"Math is Cool" Masters-2003-04

Sponsored by: ZAK DESIGNS

11th & 12th Grade - November 22, 2003

Individual Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

1	Evaluate: $8 \div 2^2 + 6 \cdot (9 - 4)$
2	What is the degree measure of one of the exterior angles of a regular 40-gon?
3	Simplify: $\sqrt{4320}$
4	What is the sum of the factors of 3720?
5	What is the measure, in degrees, of the angle supplement to the complement of an 80° angle?
6	Solve for x: $12x^2 + x = 35$
7	If $f(3x) = \dfrac{x}{x+1}$, determine $f(5)$.
8	How many ways can five students be elected to the student council if eight students run for the positions?
9	Evaluate s_{2003} given that $s_1 = 1$ and $s_{n+1} = s_n - \frac{1}{n+1} s_n$.
10	When the sum of fourteen and nine is squared, the result is eighty-seven greater than what number?
11	In ΔABC, $m\angle A = 60^\circ$, $m\angle B = 45^\circ$, and $AC = 6$ cm. What is the length of $\overline{BC}$?
12	What is the volume (in cm^3) of the largest sphere that can be inscribed in a right rectangular prism with edge lengths of 3 cm, 4 cm, and 6 cm?
13	A triangular number is a number which is equal to the sum of the first n natural numbers, for some natural number n. What is the fifth smallest triangular number?
14	Evaluate: $\log_{64} 81^{\log_{27} 16}$
15	When four fair coins are flipped, what is the probability that exactly two of them are tails?

16	A perfect number is a natural number which is equal to the sum of all of its positive integer divisors (not including itself, of course). 28 is a perfect number, because $28 = 1 + 2 + 4 + 7 + 14$. What is the smallest perfect number?
17	If $r(s) = 3s^2 - 8s + 5$ and $t(u) = \dfrac{3u}{2u+4}$, what is $t(r(2))$?
18	If $k^{\log_2 5} = 32$, evaluate $k^{(\log_2 5)^2}$.
19	Which is bigger, 3^{19} or 10^9?
20	How many four-digit numbers contain only the digits 2, 3, 5, or 7 and are multiples of 4?
21	How many of the following sets are subsets of the set of all irrational numbers? The set of all rational numbers The set of all imaginary numbers The set of all complex numbers The set of all real numbers The set of all integers
22	What is the product of all values of h for which $h^{\log h} = 1000h^2$?
23	A certain prison uses x liters of orange juice per inmate per week. At this rate, for how many weeks will y liters supply z inmates? Give your answer in terms of x, y, and z.
24	If $f(x) = 3x + 8$, evaluate $f^{-1}(-19)$.
25	Evaluate: $\begin{vmatrix} 4 & 0 & 1 & 2 \\ -1 & 0 & 2 & 0 \\ -2 & 3 & 1 & -4 \\ 3 & 0 & -1 & 5 \end{vmatrix}$
26	Evaluate: $\left(\log_5 9\right)\left(\log_{\sqrt{3}} 125\right)$
27	What is the edge length of a cube with twice the volume of a cube of edge length 1?
28	Solve for x, $0 \le x < 2\pi$, if $3\sin^2 x = \dfrac{1}{\sec^2 x}$
29	Evaluate: $\sin\left(\text{Arctan}\left(-\sqrt{3}\right) + \text{Arcsec}(2)\right)$

Challenge Questions

30	What is the area enclosed by the graph of the parametric equations: $x = 5\sin t$, $y = 3\cos t$?
31	In an urn are 17 numbered discs. Eight are red, 5 are white, and 4 are blue. In how many ways can 2 red, 1 white, and 2 blue discs be chosen without regard to order?
32	The number 555^{100} has **n** positive integer factors. How many positive integer factors does **n** have?
33	In graph theory, a graph consists of vertices (points) and edges (curves connecting two vertices). A Hamiltonian path is a route which traverses all edges exactly once. How many Hamiltonian paths are there in the graph described? [The graph is a square (four large dots connected by straight lines) with one diagonal from upper right to lower left]
34	Chord AB intercepts an arc that is 1/4 of the circumference of a circle. If chord AB is n units long, find the diameter of the circle in terms of n.
35	There are positive integers k, n, and m such that 19/20 < 1/k +1/n + 1/m < 1. What is the smallest possible value of k + n + m?
36	A parabola in $y = ax^2 + bx + c$ form passes through the points $(0,-2)$, $(4,0)$, and $(6,-2)$. Determine $a + b + c$.
37	For what value of q$(0 \le q < 25)$ is $18742 \equiv q \bmod 25$?
38	The three distinct single digit integers a, b, and c are combined to form all possible 3-digit integers which use all three of these digits. The sum of all these 3-digit integers is 1776. What is the sum of a + b + c?
39	A function of the form $w(v) = v^3 + bv^2 + cv + d$ (where b, c, d, are real) has zeros of 4 and $3+i$. Determine the sum of b, c, and d.
40	The base 10 number 1987 can be written as a 3-digit number **xyz** in some base b. If **x** + **y** + **z** = 1 + 9 + 8 + 7, determine the values of **b**, **x**, **y**, and **z**. Give your answer as an ordered quadruple (**b**, **x**, **y**, **z**).

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Individual Multiple Choice Contest

Bubble answers on colored sheet

1	Alice, Bob, and Carol repeatedly take turns tossing a die in alphabetical order by their names. Find the probability that Carol will be the first person to toss a 6. (A) 1/3 (B) 2/9 (C) 5/18 (D) 25/91 (E) 36/91
2	A palindrome is a number that remains unchanged when the digits are reversed. In base b, how many integers with (2n+1) digits are palindromes? (A) (b^{2n+1}) (B) (b^{2n}) (C) (b^{n+1}) (D) $(b-1)(b^n)$ (E) $(b-1)^{2n}$ (F) (b^n)
3	Convert the polar coordinates (3, 30°) into rectangular coordinates. (A) $\left(\frac{3\sqrt{2}}{2}, \frac{3}{2}\right)$ (B) $\left(3, \frac{3\sqrt{2}}{2}\right)$ (C) $\left(\frac{3}{2}, \frac{3\sqrt{2}}{2}\right)$ (D) $\left(\frac{3\sqrt{2}}{2}, 3\right)$ (E) none of these
4	How many prime numbers between 10 and 99 remain prime when their digits are reversed? (A) 9 (B) 10 (C) 11 (D) 12 (E) 13
5	How many pairs (m,n) of integers satisfy the equation m + n = mn? (A) 1 (B) 2 (C) 3 (D) 4 (E) more than 4
6	The altitude drawn to the base of an isosceles triangle is 8, and this triangle's perimeter is 32. The area of the triangle is: (A) not determinable without more information (B) 48 (C) 32 (D) 24 (E) none of these
7	Let n be a positive whole number. The units digit of the sum 1 + 2 + 3 + ... + n cannot be equal to: (A) 0 (B) 2 (C) 5 (D) 6 (E) 8
8	For how many integral values of n is the fraction (n + 13)/(n - 4) an integer? (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
9	The expression $1/(\log_2 36) + 1/(\log_3 36)$ is equivalent to which one of the following? (A) $2/(\log_5 36)$ (B) $\frac{1}{2}$ (C) $[(\log_{36} 2) + (\log_{36} 3)]/(\log_6 36)$ (D) 2 (E) none of these

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Team Contest

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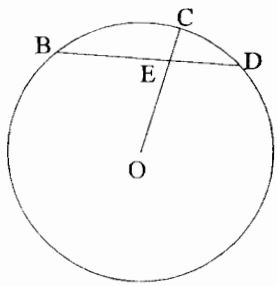
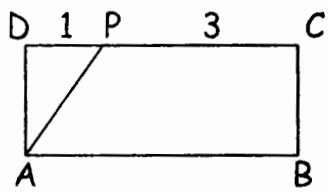
1	DEB is a chord of a circle such that DE=3 and EB=5. Let O be the center of the circle. Join OE and extend OE to cut the circle at C. Given EC=1, find the radius of the circle. 
2	Simplify $[(1\cdot2\cdot4 + 2\cdot4\cdot8 + \ldots + n\cdot2n\cdot4n)/(1\cdot3\cdot9 + 2\cdot6\cdot18 + \ldots + n\cdot3n\cdot9n)]^{1/3}$
3	Let n be an integer. If the tens digit of n^2 is 7, what is the units digit of n^2?
4	Two Martians enter a math contest in which all the other competitors are from Venus. Contestants compete pair-wise, with each contestant playing once against each of the other contestants. A contestant receives one point for a win, $\frac{1}{2}$ point for a tie, and zero points for a loss. The two Martians together score 8 points, and each Venusian scores the same number of points as every other Venusian. How many Venusians participated in the contest?
5	How many arrangements of JUPITER are there with the vowels occurring in alphabetical order?
6	What is the shape of the polar graph $r = \sin\theta$?
7	Consider the rectangle ABCD with DP = 1 inch and PC = 3 inches. If a point is picked at random in the interior of rectangle ABCD, what is the probability it is also in the interior of triangle APD? 
8	In the expansion of the binomial $(3x + 2y)^8$, what is the coefficient of the term with x^2 in it?
9	Twenty-two points are equally spaced around a circle and numbered consecutively. A line is drawn through the center of the circle and passes between points 7 and 8. What is the sum of the other two consecutive numbers the line passes between?
10	Find the domain of the function $f(x) = \ln(2 - x)$.

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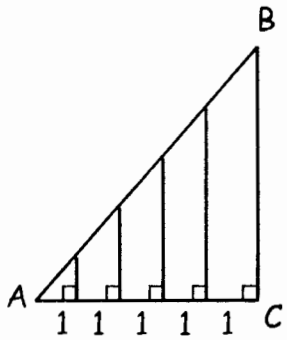
Pressure Round

1	Let p and q stand for distinct positive prime numbers, each less than 100. The sum of p + q has a units digit of 0. How many distinct values are possible for the sum of p + q?
2	What shape is the graph of the parametric equations $x = 2 - \tan t$ $y = 4\sec t + 1$?
3	Find the number of ways to order 10 books in one row on a shelf given the following conditions: The books are numbered 1-10. The books numbered 4 through 6 must be together, but not necessarily in that order. Book number 5 must have at most one book between it and #9.
4	What is the sum of the lengths of the five vertical segments? The measure of angle A equals 60°. 
5	Three people are sitting around a table. A fourth person goes to each person and paints either a red dot or a black dot on that person's forehead. Assume that the probability of either color is $\frac{1}{2}$ for each person. Each person can see the dots on the other people's foreheads but not their own. Furthermore, no communication among these people is allowed. Each person must guess the color of their own dot based solely on what they see. If they have worked out the best possible strategy beforehand, what is the probability they will all guess correctly?

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Mental Math

Express all answers as reduced fractions in terms of radicals and π, where applicable, unless otherwise instructed.

Person 1		
1	Evaluate: tangent of 135 degrees.	-1
2	The sum of two numbers is 1/2 while the difference between the two numbers is 2. What is the smaller of the two numbers?	-3/4
3	Jim drives 40 miles to his mother's house and then drives back on the same road. On the way there, he drives 30 miles per hour, but on the way back he drives 20 miles per hour. What is his average speed in miles per hour for the whole trip?	24[mph]
4	What is the farthest straight-line distance between any two vertices of a cube of edge length 9?	$9\sqrt{3}$
Person 2		
1	Find the sixth term of a geometric sequence whose first term is 4, and for which the constant ratio is $\frac{1}{2}$.	1/8
2	The sum of 5 consecutive odd integers is 575. Find the difference between the largest and smallest of the numbers.	8
3	How many diagonals are in a decagon?	35
4	What is the sum of the solutions of the equation $x^3 + 5x^2 - x - 6 = 0$?	-5
Person 3		
1	What is the period of $f(x) = -3 \sin 2x$. Read as: (What is the period of f of x = -3 times the sine of 2x.)	π
2	What is the slope of the line perpendicular to $3x + 4y = 7$?	4/3
3	Eight to the power x equals 32. What is x?	5/3
4	A sphere of radius 3 is inflated so it has a surface area of 144π. What is the ratio of the new volume to the old volume?	8:1
Person 4		
1	Simplify $\log_4 8$. (Read: log to the base 4 of 8.)	3/2
2	When a card is drawn from a standard 52-card deck, what is the probability it is a red nine?	1/26
3	Evaluate $3x^2 - 5x + 1$ when x = 5.	51
4	What is the equation of the axis of symmetry of the graph of $y = 3x^2 - 12x + 14$?	$x = 2$

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School Name_____Team #_____

Proctor Name_____Room #_____

Key

Full Name:_____

1st Score

Out of 40

Individual Contest - Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	32		
2	9 [degrees]		
3	$12\sqrt{30}$		
4	11520		
5	170[°]		
6	x = {5/3, -7/4}		
7	5/8		
8	56[ways]		
9	1/2003		
10	442		
11	3√(6)		
12	9π/2[cm^3]		
13	15		
14	8/9		
15	3/8		
16	6		
17	1/2		
18	3125		
19	3^{19}		
20	48		

	Answer	1 or 0	1 or 0
21	0		
22	100		
23	y/(xz)		
24	-9		
25	-105		
26	12		
27	cube root of 2		
28	π/6, 5π/6, 7 π/6, 11 π/6		
29	0		
30	15 π		
31	840[ways]		
32	4[factors]		
33	12[paths]		
34	n√2		
35	12		
36	-3/4		
37	17		
38	8		
39	-16		
40	(19, 5, 9, 11) order matters		

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Proctor Name_____Room #_____

Key

Individual Multiple Choice Contest-Score Sheet

1st Score

Out of 18

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer	-1, 0 or 2	-1, 0 or 2
1	D		
2	D		
3	E		
4	A		
5	B		
6	B		
7	B		
8	E	4	
9	B		

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School Name_____Team #_____

Proctor Name_____Room #_____

Key

Team Contest-Score Sheet

1st Score

Out of 10

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	8		
2	2/3		
3	6		
4	15[Venusians]		
5	840[arragements]		
6	circle		
7	1/8		
8	16128		
9	37		
10	$x < 2$ or $(-\infty,2)$		

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School Name_____Team #_____

Proctor Name_____Room #_____

Key

Pressure Round - Score Sheet

	Answer		
1.	18		
2.	hyperbola		
3.	57,600		
4.	$15\sqrt{3}$		
5.	1/2		