"Math is Cool" Masters-2003-04

Sponsored by: Western Polymer Corporation

7^{th} - 8^{th} Grade - November 22, 2003

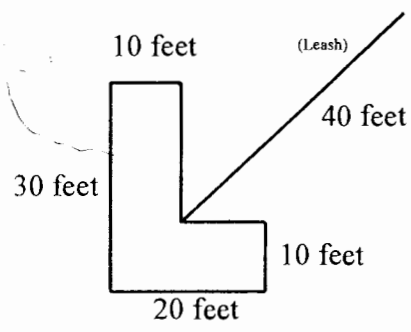
Individual Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

1	How many sides does a nonagon have?
2	Evaluate: $5 + 4 \div 2 + 3(4)^3$
3	The sum of two numbers is 29, while the difference is 15. What is the smaller of the two numbers?
4	Express $.\overline{81}$ as a reduced fraction.
5	The Lewis and Clark Math Team has 130 members. The Lewis and Clark football team has 12 members. The combined teams total 141 members. How many students are on both the football team and the math team?
6	Find the sum of the first 10 odd numbers.
7	Find the sum of the mean, median, and mode of the following data set: {11, 15, 18, 17, 17, 14, 13}.
8	Penguin Computing has three different types of computers: Basic, Intermediate, and Advanced. Each type supports the "Linux", "Windows XP", and "Windows 2000" operating systems. Customers also have a choice of 6 different accessory packages. How many different arrangements of type, operating system, and accessory packages are there?
9	There are three red marbles, four yellow marbles, eight green marbles, and seventeen blue marbles in a basket. What is the probability that the first two marbles, drawn without replacement, are green?
10	In 1972, the Dole company produced a pineapple weighing a record high of 9 pounds. If the pineapple was a cylinder, how many ways could the pineapple be cut into congruent sections if only four straight cuts are made?
11	Bill's car consumes 1 gallon of gas for every 26 miles he drives, regardless of driving conditions. It also uses a fixed amount of gas whenever Bill starts the car. If Bill starts his car 3 times during a 39-mile trip and uses 2 gallons of gas, how many gallons of gas are used to start his car?
12	At the Penguinland National Canoeing Meet, contestants have to canoe both up and down a 5 mile long river. The average time to go upstream is 20 minutes, and the average time to go downstream is 5 minutes. What is the average speed of the contestants in mph?

13	If $f(x)=x^3+4x^2-5x+2$, what is $f(5)$?
14	What is the probability of rolling 3 fives on a 7 sided fair die in 3 rolls?
15	What is the volume of a sphere with a radius of 15/2?
16	In a closed bottle, the product of the pressure and the volume is constant. By what percent must the volume be decreased to increase the pressure by 25%?
17	What is the 51^{st} term of the arithmetic sequence -19, -16, -13...?
18	Let a#b indicate the number yielded by $\frac{ab-a!}{b!(a^b)}$. What is 2#3?
19	The elevator in the 100-floor George Tower is broken. The buttons are numbered 1 through 100 for each floor in the building. When a button is pushed, the elevator goes to the square of the quantity of the square root of the number on the button plus one. How many different buttons would get you to whole numbered floors between 1 and 100 inclusive?
20	How many diagonals can be drawn in a polygon with 13 sides?
21	Five friends purchase a gourmet pizza and decide to split the cost evenly. However, one can only pay half of his share. This means the other four friends must each pay an extra $1.00 to cover the cost. How much did the pizza cost? Answer in dollars.
22	Find an expression for the arithmetic mean of 4x+ 3, 3x -2, 9x, and 7.
23	David and Andy are writing math problems for next year's "Math is Cool" competition. If David can write 5 questions in 25 minutes and Andy can write 6 questions in 15 minutes, how long, in minutes, will it take both of them together to write 30 questions?
24	A ribbon is tied tightly around the earth at the equator. How much more ribbon, in feet, would be needed if the ribbon was raised 12 feet above the equator everywhere. Assume the earth is a perfect sphere.
25	The volume of a certain sphere is $4\pi/3$. What is the circumference of a great circle on this sphere?
26	In the equation U + ME = US, U, M, E, and S each represent a different digit. If S represents zero and M represents six, what does E represent?
27	Tom is rolling 2 regular 12-sided dice. If, after each roll, he records the sums of the two numbers on the dice, what number is most likely to be the mode of the numbers he records after 100 rolls?
28	On her last 8 tests, Kali earned scores of 61, 72, 60, 77, 84, 80, 90, and 76. She wants her test average to be 80 or greater after she takes the next two tests. What is the greatest product of the least integral scores she needs to get in order to achieve this?
29	If Luke's math class stands in a circle and starts numbering off starting with one, Luke says the number 5 the first time around and 59 the third time around. How many people are in his class?

Challenge Questions

30	A semicircle is folded into the shape of a cone. If the area of a semicircle is 16π, find the volume of the cone.
31	Let $i = \sqrt{-1}$. Evaluate $(1+i)^2$. Leave answer in terms of i.
32	The propellers on an airplane rotate at a speed of 200 revolutions per minute. What length, in feet, must the blades be so that they rotate at 100 miles per hour at the tip?
33	How many integer values of x satisfy $\lvert x^2 - x - 25 \rvert \leq 5$?
34	Austin eats daisy chains. When it is sunny outside, he eats 3 per day. Otherwise, he eats 1 per day. For a certain week, the probability it will be sunny each day is 50%. What is the probability that, at the end of the week, Austin has consumed between 14 and 18 daisy chains?
35	Penguin High School's ASB is trying to choose new school colors. They can choose either two or three colors from twelve different colors. What is the total number of choices they have?
36	Simplify: $\dfrac{\frac{x}{y} - \frac{y}{x}}{\frac{1}{x^2} - \frac{1}{y^2}}$
37	Simplify $\sqrt{9 + \sqrt{9 + \sqrt{9 + \cdots}}}$
38	A dog is attached to a house to a leash with a length of 40 feet. What is the total possible area, in square feet, the dog has access to while on his leash? 
39	A fly is standing on a Cartesian plane at the origin. He walks in a straight line for 13 units and finds himself in Quadrant I, five units away from the line x=0. Next, he walks directly to the point (1,9). He then walks 10 units directly towards the point (-11,-7). Finally, he walks in a complete circle with (0,1) as the center. How many units has the fly walked in its entire trip?
40	Bag A contains 8 green marbles and 1 yellow marble, and Bag B contains 5 green and 3 yellow marbles. A bag is picked randomly, and a marble is drawn from it and placed in the other bag. What is the probability that a marble drawn from the second bag (the bag that received a marble) is green?

"Math is Cool" Masters-2003-04

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7^{th} - 8^{th} Grade - November 22, 2003

Individual Multiple Choice Contest

Record answers on colored sheet

One day, Peter Parker, William Henry Harrison, Luke Skywalker, and Bruce Wayne decided to build remote control toy cars. After building their cars, these superheroes thought it would be fun to race them, but before they raced, each superhero tested his car by finding the distance his car traveled for a given time. The results of the tests are recorded in the table below. Assume that each car starts at a time and distance of 0, and that the trends presented in the table below stay consistent. The bottom row contains the equations that model distance as a function of time. Some data and equations are missing from the table.

Peter's Car		William's Car		Luke's Car		Bruce's Car	
Time (seconds)	Distance (meters)	Time (seconds)	Distance (meters)	Time (seconds)	Distance (meters)	Time (seconds)	Distance (meters)
3	21	4	18	1	10	?	12
6	42	8	36	4	20	4	24
9	?	14	63	?	40	6	36
12	84	16	72	25	50	8	48
$d = 7t$		?		$d = 10\sqrt{t}$		?	

Questions

1	How many meters has Peter's car traveled in 9 seconds? (A) 12 (B)63 (C) 112 (D) 84 (E) none of these
2	For William's car, find an equation that models distance as a function of time. (A) d = (9/2)t (B) d = .4t (C) d = (18/7) t (D) d = (9/4)t (E) none of these
3	For Bruce's car, find an equation that models time as a function of distance. (A) d = 6t (B) t = 6d (C) d = 11t (D) t = d/6 (E) none of these
4	Whose car has the slowest average speed for the first second of travel? (A) Peter's (B) William's (C) Luke's (D) Bruce's
5	Peter and William decided to race their cars before racing with the other two superheroes. How far apart, in meters, will Peter's car be from William's car after 16 seconds? (A) 112 (B) 91 (C) 32 (D) 58 (E) none of these
6	Whose car can travel 50 meters the fastest from a stand still? (A) Peter's (B) William's (C) Luke's (D) Bruce's
7	Instead of launching William's car and Bruce's car from two tracks side by side, the race car judge decided to launch the cars from opposite ends of one track, with William's car being on the left side and Bruce's car being on the right. What fraction of the entire track will William's car travel before being hit by Bruce's car? (A) 2/5 (B) 11/12 (C) 1/2 (D) 3/7 (E) none of these
8	How many meters from the starting line will Peter's car and Luke's car be tied in the race after the beginning of the race? (A) 100/7 (B) 100/49 (C) 1/7 (D) 10 (E) none of these
9	Feeling left out, Scott Summers decided to build his own remote-controlled toy car. How fast, on average, in meters per second, would his car need to go in order to win a 300 meter race by 10 seconds? (A) 8 (B) 9 (C) 10.5 (D) 11 (E) none of these

"Math is Cool" Masters-2003-04

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7^{th} - 8^{th} Grade - November 22, 2003

Team Contest

Express all answers as reduced fractions unless stated otherwise.
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Record all answers on colored cover sheet.

1	Keisha and Brian have respectively 20 cents and 60 cents more than Elizabeth. Together, the three of them have $X. If X is an integer, what is the smallest possible value of X?
2	A bag contains red, blue, and yellow cough drops. Each variety of cough drop has a different mass, but cough drops of the same variety have the same mass. Kendra leaves only 10 blue and 2 red drops in the bag and finds that the mass of the bag and the drops is 128 grams. With 9 red and 6 yellow drops in the bag, the total mass is 174 grams. With 6 each of red, yellow, and blue drops in the bag, the total mass is 210 grams. The bag alone weighs 12 grams. What is the total mass of one cough drop of each variety?
3	What is the sum of all values of y such that the point (8,y) is 5 units away from the point (4,8)?
4	Grampy Sampy's Discount Produce sells tomatoes for half price of what tomatoes cost at Silas Stupendous Souk during the week. However, on weekends Silas discounts his tomatoes by 20 cents per pound. If Libber buys 3 pounds of tomatoes on a weekend from Silas' Stupendous Souk and pays $5.40, how much, in dollars, would 5 pounds of tomatoes cost at Grampy Sampy's during the week?
5	Keisha is twice as old as Tassie. Three years ago Keisha was three times as old as Tassie. How old was Tassie three years ago in years?
6	Evaluate: $100_2 + 100_3 + 100_4 + \ldots + 100_{10}$ in base 10.
7	What is the largest prime factor of 95 factorial?
8	What is the largest 2-digit composite number "n", such that, the square root of "n" can only be written as "a" square root of "b" where "a" is one, and "b" is "n"?
9	A water system hook up for a house with a pipe of diameter $\frac{1}{2}$ inch costs 16,000 dollars. What would the price be if the pipe was 3/4 of an inch in diameter assuming that cost varies directly with cross sectional area and the same total length of pipe is used?
10	Silas rolls three dice. What is the probability that the sum of the three values is 13?

"Math is Cool" Masters-2003-04

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7^{th} - 8^{th} Grade - November 22, 2003

Pressure Round

1	A certain type of bacteria divides once every 6 minutes. If it takes one bacterium 2 hours to completely fill a jar, how many minutes will it take 4 bacteria to completely fill the same jar?
2	A square which contains an inscribed circle is inscribed within a different circle. If the side length of the square is 4 units, what is the area of the region bounded by the two circles?
3	Let b be an integer such that 2614 in base b equals 1420 in base 10. What is b?
4	Let an "abundant" number be defined as a composite number whose divisors (excluding itself) add up to create a sum greater than the number itself. What is the sum of the first six abundant numbers?
5	When a number that is four greater than my age is squared and then added to my age, the result is 1118. What is my age in years?

"Math is Cool" Championships-2003-04

Sponsored by: Western Polymer Corporation

7^{th} - 8^{th} Grade - October 24, 2003

Mental Math

Express all answers as reduced fractions in terms of radicals and π, where applicable, unless otherwise instructed.

Person 1		
1	Solve for "x" in the equation $5x + 3 = 13$	2
2	What is the difference between 9/8 and 8/7?	-1/56
3	The hypotenuse of an isosceles right triangle is $4\sqrt{2}$. Find the area of the triangle.	8
4	For how many distinct positive integers n will 60/n also be an integer?	12
Person 2		
1	What is the least common multiple of 12 and 18?	36
2	What is the volume of a square pyramid of height 8 and base area 3?	8
3	Find the smallest integer value of x for which two times the absolute value of x is less than 17.	-8
4	What is the ratio of the number of odd factors of 36 to the number of its even factors? Express as a fraction.	$\frac{1}{2}$
Person 3		
1	What is the smallest prime number greater than 53?	59
2	Evaluate 3 to the 4^{th} power minus 4 cubed.	17
3	Tom went to the pet store and bought a cat and a rat for a total cost of $200. The cat cost $100 more than the rat. How much, in dollars, did the rat cost?	[$]50.00
4	What is the area of a regular hexagon with sides of length 1?	$\frac{3\sqrt{3}}{2}$
Person 4		
1	What is the greatest common factor of 14 and 42?	14
2	After picking apples from my apple tree, I gave 7 pounds to Jimbob, then one quarter of what was left to Jimmyjames. If I end with 42 pounds, how many pounds did I have to begin with?	63[pounds]
3	What is the volume of a cylinder with height 2 and radius 3?	18π
4	Six students split up work on a school assignment. If 3 students do their fair share on the project, one student does twice his fair share, one student does 1/3 of his fair share, how much of his fair share does the last student do?	2/3

"Math is Cool" Masters-2003-04

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7^{th} - 8^{th} Grade - November 22, 2003

College Knowledge Bowl Questions #1

1	Mr. Ed has a 2 out of 7 chance of winning a certain race. What are the odds against Mr. Ed for this race?	5 to 2
2	A log is cut into 4 pieces in 12 seconds using parallel slices. At this rate, how many seconds will it take to cut the log into 15 pieces?	56[seconds]
3	How many 2-digit integers consist of two distinct even digits?	16[integers]
4	What is the reciprocal of the square root of the reciprocal of 4/9?	2/3
5	Evaluate: The quantity negative one-half raised to the power negative 2.	4
6	What is the units digit of 3 raised to the 107^{th} power?	7
7	How many days are in 3600 minutes? Express your answer as a decimal.	2.5[days]
	Extra Question: Only use it if needed	
8	What is the length of the diagonal of a square with side length 8?	$8\sqrt{2}$

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College Knowledge Bowl Questions #2

1	Express one-fourth of the quantity 2 to the 9^{th} power as a power of 2.	2^7
2	What is the sum of the first 20 even natural numbers?	420
3	How many ways can 8 people be seated at a round table?	5040[ways]
4	What is the sum of all distinct positive prime factors of 2004?	172
5	Penguin High School is choosing a new ASB President, Vice President, and Treasurer. 12 people are applying for any of the positions. How many ways can the President, Vice President, and Treasurer be elected?	1320[ways]
6	How many real values of x satisfy the equation $x^2 = -x$?	2
7	The sum of the interior angles of a certain polygon is 3240 degrees. How many sides does this polygon have?	20[sides]
	Extra Question: Only use it if needed	
8	For what values of x is the inequality $x^2 > x^3$ true?	$x<1$, $x\neq 0$, [both must be stated]

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7^{th} - 8^{th} Grade - November 22, 2003

College Knowledge Bowl Questions #3

1	The sum of two integers is 61. What is the largest possible product of these two integers?	930
2	What is the sum of all values of x for which $\|5x+20\|=18$? (Read as the absolute value of the quantity 5x+20 equals 18)	-8
3	Find the slope of the line parallel to a line passing through the points (3, 7) and (-2, -3).	2
4	What is the largest integer less than 100 that has a remainder of 3 when divided by 5?	98
5	What is the maximum number of pieces that can be cut from a circular pizza with four straight cuts without stacking or folding the pizza?	11[pieces]
6	Nine distinct points are placed around a circle. Each point is connected to every other point by a line segment. How many line segments are there total?	36[line segment]
7	Evaluate: 1011_2+1101_2 in base two	$11000_{[2]}$
	Extra Question: Only use it if needed	
8	A 12 foot by 6 foot floor is completely covered with 4 inch by 3 inch tiles. How many tiles are there on the floor?	864[tiles]

"Math is Cool" Masters -- 2003-04

7th & 8th Grade - November 22, 2003

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Full Name:_____

1st Score

Out of 40

Individual Contest - Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	9		
2	199		
3	7		
4	9/11		
5	1[student]		
6	100		
7	47		
8	54[arragments]		
9	7/124		
10	5 [ways]		
11	1/6 [gallon]		
12	24 [mph]		
13	202		
14	1/343		
15	$1125\pi/2$		
16	20[%]		
17	131		
18	1/12		
19	9[buttons]		
20	65[diagonals]		

	Answer	1 or 0	1 or 0
21	[$]40		
22	$4x + 2$		
23	50 [min]		
24	24π [feet]		
25	2π		
26	3 or three		
27	13		
28	10000		
29	27[people]		
30	$16\pi\sqrt{6} / 3$ [units cubed]		
31	2i		
32	$22/\pi$ [feet]		
33	4		
34	7/16		
35	286[choices]		
36	-xy		
37	$(1 + \sqrt{37})/2$		
38	850π [square feet]		
39	$28 + 10\pi$[units]		
40	9829/12960		

"Math is Cool" Masters -- 2003-04

7th & 8th Grade - November 22, 2003

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Individual Multiple Choice Contest-Score Sheet

1st Score

Out of 18

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer	-1, 0 or 2	-1, 0 or 2
1	B		
2	A		
3	D		
4	B		
5	E		
6	A		
7	D		
8	A		
9	E	210/23 meters/sec	

"Math is Cool" Masters -- 2003-04

7th & 8th Grade - November 22, 2003

School Name_____Team #_____

Proctor Name_____Room #_____

Key

1st Score

Out of 10

Team Contest-Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	[$]2.00 or 2		
2	33 [grams]		
3	16		
4	[$]5		
5	3[years]		
6	384		
7	89		
8	95		
9	36,000 [dollars]		
10	7/72		

"Math is Cool" Masters -- 2003-04

7th & 8th Grade - November 22, 2003

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Pressure Round - Score Sheet

	Answer		
1.	108 [minutes]		
2.	4π [un^2]		
3.	[b =] 8		
4.	140		
5.	29[years]		