"Math is Cool" Championships – 2004-05

Sponsored by: Basic American Foods

11^{th} – 12^{th} Grade – October 13, 2004

Individual Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

1	What is the sum of the roots of the polynomial: $3x^3 + 2x^2 - 5x + 7$?
2	Evaluate: $\log_2 64$
3	Convert 2817_9 to base 3
4	Evaluate: $\log_3 32 * \log_2 81$
5	What is the slope of the line perpendicular to $y = x\sqrt{2} + 7\sqrt{3}$?
6	What is the largest number less than 100 with exactly 12 factors?
7	How many more arrangements of the word 'squares' are there than of the word 'square'?
8	What is the determinant of the matrix $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 3 & 4 & 2 \\ 1 & 0 & 2 & 17 \\ 3 & 0 & 7 & 33 \end{bmatrix}$?
9	A circle is inscribed in a square with perimeter 16, what is the circumference of the circle?
10	What is the prime factorization of 180?
11	What is the sum of the series 2+5+8 . . . + 44 ?
12	If z varies directly with the sum of x and y, and z=12 when x=1 and y=5, what is z equal to when x=7 and y=-12?
13	How many positive palindromes less than 10,000 have at least three identical digits?
14	How many lattice points (points with integer coordinates) satisfy both $y \geq 0$ and $y \leq -x^2 + 4x$?
15	What is the sum of the infinite geometric series $3 + 2 + 4/3 + 8/9 \ldots$?
16	If the surface area of a sphere is $\frac{25\pi}{4}$ square centimeters, what is its volume, in

	cubic centimeters?
17	What is the area of the ellipse defined by the equation $\frac{x^2}{20}+\frac{y^2}{45}=1$?
18	A cube is constructed such that its volume in cubic centimeters is half its surface area measured in square centimeters, what is the space diagonal of the cube, in centimeters?
19	How many positive numbers less than 24 are relatively prime to 24?
20	The vector $\langle 2,7\rangle$ is rotated $\frac{3\pi}{4}$ radians counter clockwise. What is the result?
21	After the 2004 'Math is Cool Masters' competition, a small party took place. If the 14 people there all shook hands with each other, how many handshakes took place?
22	Evaluate the infinite expression: $2\sqrt{3+2\sqrt{3+2\sqrt{3+\ldots}}}=?$
23	Solve for all values of θ between 0 and 2π which satisfy $\sec^2\theta-\tan^2\theta+\cos^2\theta=\frac{5}{4}$.
24	How many positive numbers less than 100 have a remainder of 3 when divided by 5, and a remainder of 4 when divided by 7?
25	What is the magnitude of the vector $(1,2,3)\times(4,5,6)$?
26	Jack is currently running laps around a track defined by the equation $x^2+y^2=16$, while Sierra is walking along a path defined by the parabola $y=x^2-4$. If Sierra stops and waits for Jack to come by each time she crosses the track, at what point(s) will they meet?
27	What is the maximum value of the function $f(x)=\cos(x)-x^2+4\pi x+7$?
28	A fish is randomly swimming around in a fishbowl in the shape of a pyramid with a square base of side length 8, and a height of 14. At any given time, what is the probability that the fish is within 7 vertical units of the base of the pyramid?
29	Evaluate: $7^{\log_{49}3}$

Challenge Questions

30	What is the sum of the values of x such that $\left(x^2-9x+14\right)^{\left(x^2-25x+46\right)}=0$?
31	One afternoon five friends meet up to play ultimate frisbee, all bringing their own Frisbee. In how many ways can all but one person leave with someone else's Frisbee?
32	What is the sum of the following numbers in base (-2): $10111_{-2}+11010_{-2}$?
33	What is the remainder when 2^{101} is divided by 13?
34	A certain right rectangular prism has surface area equal to 148 and its side lengths form an arithmetic series with difference 1 between terms. What is the volume of the shape?
35	A pyramid has height 8 and a hexagonal base inscribed in a circle with radius 6. What is the surface area of the pyramid?
36	What is the largest possible area of a triangle inscribed in a circle with area 25π ?
37	What is the area bounded by the x-axis and the parabola which passes through the points (-2,0) , (6,0) , and (2,9) ?
38	If x is a randomly chosen point on the interval $[-1,1]$, what is the expected value of $f(x)=\sqrt{1-x^2}$?
39	If the harmonic mean of x and y is $\frac{1}{2}$, and the harmonic mean of x^2 and y^2 is $\frac{1}{5}$, what is the harmonic mean of x^3 and y^3 ?
40	What is the maximum number of terminal zeros when the function $f(x)=x^4+4x^3-12x^2$, for integral values of $x\geq 2$, is evaluated and expressed in base x?

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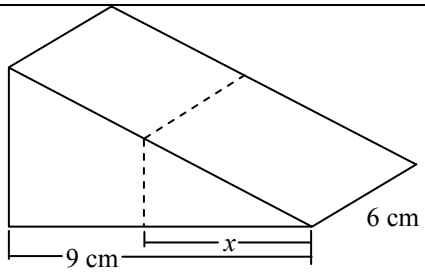
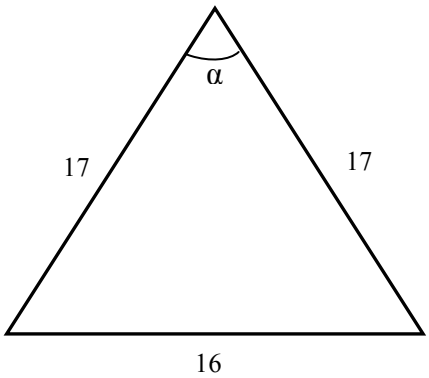
Individual Multiple Choice Contest

Record only a letter as your answer on the colored sheet.

1	What is the domain of $f(x)=\sqrt{x+2}$? A) $x \le -2$ B) $x \ge -2$ C) $x > -2$ D) $x < -2$ E) answer not given
2	A wedge (a right triangular prism) of cheese is cut with a single vertical cut to remove a smaller wedge, leaving 2/3 of the original volume of cheese. What is length **x**, in cm?  A) 3 B) 4 1/2 C) $3\sqrt{3}$ D) $3^{5/3}$ E) answer not given
3	Solve for **x**: $\log_{\sqrt{2}} (1/4)^{-2} = x$. A) 16 B) $\sqrt{2}$ C) 2 D) 4 E) answer not given
4	For the triangle shown, find sin α. A) 64/289 B) 256/289 C) 240/289 D) 16/17 E) 15/17 F) answer not given 
5	How many real solutions does the equation $x^2 - 4\lvert x \rvert + 2 = 0$ have? A) 0 B) 1 C) 2 D) 3 E) 4 F) answer not given

6	Over the past d days, Rebecca watched 7 movies in all, either in the afternoon or at night. She never watched a video in the afternoon on a day when she watched one at night. There were 6 afternoons and 5 nights when she didn't watch a video. Find d. A) 7 B) 9 C) 10 D) 11 E) answer not given
7	When Linda started working a math problem, it was between 8 AM and 9 AM and the hands of her watch coincided. She finally solved the problem that afternoon between 2 PM and 3 PM, when the hands of her watch were 180° apart. How long did it take her to solve the problem? A) 6 hours B) 6 hours and 43 $7/11$ minutes C) 5 hours and 16 $4/11$ minutes D) 6 hours and 30 minutes E) answer not given
8	If w wabbits can wepair a warren in w weeks, then how many weeks would it take w+3 wabbits to wepair an identical warren? A) w+3 B) $w/(w+3)$ C) w-3 D) $w^2/(w+3)$ E) $(w-3)/2w$ F) answer not given
9	What type of number is $(\sqrt{50}^{\sqrt{50}})^{\sqrt{50}}$? A) an irrational number B) a perfect fifth power C) a perfect square D) a triangular number E) a perfect cube

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Team Contest

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1	What is the area of the region satisfying $x - y \geq -1$, $6x + y \leq 22$, and $x + 6y \geq -8$?
2	How many ways can you make change for \$100 using only \$1, \$5, \$10, and \$20 bills? Note: you are not required to use each type of bill in each way of making change; i.e. 100 \$1 bills is an acceptable way to make change for \$100.
3	An equilateral triangle is circumscribed about a regular hexagon with two-centimeter sides such that three of the sides of the hexagon are coincident with portions of the sides of the triangle. What is the area of the triangle, in square centimeters?
4	The Iccanobif sequence is defined recursively as $I_1 = I_2 = I_3 = 1$ and $I_n = I_{n-2} + I_{n-3}$ for $n > 3$, beginning 1, 1, 1, 2, 2, 3, 4, 5, 7, 9, 12, … . Determine the value of I_{25}.
5	If $\sin(r) = \frac{2}{5}$, evaluate $\cos(2r)$.
6	What is the area of a circle that shares a center with the hyperbola $16(x-3)^2 - 9(y+4)^2 = 1$ and passes through the foci of that hyperbola?
7	A sphere and a right circular cylinder have the same volume, but the radius of the sphere is three times that of the cylinder. If the height of the cylinder is 90 centimeters, what is the radius of the sphere, in centimeters?
8	In order to randomly choose a day of the week, I roll two dice and make my decision based on the sum of the numbers on their upper faces. If the sum is 4, 5, 6, 7, 8, 9, or 10, I choose Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, or Saturday respectively. If any other sum is rolled, I ignore that sum and re-roll the dice, continuing to re-roll as necessary until I get a sum between 4 and 10 inclusive. What is the probability that I end up choosing Tuesday?
9	Determine the ordered triple, (a,b,c), which satisfies the equations $4a + 2b - c = -6$, $3a - 3b - 2c = -11$, and $-a + b + 4c = -3$.
10	In square ABCD, line segments $\overline{BE}$ and $\overline{CF}$ are drawn, with F being the midpoint of side $\overline{AD}$ and E lying on $\overline{AD}$ such that $AE = 2ED$. If G is the point of intersection of $\overline{BE}$ and $\overline{CF}$, what is the ratio of the area of triangle EFG to that of square ABCD?

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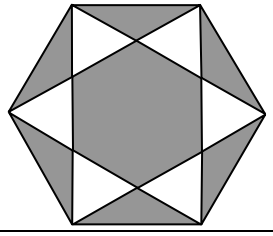
Pressure Round Contest

1	The perimeter of a certain regular hexagon is $12\sqrt{3}$ units. This hexagon is subdivided by connecting its vertices as shown. What is the total shaded area, in square units? 
2	Find the sum of $i^{(0!)} + i^{(1!)} + i^{(2!)} + \ldots + i^{(2002!)} + i^{(2003!)} + i^{(2004!)}$, where $i^2 = -1$.
3	A lattice point is a point whose coordinates are integers. How many lattice points are on the boundary or inside the region bounded by the **x**-axis, the line **x** = 4, and the parabola $\mathbf{y} = \mathbf{x}^2$?
4	For a collection of five positive integers (**n**, 10, 3, 9, 6), the mean of the collection is equal to its median. Find the sum of all possible values for **n**.
5	Four white unit cubes are glued together face to face in an arrangement selected at random. All 8 distinct arrangements of four cubes are equally likely. The structure is then painted red on all exposed surfaces. Next it is broken apart into the unit cubes from which it was made. One cube is selected at random and rolled. As a common fraction, find the probability that the cube will land with a red face up.

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Mental Math Contest

Express all answers as reduced fractions in terms of radicals and π where applicable, unless stated otherwise.

PERSON 1		
1	What is the maximum product that can be obtained from two real numbers that sum to one-hundred?	2500
2	If "x" is an angle in the second quadrant with a sine of five-thirteenths, what is the cosine of x?	-12/13
3	I toss a coin three times; given that there is at least one tail, what is the probability that I toss exactly two heads?	3/7
4	What is the slope of the line with the equation y = 3x-5?	3
PERSON 2		
1	What is the sum of the first one-hundred positive integers?	5050
2	What is the log base 2 of 12, minus the log base 2 of 3?	2
3	What is the lateral surface area, in square centimeters, of a right circular cone with a base diameter of six centimeters and a height of four centimeters?	15π [cm^2]
4	What is the area of a circle with diameter 10?	25π
PERSON 3		
1	If a car traveled at 60 mph for 3 hours, how many miles did it travel?	180 [miles]
2	The sun is 60 degrees above the horizon, and a tree casts a thirty-meter shadow on level ground. How many meters tall is the tree?	$30\sqrt{3}$
3	A circle of radius five centimeters is inscribed in a triangle that has a perimeter of sixty centimeters. What is the area of the triangle, in square centimeters?	150 [cm^2]
4	What is the result when the largest four-digit base five number is expressed in base ten?	624
PERSON 4		
1	What is the arithmetic mean of 81 and 17?	49
2	What is log base 2 of 3, times log base 3 of 4?	2
3	One leg of a right triangle is nine centimeters and the other leg and hypotenuse differ by one centimeter. What is the length of the hypotenuse, in centimeters?	41
4	How many zeros does twenty factorial end in?	4

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COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	Captain Sparrow and Captain Barbossa have decided to play a game of chicken on the high seas. The two ships are 200 meters apart. Sparrow's ship travels at 20 meters per second while Barbossa's travels at 30 meters per second. The ships parrot flies back and forth between the two ships the entire time. If the parrot flies at 70 meters per second, what is the total distance traveled by the parrot?	280 [meters]
2	What is the sum of the squares of the roots of the polynomial: x cubed minus three x squared minus thirteen x plus 15?	35
3	What is the amplitude of the function: f of x equals three sine of the quantity, 2x plus 1, minus 5.	3
4	If the sum of a number and its reciprocal is three, what is the sum of the cube of the number and the cube of its reciprocal?	18
5	What is the perimeter of an ellipse with an axis length of four and eccentricity zero?	4π
6	How many ways can three couples be seated in a row so that wives and husbands are seated next to each other?	48 [ways]
7	The geometric mean of five numbers is four. If the geometric mean of two of the numbers is three, what is the product of the other three numbers?	1024/9
	Extra Problem – Only if Needed	
8	How many radians does the hour hand of a clock move between noon today and 3 pm tomorrow?	$9\pi/2$ [rad]

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COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	If there are three waggles in a wiggle and seven woggles in a wiggle, how many woggles are in two waggles?	14/3 [woggles]
2	What is the remainder when x to the sixth power minus two times x to the fourth power plus three times x cubed plus 8 is divided by x minus 1?	10
3	What is the sum of the common logarithms of the first and third prime numbers?	1
4	When a two-digit positive integer is multiplied by the sum of its digits, the result is thirty-six. What is the number?	12
5	If "y" equals five, what is y to the fifth power divided by twenty-five, plus y?	130
6	The roots of the equation: the quantity x to the seventh power minus one divided by the quantity x minus one equals zero, when graphed in the complex plane, form the vertices of what convex polygon?	Hexagon
7	If six woodchucks can chuck 9 cords of wood in 3 days, how much wood could a woodchuck chuck in a day if woodchucks could chuck wood, in cords?	1/2 [cord]
	Extra Problem – Only if Needed	
8	Evaluate the tangent of the quantity arctangent of three plus arctangent of four.	-7/11

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COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	What is the area of a triangle formed on a sphere of radius four if the triangle has angles of 75, 90, and 60 degrees?	4π
2	Nathan and Adam plan to meet between 4 pm and 5 pm on Tuesday to discuss the merits of a salary cap in Major League Baseball. Both of them arrive at random times, wait ten minutes, and leave. What is the probability they miss each other?	25/36
3	Express one-two-six base 10 in base 3.	11200_3
4	What is the x-coordinate of the vertex of the parabola y equals x squared minus two x plus one?	1
5	What is the y-coordinate of the center of the ellipse with equation: four times x-squared plus nine times y-squared minus twenty-four x plus thirty-six y plus thirty-six equals zero?	-2
6	What is the radius of the inscribed circle in a right triangle with legs 8 and 15?	3
7	What is the area of a rectangle whose length is three times its width and whose perimeter is thirty-two?	48
	Extra Problem – Only if Needed	
8	Find the limit as x approaches infinity in the following expression: the quantity three x minus four divided by the quantity x minus one.	3

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Final Score: KEY

First Score

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	$-2/3$		
2	6		
3	$2220121_{(3)}$		
4	20		
5	$\frac{-\sqrt{2}}{2}$		
6	96		
7	1800		
8	3		
9	4π		
10	$2^2 \bullet 3^2 \bullet 5$		
11	345		
12	[z=] -10		
13	18 [palindromes]		
14	15 [points]		
15	9		
16	$\frac{125\pi}{48}$		
17	30π		
18	$3\sqrt{3}$ [cm]		
19	8 [numbers]		
20	$\left\langle -\frac{9\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2} \right\rangle$		

	Answer	1 or 0	1 or 0
21	91 [handshakes]		
22	6		
23	$\theta = \left(\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right)$		
24	3		
25	$3\sqrt{6}$		
26	$(\sqrt{7},3), (-\sqrt{7},3), (0,-4)$		
27	$8 + 4\pi^2$		
28	7/8		
29	$\sqrt{3}$		
30	7		
31	45 [ways]		
32	$1101001_{(-2)}$ or $-111011_{(-2)}$		
33	6		
34	120		
35	$18\sqrt{91} + 54\sqrt{3}$		
36	$\frac{75\sqrt{3}}{4}$		
37	48		
38	$\pi/4$		
39	$1/14$		
40	4		

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School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score

(out of 18)

STUDENT NAME_____

Individual Multiple Choice Contest – Score Sheet

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer	-1, 0 or 2	-1, 0 or 2
1	B		
2	C		
3	E (8)		
4	C		
5	E		
6	B		
7	A		
8	D		
9	B		

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School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score

(out of 10)

STUDENT NAME_____

Team Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	$\frac{35}{2}$, $17\frac{1}{2}$, or 17.5		
2	286 [ways]		
3	$9\sqrt{3}$		
4	616		
5	$\frac{17}{25}$		
6	$\frac{25\pi}{144}$		
7	$\frac{15}{2}$ [cm]		
8	$\frac{1}{6}$		
9	$(-3,2,-2)$ (must be as an ordered triple)		
10	$\frac{1}{84}$		

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School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score

STUDENT NAME_____

Pressure Round Answers

	Answer
1	$12\sqrt{3}$ [un^2]
2	1999 + 2i
3	35 [points]
4	26
5	71/96