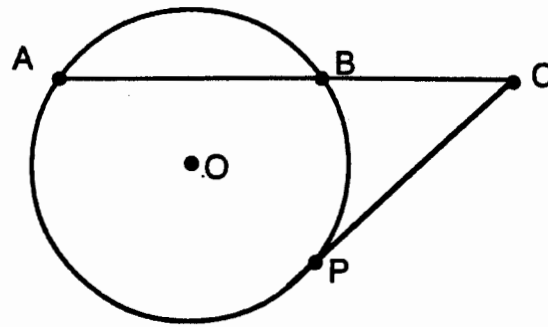
$\overline{CP}$ is tangent to circle O. $\overline{CA}$ is a secant with $\overline{AB} = 12$ and $\overline{BC} = 4$. Determine $\overline{CP}$.

17. Solve for x

$$x+1 = \sqrt{1+\sqrt{1+\sqrt{1+...}}}$$

18. What points do the following functions have in common?

$$f(x) = 2x+1$$
$$f(x) = x^2 - 2$$

19. Expand and simplify
$(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2})^2$ where $i = \sqrt{-1}$ so that $i^2 = -1$.

20. Holly-Mason students are selling chocolate models of their school as a fund-raiser. A small replica sells for $1.50. What should be the price of a replica with all dimensions doubled if the cost per gram is to remain unchanged?

21. George lives in Seattle and wants to visit his parents in Spokane (280 miles away). He drives to Spokane at a constant rate of 120 mph. After his visit he returns to Seattle traveling at a constant rate of 80 mph. What is his average speed in mph?

22. Determine the principal value of

$$Arc\cos(\cos(\frac{4\pi}{3}))$$

23. Solve for x over the reals: $16^x - 5 \cdot 4^{x+1} = -2^6$

24. Bill and Robert are playing a game. Bill has a fair 8 sided die and Robert has a fair coin. Bill goes first. If Bill rolls a 1,3,5,7 or 8 he wins. Otherwise it is Robert's turn. If Robert flips a head he wins, otherwise it is Bill's turn again. Play continues until someone wins. What is the probability Robert wins?

25. What is the area of a regular hexagon inscribed in a circle with diameter 1?

26. What is the period of $f(x)=3\tan(5x-1)+4$?

27. Consider $f(x)=-x^4+3x^3+2x^2+1$. If a,b,c,d are the four zeros of this polynomial, then $a^2+b^2+c^2+d^2 = ?$

28. Which is largest $\sqrt[9]{215}$ or $\sqrt[3]{6}$?

29.
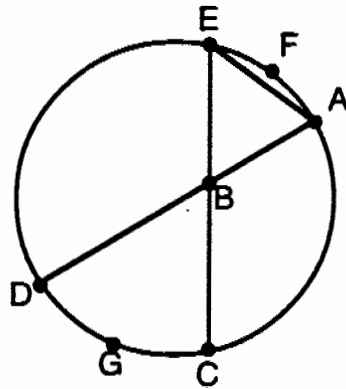
$\overline{AB}=8$, $\overline{BD}=18$, $\overline{CE}=24$, $\overarc{EFA}+\overarc{CGD}=120°$. What is the area of $\triangle ABE$?

30. Simplify to a polynomial of decreasing powers of x:

$$7+x+4(x^2+7(x-3(x^2+8))+6x(3x^3-1))$$

31. You have two homogeneous mixtures. Mixture A is 20% water and 80% wine. Mixture B is 10% water and 90% wine. You want a 10 oz. drink that is 83% wine, 17% water. How much of mixture A do you need to mix with mixture B to get the desired mixture?

32. Evaluate: $\log_3(\ln 8)-\log_3(\ln 2)$

33. Gregg, Karen and Mike each toss a fair coin. If one of them gets a different outcome from the other two, that person wins. If they all get heads or all get tails, they toss again. What is the probability they must all toss more than twice before someone wins?

34. How many solutions does the equation

$$(\sin x)\sqrt{\sin^2 x - \sin^4 x} - (\cos^2 x)(\sin^2 x) + (\sin^2 x)(\cot^2 x) = \frac{1}{4}$$

have on the interval $[0, 2\pi)$?

35. We have 17 pennies to divide among Spencer, Nadav, Ryan and Jude. If Spencer must get at least 4 pennies, Nadav at least 3, Ryan at least 2 and Jude at least 1, how many ways can we distribute the pennies?

36. Let $\triangle ABC$ be a 30-30-120 triangle. Let D be the intersection of the altitudes of $\triangle ABC$. Let E be the intersection of the medians of $\triangle ABC$. Let F be the center of the circumscribed circle of $\triangle ABC$. If $\overline{DE} = 6\sqrt{3}$, what is $\overline{EF}$?

37. Jina can mow a lawn in 3 hours, Josh can mow a lawn in 2 hours and Gregg can mow a lawn in 1 hour. If they work together, how many minutes (rounded to the nearest minute) will it take them to mow 2 lawns?

38. How many counting numbers from 1 to 500 (inclusive) are not divisible by either 3,4,8,9, or 12?

39. $\displaystyle\sum_{n=1}^{1000} \frac{1}{n^2 + n} = ?$

40. How many ways can you arrange the letters in the word Tennessee, if the two n's must be consecutive?

"Math Is Cool" Championships 98/99

March 5, 1999
Multiple Choice, High School

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1. $5x^0 + 6(x+y)^0 + y^0 - 4 = ?$

   A) $11x + 7y - 4$ B) 9   C) 8   D) -4   E) $11x + 2y - 4$

2. What is the result of this base 8 multiplication
   $2607_8 \cdot 32_8 = ?$

   A) 107666_8 B) 83424_8 C) 105424_8 D) 36790_8 E) None of the above

3.
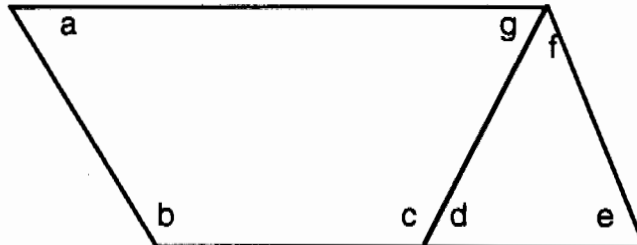
   a,b,c,d,e,f and g are the measures of the 7 non-overlapping angles shown. What is their sum in degrees?

   A) $360°$ B) $450°$ C) $540°$ D) $630°$ E) Not enough information given.

4. What is the remainder when 1111...11 (1999 ones) is divided by 11?

   A) 1   B) 2   C) 3   D) 6   E) 10

5. 7 people sit in a circle. Whenever someone has a coin, he passes it to the person on his left (clockwise around the circle). If the people are numbered 1 to 7 in a clockwise fashion and person 3 starts with the coin, who has it after 1999 passes?

A) person 1   B) person 3   C) person 4   D) person 6   E) person 7

6. Which of the following expressions is equal to $\cos(3x)$?

A) $2\cos^3 x - \cos^2 x$ B) $4\cos^3 x - 3\cos x$ C) $(\sin x)(\cos^2 x - \sin^2 x)$
D) $3\cos x$ E) None of the above

7. Define $(a,b)\oplus(c,d)=(a\cdot c,\ b+d)$. What is an element (x,y) such that $(x,y)\oplus(a,b) = (a,b)\oplus(x,y) = (a,b)$ for all choices of (a,b)?

A) $(1/a,-b)$ B) $(1,1)$ C) $(0,0)$ D) $(1,0)$ E) No such element exists

8. What is the largest prime factor of x if
$\log_{22}(\log_{39}(\log_{35} x))=11?$

A) 13 B) 11 C) 5 D) 2 E) None of the above

9. Consider an equilateral triangle with side length a. In terms of a, what is the difference between the area of the circumscribed circle and the area of the inscribed circle?

A) $\dfrac{\pi a^2}{9}$ B) $\dfrac{\pi a^2}{3}$ C) $\dfrac{\pi a^2}{4}$ D) $\dfrac{2\pi a^2}{9}$ E) $\dfrac{\pi a^2}{3}-\dfrac{\pi a^2\sqrt{3}}{2}$

"Math Is Cool" Championships 98/99

1. In $\triangle ABC$, $\overline{AB} = 4$ and $\overline{BC} = 12$. $\overline{AC}$ is also an integer. What is the sum of all possible values of $\overline{AC}$?

2. Nadav and Spencer each have a Grecian Urn. In Nadav's urn he has 4 black marbles and 5 red marbles. Spencer's urn contains 7 black marbles and 3 red marbles. If Nadav draws a marble from his urn and places it in Spencer's urn and then Spencer draws a marble out of his urn, what is the probability that Spencer draws a red marble?

3. Six people: Laura, Larry, Dora, Durry, Maura and Murray are to be divided into two groups. Each group is to have a president, a vice-president and a third-executive. In how many ways can this be done?

4. $1100 was collected in ticket sales to a concert. 370 people attended. Some paid $2 each for student tickets while the rest paid $5 each for adult tickets. How many student tickets were sold?

5. x varies directly with y^2 and inversely with z. If $y=4$ and $z = 2$, then $x = 12$. If $y=7$ and $z=4$ what is x?

6. $\displaystyle\sum_{k=1}^{12}\binom{12}{k} = ?$ (Note that $\binom{n}{k}$ is the number of ways to choose k objects out of n total objects.)

7.
$g(x) = 5x + 2$
$f(y) = y^2 + y$
$h(z) = z$

What is $g(f(y) + h(f(y)))$ as a polynomial of decreasing powers of y?

8. $A = 1 + \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + ...$

   $B = 1 + \dfrac{1}{2} + \dfrac{1}{4} + ... + \dfrac{1}{256}$

What is A - B?

9. A group of 7 people consists of Jude, Mike, Sam, Robert, Mark, Ryan and Josh. The first 3 are left-handed while the other 4 are right-handed. 3 people are chosen at random from this group. If it is known that at least 2 of the people chosen are left-handed, what is the probability that the three people chosen are Jude, Sam and Robert?

10. Consider an ellipse centered at the origin with a semi-major axis of length 4. Let point $p = (1, \dfrac{3\sqrt{5}}{4})$ be on the ellipse. What is the sum of the distances of p to the foci of the ellipse?

"Math Is Cool" Championships 98/99

March 5, 1999
Pressure Round, High School

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1. Where do these lines intersect ?

   $y = 2x + 1$
   $y = 5x - 5$

2. What is the harmonic mean of 2,4,8 ?

3. What is the 6th smallest prime number that appears in the Fibonacci sequence. (The Fibonacci sequence is given by $a_1 = 1$, $a_2 = 1$, and $a_n = a_{n-1} + a_{n-2}$).

4. What is the largest prime factor of 1999?

5. $(\sin 165°)(\cos 15°) = ?$

"Math Is Cool" Championships 98/99

March 5, 1999
Mental Math, High School

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Person 1

1. The sum of two consecutive integers is -1999. What is the smallest of the two integers?

2. What does $(\sqrt[3]{64})^2 + (\sqrt{64})^3$ equal?

3. If $x + y = 14$ and $x - y = 10$. What is $3x$?

4. The surface area of a cube is 12 sq meters. What is the volume of the cube?

Person 2

1. What is the slope of the line with equation $x + 4y = 3$?

2. What is the smallest non-negative integer divisible by 2,3,4,5 and 6?

3. Suppose $8^x = 7$. What is 2^{6x}?
   [*Read as: "Suppose 8 to the power x equals 7. What is 2 to the power 6x?"*]

4. A dodecagon has 12 sides. How many diagonals does a regular dodecagon have?

Person 3

1. 6791 and A are both prime numbers, it is also true that 6791 + A is prime. What is A?

2. What is the difference between the sum of the interior angles and the sum of the exterior angles of a regular pentagon?

3. What is the geometric mean of 2, 4 and 8?

4. What is $6+7+8+9+10+...+20$?

Person 4

1. Using a fair coin, heads is flipped five times in a row. What is the probability that the next flip is a heads as well?

2. Let x, y be two distinct positive integers such that $x^y = y^x$
   What is x times y?

3. How many even integer values of x satisfy $27 < x \le 56$?

4. What is the area of a triangle with side lengths 4, 6 and 8?

"Math Is Cool" Championships 98/99

March 5, 1999
College Knowledge Bowl 9-10, High School

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Divine's Auto Service

College Knowledge Bowl 9-10, Round #1

1. What is $(8+3)^2 - 8^2 - 3^2$?   [*Read as "What is the quantity 8 plus 3 squared minus 8 squared minus 3 squared?"*]   Answer: 48

2. Bill rolls two, fair six-sided dice. What is the probability the sum of the rolls is 8?

    Answer: 5/36

3. Give, in slope-intercept form, the equation of the line which is the perpendicular bisector of the segment whose endpoints are (1,1) and (5,3).

    Answer: y = -2x + 8

4. Solve for x: $x^2 - 7x + 12 = 0$ Answer: x=3 and x=4
    [or, the solutions are 3 and 4]

5. Consider a triangle with sides 5,12 and 14. Is the triangle acute, obtuse or right?

    Answer: obtuse

6. Factor completely:
    $x^3 + 2x^2 - 5x - 6$

    Answer: (x-2)(x+1)(x+3)

7. What is the measure, in degrees, of a single interior angle of a 15 sided regular polygon?

    Answer: 156°

Extra Question if Needed:
    What is the sum of the first 20 positive even integers?

    Answer: 420

1. How many ways can you pick 5 objects out of 7 different objects?

   Answer: 21

2. What is the volume of a circular cone with a slant height of 13 if the base of the cone has a perimeter of 24π?

   Answer: 240π

3. What is the area of a rhombus with diagonals length 10 and 12?

   Answer: 60

4. If log x = .3 and log y = .4, what is $\log(xy^2)$?
   [*Read* "$\log(xy^2)$" *as log of the quantity* xy^2.]

   Answer: 1.1

5. What is the reciprocal of the sums of the reciprocals of 2,3 and 4?

   Answer: 12/13

6. Given a right triangle with a hypotenuse of length 100 and one leg of length 28, what is the length of the other leg?

   Answer: 96

7. Bill rolls a pair of fair, 4 sided dice. What is the probability that he rolls a total of 4?

   Answer: 3/16

Extra Question if Needed:
   Simplify: $(2x^3y^3)^2 + (3x^2y^2)^3$
   [*Read as* "*the quantity* $2x^3y^3$ *squared plus the quantity* $3x^2y^2$ *cubed*".]

   Answer: $31x^6y^6$

1. Factor completely: $x^2 - 784$

   Answer: $(x - 28)(x + 28)$

2. What is the smallest prime number larger than 31?

   Answer: 37

3. For what real values of x is the following function non-negative:
   $\dfrac{x^2 - 15x + 56}{x - 1}$?

   [*Read as "the quantity* $x^2 - 15x + 56$ *divided by the quantity* $x - 1$".]

   Answer: $x \geq 8$ or $1 < x \leq 7$ (need both)

4. A certain cube has a volume of 2 cubic units. If we triple its side length, what is its new volume?

   Answer: 54 (cubic units)

5. Given a 3-4-5 right triangle, what is the cosine of the angle opposite the side of length 4?

   Answer: 3/5

6. An integer is chosen randomly from 1 to 10 inclusive. What is the probability that the number is prime?

   Answer: 2/5

7. Bill's age is twice the difference between Gregg's age and Robert's age. When Robert is three times as old as he is now, Gregg will be 71. Currently, the sum of all three of their ages is 73. How old is Bill now?

   Answer: 22

Extra Question if Needed:

What is the equation of the line, in slope intercept form, of the line passing through the point (7,0) and perpendicular to the line $x + y = 1$.

   Answer: $y = x - 7$

"Math Is Cool" Championships 98/99

March 5, 1999
College Knowledge Bowl 11-12, High School
Sponsored by
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Divine's Auto Service

College Knowledge Bowl 11-12, Round #1

1. What is the smallest positive integer that has remainder 1 when divided by 7 and remainder 4 when divided by 3?

    Answer: 43

2. Simplify: $i^{234} + i^{15} + i^{-7}$

    Answer: -1

3. What is $\cos^2(-40°) + \sin^2(-40°) = ?$

    Answer: 1

4. In a given triangle, one angle is of measure 45° and the opposite side has length $\frac{1}{2}$. Another angle in the triangle is 75° and has opposite side of length $\frac{1+\sqrt{3}}{4}$. What is the length of the remaining side?

    Answer: $\frac{\sqrt{6}}{4}$

5. Bill wants to buy 6 flowers. If he can choose from roses, tulips and carnations, each in unlimited supply, how many different collections of flowers can he buy?

    Answer: 28

6. What is the remainder when $x^{101} - 2$ is divided by $x + 1$?

    Answer: -3

7. What is the radius of the circle given by the equation $x^2 + y^2 + 4x - 8y = 81$?

    Answer: $\sqrt{101}$

Extra Question if Needed:
    What is the 9th digit after the decimal point in the decimal representation of 1/7?

    Answer: 2

1. Give the equations of all vertical asymptotes of the function:
   $$f(x) = \frac{x^3 - 3x^2 + 3x - 1}{2x^2 - 18x + 36}.$$

   Answer: x = 3 and x = 6 (either order)

2. If x = ln(10) and y = log$e$, what is x times y?
   [*Read* "ln(10)" *as "the natural log of n".*]   Answer: 1

3. $1^2 + 2^2 + 3^2 + ... + k^2 = 285$. What is k?

   Answer: 9

4. At Big Kahuna Pizza there are up to seven different toppings you can
   order. Two pizzas are considered the same if they are the same size
   and have the exact same toppings. If Big Kahuna pizza wants to
   advertise that they can make over 500 different types of Pizzas, what
   is the smallest number of different sizes of pizza they must they sell?

   Answer: 4

5. What is the length of the longest diagonal in a box with width 5, height
   8 and length 11?

   Answer: $\sqrt{210}$

6. Express $\cos^2 \frac{\pi}{8}$ in the form p + q$\sqrt{2}$ where p and q are rational numbers.

   Answer: $\frac{1}{2} + \frac{1}{4}\sqrt{2}$

7. Evaluate: $\lim\limits_{x \to \infty} \frac{(3x^2 + 1)(2x + 4)}{8x^3 + 6x^2 + 1}$. [*Read as "the limit as x approaches infinity of the*
   product of the quantities $3x^2 + 1$ *and* $2x + 4$ *divided by the quantity* $8x^3 + 6x^2 + 1$".]

   Answer: $\frac{3}{4}$

Extra Question if Needed:
   Bill draws two cards from a standard deck of 52 cards. What is the
   probability that one is black and the other is red?

   Answer: $\frac{26}{51}$

1. What is the maximum area that can be enclosed with 10 meters of fence?

    Answer $\dfrac{25}{\pi}$ square meters

2. Let $f(x) = x^6 + x^5 - 4x^4 - 6x^4 + x^2 + 5x + 2$. It is given that $x = 1$ is a zero of this polynomial. How many positive, real zeros does $f(x)$ have?
   [*Read* "$f(x)$ as f of x."]

    Answer: 2

3. Calculate $\displaystyle\sum_{k=0}^{5} 3 \cdot (\tfrac{1}{3})^k$.   [*Read as "the sum as k goes from 0 to 5 of 3 multiplied by the quantity one-third to the k power."*]

    Answer: $\dfrac{364}{81}$ or $4\dfrac{40}{81}$

4. If p is a positive integer, what are the possible remainders when p^3 is divided by 9?

    Answer: 0,1 and 8

5. For what values of x does $\ln(x^2 - 25) - \ln(x + 5) = \ln(x - 5)$?
   [*Read "ln" as "the natural log of".*]

    Answer: $x > 5$

6. Evaluate e^x where $x = 4\ln(5)$. [*Read "$4\ln(5)$" as "4 times the natural log of 5".*]

    Answer: 625

7. What is the vertex of the parabola given by $x = 2y^2 + 4y - 1$?

    Answer: (-3,-1) order matters

Extra Question if Needed:
   Express in the form a+bi, where a and b are reduced fractions:
   the reciprocal of 4 - 3i.

    Answer: $\dfrac{4}{25} + \dfrac{3}{25}i$

Math Is Cool" Championships -- 1998-9

High School - March 5, 1999

School Name_____Team #_____

Proctor Name_____Room #_____

Full Name:_____

Individual Contest - Score Sheet
DO NOT WRITE IN SHADED REGIONS

	Answer		
1.	8(sq inches)		
2.	19		
3.	36(sq inches)		
4.	21_3		
5.	5/36		
6.	66(°)		
7.	15		
8.	55(°)		
9.	9		
10.	25		
11.	($)5.50		
12.	15		
13.	3		
14.	y-x or -x+y		
15.	3250		
16.	8		
17.	$(-1+\sqrt{5})/2$		
18.	(-1,-1),(3,7) (order doesn't matter)		
19.	i		
20.	($)12.00		

	Answer		
21.	96(mph)		
22.	$2\pi/3$		
23.	x=2 or x=1 (Need both)		
24.	3/13		
25.	$3\sqrt{3}/8$		
26.	$\pi/5$		
27.	13		
28.	$\sqrt[3]{6}$		
29.	$24\sqrt{3}$		
30.	$72x^4-80x^2+5x-665$		
31.	7 (oz)		
32.	1		
33.	1/16		
34.	8		
35.	120		
36.	$3\sqrt{3}$		
37.	65 (Minutes)		
38.	250		
39.	1000/1001		
40.	840		

Math Is Cool" Championships -- 1998-9

High School - March 5, 1999

School Name_____Team #_____

Proctor Name_____Room #_____

Individual Multiple Choice Contest-Score Sheet

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

1st Score

Out of 18

DO NOT WRITE IN SHADED REGIONS

	Answer		
1.	C		
2.	A		
3.	C		
4.	A		
5.	E		
6.	B		
7.	D		
8.	E		
9.	C		

Math Is Cool" Championships -- 1998-9

High School - March 5, 1999

School Name_____ Team #_____

Proctor Name_____ Room #_____

Key

Team Contest-Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer		
1.	84		
2.	32/99		
3.	360		
4.	250		
5.	$18\frac{3}{8}$ or $\frac{147}{8}$		
6.	4095		
7.	$10y^2 + 10y + 2$		
8.	1/256		
9.	1/13		
10.	8		

Math Is Cool" Championships -- 1998-9

High School - March 5, 1999

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Pressure Round - Score Sheet

	Answer		
1.	(2,5)		
2.	$\frac{24}{7}$ or $3\frac{3}{7}$		
3.	233		
4.	1999		
5.	1/4		

Math Is Cool" Championships -- 1998-9

High School - March 5, 1999

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Mental Math - Score Sheet

A. 1. -1000

2. 528

3. 36

4. $2\sqrt{2}$ (Meters³)

B. 1. -1/4

2. 0

3. 49

4. 54

C. 1. 2

2. 180

3. 4

4. 195

D. 1. $\frac{1}{2}$ or .5

2. 8

3. 15

4. $3\sqrt{15}$