"Math is Cool" Master's – 2004-05
Sponsored by: Zak Designs, Inc.
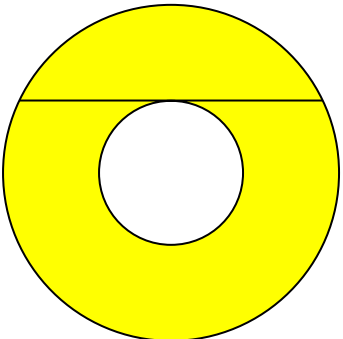### 11th – 12th Grade – November 20, 2004
Individual Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

1	What is the slope of the line perpendicular to the line passing through (-1,-4) and (0,7)?
2	Evaluate: 5 - 2(3-4)-8(6-11)
3	What is the perimeter of a regular hexagon with a distance from the center to a vertex of 7?
4	Reduce the following fraction and write as an improper fraction. (-81/-54)
5	Evaluate: $\log_{27}9$
6	Solve for all values of x: $(x+3)^2 = 1$
7	What is the area of the triangle formed by the lines y=2, y=x+1, and x=6?
8	Two lighthouses start pointing their beams in the same direction at 11:11 PM. One completes its rotation every 18 minutes and the other every 28 minutes. What is the next time they will both be pointing in the same direction? (Express your answer in terms of AM or PM.)
9	What is the vertex, expressed in the form of (x,y), of the parabola: $f(x) = 3x^2 -6x + 11$?
10	Solve for x in base 7: $41_6 + 10011_5 + x_7 = 1961_{10}$
11	Evaluate: Tan 60E
12	The measure of the interior angles of a pentagon are (x+3)°, (2x-8)°, (6x+1)°, (4x-3)°, and (x+9)°. Solve for x. Write your answer as an improper fraction.
13	What is the square root of the product of the first five positive perfect squares?
14	What is the equation of the horizontal asymptote of the rational function $y = \dfrac{-2x^2 + 5x - 11}{3x^2 + 5}$?
15	What is the period of the function $y = 2\tan(\dfrac{3}{8}x + \dfrac{1}{2})$?

16	What is the probability of rolling three six-sided, fair dice and obtaining a sum of 9?
17	How many values of θ, for $0 < \theta < 15\pi$ make the following equation true? $121\cos(2\theta + \pi/4) = 0$
18	Jarret's license plate has 4 digits and 2 letters, with the digits all before the letters. Exactly three of the four digits are the same. The two letters are identical. Knowing all this, how many possible license plates are there for Jarret?
19	What is the surface area of an ice cream cone, which is made of a hemisphere placed on top of a cone with equal radii, with a total height of 8 and a radius of 2?
20	What is the slant asymptote, g(x), of the rational function given by $f(x) = \dfrac{x^2 + 5x - 11}{x + 5}$
21	Evaluate $\sum\limits_{n=0}^{\infty} 3 \cdot (\frac{1}{4})^n$
22	How many positive integers less than 1200 have an odd number of distinct factors?
23	A group of 6 friends went to the movies. Colin and Megan insist on sitting next to each other and Abe wants to sit next to them as well. Lee, Dani, and Libbey also want to sit together. How many ways can they do this?
24	Colin has a $\frac{1}{2}$ probability of waking up each time his alarm rings in the morning. His alarm continues going off at regular intervals. How many times must his alarm ring for his mom to figure there is at least an 85% probability that he is awake?
25	A mad soda mixer wants a drink that is 30% cranberry juice, 50% club soda, and 20% orange juice. He currently has a 24 ounce drink that is 50% cranberry juice and 50% orange juice. If his assistant spilled all the orange juice so all he has left to add is more cranberry juice and some club soda to the mix, how many additional ounces of fluid does he need to add?
26	What is the area of the shaded region of the concentric circles? Line tangent to inner circle has length 16.
27	What is the acute angle, in degrees, between the minute and second hands of a clock at precisely 30 seconds after 3:15?
28	Solve the following absolute value inequality: $3|2x - 1| \le x + 4$
29	If $\log_8 3 + \log_8 x + 2\log_8(4x) = \log_8 162$, then x=?

Challenge Questions

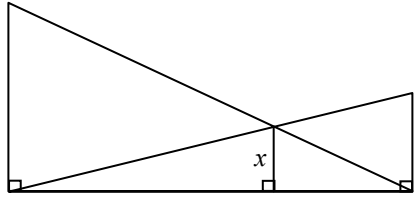|	
30	When all the 5-digit numbers that can be made by arranging the digits 2, 9, 7, 8, and 4 (each used exactly once) are listed in increasing order, what will be the 50th number on the list?
31	What is the sum of x, y, and z in the solution to this system of equations? $\quad$ 3x + 2y - 4z = 15 $\quad$ -6x + y + 2z = -16 $\quad$ x – 4y – 8z = -5
32	Evaluate $\displaystyle\sum_{n=2}^{5}(\frac{n^2}{n!}-\frac{(n-1)^2}{(n-1)!})$
33	Evaluate the finite geometric series: $\dfrac{2}{3}+\dfrac{4}{9}+\dfrac{8}{27}+\dfrac{16}{81}+\dfrac{32}{243}$
34	Mathitis is a disease that affects 10% of the population. There is a simple test for mathitis. This test results in a false positive 10% of the time and 1% of the people with the disease will test negative for it. What is the probability that someone who tests positive for mathitis actually has the disease?
35	What is the sum of the squares of the fourth roots of unity?
36	A gas tank formed by hemispheres attached to each end of a right cylinder turned on its side is filled to 2/3 capacity. The surface area of the entire tank is 84π square feet and the total length of the tank is 14 feet. What is the volume of gas, in cubic feet, currently contained in the tank?
37	What is the largest possible sum of two relatively prime factors of 4680 (not including 1 and 4680)?
38	A Pythagorean triple is a set of 3 positive integers that could form the sides of a right triangle. How many Pythagorean triples with all terms less than 50 exist such that at least two of the terms of the triple differ by exactly one?
39	The polynomial $x^3 + ax^2 + bx + 192 = 0$ has three real roots in geometric sequence. Determine the value of $\dfrac{b}{a}$.
40	Find three positive integral values for x that make $4^x + 4^{51} + 4^{54}$ a perfect square.

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11th – 12th Grade – November 20, 2004
Individual Multiple Choice Contest

Record only a letter as your answer on the colored sheet.

1	For the figure at right, which of the following statements about x is true? A) x is half the arithmetic mean of 9 and 4. B) x is half the geometric mean of 9 and 4. C) x is half the harmonic mean of 9 and 4. D) There is not enough information to determine x. E) None of the statements above is correct.
2	Curve A is the set of points (x, y) such that $x = m + 1$ and $y = -2m + 3$. Curve B is the set of points (x, y) such that $x = -2n + 2$, $y = 4n + 1$. Both m and n are real numbers. How many points do curves A and B have in common? A) 0 B) 1 C) 2 D) infinitely many E) answer not given
3	For triangle ABC, the point equidistant from vertices A, B, and C is A) the point at which the 3 altitudes intersect B) the point at which the 3 perpendicular bisectors of the sides intersect C) the point at which the 3 angle bisectors intersect D) the point at which the 3 medians intersect E) none of the above

4	Biff can climb up 14 steps of a 65-step stairway in one minute. He rests for a minute after each minute of climbing. During each minute of rest, Eho pushes Biff down **n** steps, where **n** is a whole number. If Biff first reaches the top of the stairway sometime during the 15th minute after he starts to climb, find **n**.
	A) 5 B) 6 C) 7 D) 8 E) 9 F) answer not given
5	Which of the following could not be the measure of an interior angle of a regular polygon?
	A) 171° B) 173° C) 175° D) 177° E) 179° F) answer not given
6	What is the value of x so that the line passing through $(x, 5)$ and $(4, 11)$ has a slope of 6?
	A) 0 B) 1 C) 2 D)3 E) answer not given
7	For each real number **x**, **[x]** denotes the greatest integer which is less than or equal to **x**. Which of these equations, if any, are true for all real numbers **x** and **y**? I) **[x + 3] = [x] + 3** II) **[x + y] = [x] + [y]** III) **[5x] = 5[x]** IV) **[x·y] = [x] · [y]**
	A) I, II, III, and IV B) I and III only C) II and III only D) I and II only E) answer not given
8	$y = x^3$ is symmetric to the
	A) x - axis B) y- axis C) origin D) the line y = 2x + 3

9. The table on the right shows the result of the operation $r \pounds c$, where r is the row entry and c is the column entry. For example, D £ B = A. What is A £ B £ C £ D £ E?

£	A	B	C	D	E
A	B	C	D	E	A
B	C	D	E	A	B
C	D	E	A	B	C
D	E	A	B	C	D
E	A	B	C	D	E

A) A   B) B   C) C   D) D
E) E   F) answer not given
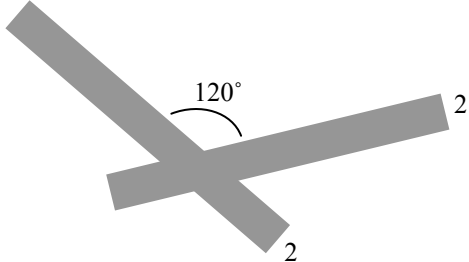
"Math is Cool" Master's – 2004-05
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11th – 12th Grade – November 20, 2004
Team Contest

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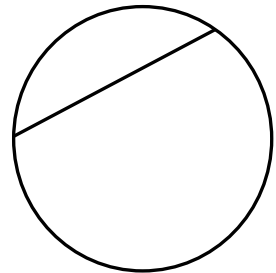Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

1	When two distinct numbers are chosen from the first ten positive integers, what is the probability that 4 is the smaller of the two numbers?
2	Two 8 by 2 inch rectangles are overlapped as shown, such that the obtuse angle between the two is 120°. What is the total shaded area, in square inches?
3	When expressed in base four, my secret number (which is a positive integer) is between four and seven digits long (inclusive). When expressed in base six, my secret number is between three and five digits long (inclusive). How many possible values are there for my secret number?
4	Find all values for w that make the following statement true. $C(w+1,2) = 9 \cdot C(w,1)$
5	Evaluate the following determinant: $$\begin{vmatrix} 2 & 1 & 0 & 1 \\ -1 & 0 & -2 & -1 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & -2 & 2 \end{vmatrix}$$
6	If $a + b = 2$ and $a^3 + b^3 = 4$, determine the value of ab.
7	The first time Jorge sees a new kind of math problem, it takes him sixteen minutes to solve it. Each time he solves a problem of a type he has previously solved, he solves it in 60% of the time he spent on the previous one. How many minutes would it take Jorge to solve an infinite number of similar problems of a kind he has not seen before?
8	A game is played using two special cubical dice: one die has faces labeled 1, 1, 1, 2, 2, 3; the other die has faces labeled 1, 2, 3, 5, 5, 6. When these dice are rolled, what is the expected value of the sum of the numbers shown on their upper faces?
9	What is the sum of the 25 smallest perfect squares which are not perfect cubes?
10	Find the sum of all solutions in the interval [0,2π]: $\tan^2 x + \sec^2 x = 3$

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11th – 12th Grade – November 20, 2004

Pressure Round Contest

1	Evaluate: $8^{(2\log_8 3 + \log_8 2)}$
2	The sum of the digits of a three-digit number is 20. The tens digit exceeds twice the units digit by 1. The hundreds digit is one less than twice the units digit. Find the number.
3	Find the product of x and y if: $8^{(3x+2)} \cdot 3^{2y} = 2^x \cdot 27^{(y+1)}$
4	Both 81 and 64 are perfect squares, and their difference is 17, which is a prime number. How many primes less than 100 can be expressed as the difference between two perfect squares?
5	One chord cuts a circle into two regions, as shown. Find the median of all possible numbers of regions that could be created by 5 chords, no two collinear. (It may be possible to create a given number of regions in more than one way. In calculating median, ignore these duplicates and consider only the distinct numbers of regions possible.)

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11th – 12th Grade – November 20, 2004

Mental Math Contest

Express all answers as reduced fractions in terms of radicals and π where applicable, unless stated otherwise.

PERSON 1		
1	Simplify: sine of "x" plus cosine of "x" times cotangent of "x"	1/sinx or cscx
2	During a season in a certain baseball league, every team plays every other team ten times. If there are five teams in the league, how many games are played in one season?	100 [games]
3	A right triangle has integral side lengths x and x+2 and a hypotenuse of 2x-2, all measured in centimeters. What is x?	6
4	Two coins are drawn out of a jar containing four pennies, five nickels, and two dimes. What is the probability their combined value is eleven cents?	8/55

PERSON 2		
1	If four to the quantity x-plus-two power equals thirty-two, evaluate sixteen to the "x" power.	4
2	The sum of the interior angles of a polygon is 540 degrees. How many sides does the polygon have?	5 [sides]
3	If the area of an equilateral triangle is nine root three square centimeters, what is the side length of the triangle, in centimeters?	6 [units]
4	Three coins are weighted such that for each of them the probability of getting heads is three-fourths. If the three coins are tossed simultaneously, what is the probability of getting exactly two heads?	27/64

PERSON 3		
1	If the square of the cosine of "x" plus the square of the sine of "x" equals root three times the tangent of "x", what is the square of the sine of "x"?	$\frac{1}{4}$
2	An isosceles triangle has base angles each measuring x degrees and the other angle is of measure 2x degrees. What is the measure of one of the base angles in degrees?	45 [°]
3	The perimeter of a rectangle is eight "c". If one side has length one-half "c", what is the area of the rectangle?	7c²/4
4	If "a" plus "b" equals six and "a" squared plus "b" squared equals twenty-two, what is the value of the product of "a" and "b"?	7

PERSON 4		
1	The lines y-equals-negative-four-fifths-x-plus-four, the y-axis, and the x-axis form a triangle that is rotated about the x-axis. What is the volume of the solid generated?	80π/3
2	A rectangle has a side length of measure 5 and diagonals of length 13. What is the measure of the other side?	12
3	A circle is inscribed in a square that is inscribed in a circle. What is the ratio of the area of the larger circle to that of the smaller circle?	2:1 [or 2 to 1]
4	Robert has three hundred sixty-four apples. He wishes he had more, so he decides to count his apples in an unusual base, giving him one zero three zero apples. What base	[base] 7

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11th – 12th Grade – November 20, 2004

COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	Solve for x: Eight minus 3 times the quantity five minus x equals 11.	6
2	A right triangle has sides of length x and x+1. The hypotenuse has a length of x+2. What is the value of x?	3
3	What is the sum of all the zeros of the function f of x equals three times the square of the sine of x over the open interval from zero to two pi?	π
4	How many four digit numbers have four distinct digits, the first three of which are prime, and the last of which is a multiple of 4?	48 [numbers]
5	What is the sum of the reciprocals of the roots of the equation y equals twenty x-squared minus x minus 1?	-1
6	A total of 925 tickets were sold for a total of $1150. If adult tickets sold for $2.00 each and children's tickets sold for $1.00 each, how many adult tickets were sold?	225 [tickets]
7	Evaluate: 125 raised to the negative two-thirds power. Write as a reduced fraction.	1/25
	Extra Problem – Only if Needed	
8	What is the interquartile range for this set of data: 13, 15, 25, 22, 18, 19	7

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COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	Evaluate seven choose three, plus six choose three.	55
2	What is the larger angle, in degrees, between the hour hand and the minute hand on a standard twelve-hour analog clock at three forty-eight PM?	186 [deg]
3	The chance of having a picnic given that it rains is five one-hundredths. The probability of having a picnic and having it rain is three one-hundredths. What is the probability that it rains? Express your answer as a fraction.	3/5
4	Evaluate: The limit, as x approaches infinity, of one over the quantity x plus 2.	0
5	z varies jointly with x and the square root of y. If z is 15 when x is 5 and y is 36, find z when x is 2 and y is 25.	5
6	Find the equation of the line perpendicular to y equals negative one half x plus two and passing through the point (2 comma 7) in slope intercept form.	Y=2x+3
7	What is the units digit in the expansion of 7 raised to the 111 power?	3
	Extra Problem – Only if Needed	
8	Given the hypotenuse of a 30-60-90 triangle is 16, what is the length of the side opposite the 60 degree angle?	$8\sqrt{3}$

COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	What is the period of the function three-plus-three-times-the-tangent-of-the-quantity-pi-x-minus-three?	1
2	What is the remainder when four x cubed minus five x squared plus six x minus eight is divided by the quantity x minus 1 ?	-3
3	Tealah likes to go shopping at American Eagle. She wants to know how much, in dollars, she will pay the cashier for a pair of pants that costs $53.20 in a town with 8% sales tax?	[$] 57.46
4	Use the equation y equals negative three times two to the x power to find the value of x when y = -48	4
5	Simplify to a single trigonometric function: the quantity two minus two sine squared of x, divided by the cotangent of x.	Sin(2x)
6	Find the radius of a sphere with a surface area of 144B.	6
7	Find the number you would add to both the numerator and the denominator of 8/11 so the result would be 6/7.	10
	Extra Problem – Only if Needed	
8	Find the circumradius, in centimeters, of a triangle with a six-centimeter side opposite an angle measuring $\frac{\pi}{4}$ radians.	$3\sqrt{2}$

"Math is Cool" Championships – 2004-05
11th – 12th Grade – November 20, 2004

School Name_____Team #_____
Proctor Name_____Room #_____

STUDENT NAME_____

Individual Contest – Score Sheet
DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0		Answer	1 or 0	1 or 0
1	-1/11			21	4		
2	47			22	34 [integers]		
3	42			23	48 [ways]		
4	3/2			24	3 [times]		
5	2/3			25	36 [ounces]		
6	[x=] -2, -4			26	64π		
7	25/2			27	87 [°]		
8	3:23 AM			28	$-1/7 \le x \le 7/5$ or [-1/7,7/5]		
9	(1,8)			29	3/2		
10	$31_{[7]}$			30	72498		
11	$\sqrt{3}$			31	21/4		
12	269/7			32	-19/24		
13	120			33	$\dfrac{422}{243}$		
14	y = -2/3			34	11/21		
15	$8\pi/3$			35	0		
16	25/216			36	72π [ft^3]		
17	30 [values]			37	941		
18	9360 [lic plates]			38	5		
19	$(8+4\sqrt{10})\pi$			39	$4\sqrt[3]{3}$		
20	[g(x)] = x			40	X= {47, 53, 56}		

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11th – 12th Grade – November 20, 2004

School Name_____Team #_____
Proctor Name_____Room #_____

STUDENT NAME_____

Individual Multiple Choice Contest – Score Sheet

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer	-1, 0 or 2	-1, 0 or 2
1	C		
2	D		
3	B		
4	B		
5	B		
6	D		
7	E (only I is true)		
8	C		
9	E		

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11th – 12th Grade – November 20, 2004

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Team Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	$\frac{2}{15}$		
2	$32 - \frac{8\sqrt{3}}{3}$		
3	7712 [values]		
4	17		
5	-4		
6	$\frac{2}{3}$		
7	40 [min]		
8	$\frac{16}{3}$		
9	6920		
10	4π		

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School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Pressure Round Answers

	Answer
1	18
2	794
3	9/4
4	24
5	11