# "Math is Cool" Master's – 2004-05

## 8th Grade – November 20, 2004

### Individual Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

| | |
|---|---|
| 1 | What is the next number in the sequence 4, 7, 11, 16, __ |
| 2 | What quadrant is the point (-8, -5) located? |
| 3 | Evaluate: 5 - 2(3-4)-8(6-11) |
| 4 | Reduce the following fraction and write as an improper fraction. (-81/-54) |
| 5 | What is the perimeter of a nonagon with side length 7? |
| 6 | Solve for x: $4x + 19 = 71$ |
| 7 | The length of a rectangle is 3 less than twice the width. The perimeter is 18 meters. Find the width in meters. |
| 8 | Ratchet left home and headed off to the football game 20 miles away. Half way to the game he realized he forgot his helmet and went back home and got it. How many total miles did he travel before he got to the game? |
| 9 | What is the sum of the first 10 even natural integers? |
| 10 | Express as a reduced common fraction: $.\overline{27}$ |
| 11 | The sum of two numbers is 44, and their difference is 18. Find the smaller of the two numbers. |
| 12 | How many factors does 108 have? |
| 13 | The sum of three consecutive odd numbers is 111. If the median of the three is equal to x, what is $4x + 7$? |
| 14 | Find the y-coordinate of the point of intersection of the two lines $3x + 2y = 0$ and $2x + y = -1$ |
| 15 | Carol is two years older than Mike. In 4 years her age will be 8 less than twice Mike's. How old, in years, is Mike now? |

| | |
|---|---|
| 16 | What is the lowest common multiple between $4x^2y^7$, $6x^4y^3$ and $12x^3y$? |
| 17 | How many ways are there to choose 17 books from a shelf that has 20 books on it? |
| 18 | What is the slope of a line passing through the points (1,7) and (-5,-2)? |
| 19 | Compute: $216^{1/3}$ |
| 20 | Evaluate and write as an improper fraction: $4^0 + 3^{-2} + 7^{-1}$ |
| 21 | Aaron is getting dressed for his next football game. His coach has told him that he must wear a helmet, a jersey, shoulder pads, pants, socks, and shoes, but an undershirt is optional. If Aaron has one helmet, 3 jerseys, one pair of shoulder pads, 2 pairs of pants, 8 different pairs of socks, two pairs of shoes, and 6 undershirts, how many ways could he follow his coach's orders and get dressed? |
| 22 | Convert $704_{10}$ to base 4? |
| 23 | A gigantic seagull decided to fly from New York to Los Angeles, and then fly back. On the first flight, it averaged 5 miles per second, and on the return flight, it averaged 17 miles per second. What was the seagull's average speed in miles per second? |
| 24 | Factor: $2x^2 - x - 10$ |
| 25 | Find the radius of a circle with circumference $24\pi$ |
| 26 | Solve for all values of x: $(x+3)^2 = 1$ |
| 27 | How many integer values make $\lvert 2x - 3 \rvert < 8$ true? |
| 28 | At James Miller High School the amount of time varsity boy's basketball members spend talking with their girlfriends varies inversely with their playing time. If a player talks with his girlfriend for 10 minutes, then he plays 20 minutes during the next game. If a player plays for 50 minutes in another game, how long, in minutes, did he talk to his girlfriend? |
| 29 | In a regular deck of cards, what is the probability of drawing a red king, then a spade, then an ace that isn't a spade without replacement? |

# Challenge Questions

| | |
|---|---|
| 30 | A ◊ B = $ab(b+a^b)^a+\sqrt{6ab}$. What is 2 ◊ 3? |
| 31 | Andy can mow a lawn in 7 hours. James can mow the same lawn in 1 hour. Charlie can mow the lawn in 4 hours. How many hours would it take Andy, James, and Charlie to mow the same lawn together? |
| 32 | Nathan, Amanda, Cameron, and Joel are trying to write a 40 question individual test. Nathan writes 2 questions every 5 minutes, Amanda writes 1 question every 5 minutes, Cameron writes 3 questions every 2 minutes and Joel writes 1 question every 4 minutes. How long will it take them to write the test rounded to the nearest minute? |
| 33 | What is the probability of getting two heads and three tails on five flips of a coin? |
| 34 | If two dice are rolled 144 times how many times would you expect a sum of 6 or 4 to occur? |
| 35 | In non-degenerate quadrilateral ABCD, $AB=12$, $BC=17$, $CD=10$, and $DA=14$. Find the sum of all possible integer lengths of diagonal $\overline{AC}$. |
| 36 | Solve for x: $3^{5x+2} = 81^{x-4}$ |
| 37 | Solve for xy where $x^2+y^2=36$ and $x+y=4$ |
| 38 | What is the area of a regular hexagon with side length 8? |
| 39 | Compute: 1024+768+576+432+324+… |
| 40 | At what point, in $(x,y,z)$ form, do the planes $3x+2y-z=1$, $-x+y-z=0$, and $2x-y+2z=-3$ intersect? |

# "Math is Cool" Master's – 2004-05

8th Grade – November 20, 2004

Final Score: KEY

First Score 8th

School Name_____Team #_____

Proctor Name_____Room #_____

**STUDENT NAME_____**

**Division: _____**

## Individual Contest – Score Sheet

## DO NOT WRITE IN SHADED REGIONS

| | Answer | 1 or 0 | 1 or 0 |
|---|---|---|---|
| 1 | 22 | | |
| 2 | 3 or 333 or 3rd | | |
| 3 | 47 | | |
| 4 | 3/2 | | |
| 5 | 63 | | |
| 6 | 13 | | |
| 7 | 6 [meters] | | |
| 8 | 40 [miles] | | |
| 9 | 110 | | |
| 10 | 3/11 | | |
| 11 | 13 | | |
| 12 | 12 [factors] | | |
| 13 | 155 | | |
| 14 | 3 | | |
| 15 | 6 [yrs] | | |
| 16 | $12x^4y^7$ | | |
| 17 | 1140 [ways] | | |
| 18 | 3/2 | | |
| 19 | 6 | | |
| 20 | 79/63 | | |
| | | | |

| | Answer | 1 or 0 | 1 or 0 |
|---|---|---|---|
| 21 | 672 [ways] | | |
| 22 | $23000_{(4)}$ | | |
| 23 | 85/11 [miles/sec] | | |
| 24 | (2x-5) (x+2) | | |
| 25 | 12 | | |
| 26 | [x=] -2, -4 | | |
| 27 | 8 | | |
| 28 | 4 [minutes] | | |
| 29 | 1/1700 | | |
| 30 | 732 | | |
| 31 | 28/39 [hours] | | |
| 32 | 17 [min] | | |
| 33 | 5/16 | | |
| 34 | 32 | | |
| 35 | 261 | | |
| 36 | -18 | | |
| 37 | -10 | | |
| 38 | $96\sqrt{3}$ | | |
| 39 | 4096 | | |
| 40 | $(1,-3,-4)$ | | |
| | | | |

# "Math is Cool" Master's – 2004-05

## 7th & 8th Grade – November 20, 2004

## Individual Multiple Choice Contest

The Sampson Motor Company is testing four new automobiles for fuel efficiency. Assume the cars only travel 30 mph or 60 mph and that acceleration is not a factor. Also assume that for each question, each car begins with a full tank of gas.

| Car | Fuel Efficiency at 30 mph | Fuel Efficiency at 60 mph | Gas Tank Capacity |
|---|---|---|---|
| Albert | 15 mpg | 45 mpg | 10 gallons |
| Hawking | 20 mpg | 40 mpg | 15 gallons |
| Tycho | 30 mpg | 30 mpg | 20 gallons |
| Gabe | 25 mpg | 35 mpg | 15 gallons |

*mpg stands for miles per gallon

| | |
|---|---|
| 1 | What is the fuel efficiency of the Hawking at 30 mph?<br>A) 15 mpg B) 20 mpg C) 30 mpg D) 25 mpg E) Answer not given |
| 2 | The Albert is driven at 60 mph until it runs out of gas. How far does it go, in miles?<br>A) 100 B) 150 C) 300 D)450 E) 600 |
| 3 | The Hawking is driven for 50% of the total time traveled at 30 mph and at 60 mph for the remaining time. How far, in miles, will it have traveled when it runs out of gas?<br>A) 300 B) 450 C) 150 D) 600 E) 200 |
| 4 | The four cars are driven at 30 mph until they have all run out of gas. How much distance, in miles, separates the first car from the last car?<br>A) 450 B) 150 C) 225 D) 300 E) 600 |
| 5 | The Sampson Motor Company is comparing the Hawking and the Tycho. They were both driven for one hour at 30 mph. How many hours must they both be driven at 60 mph to have burned the same amount of gas?<br>A) 4 B) 3 C) 2 D) 1 E) 0 |
| 6 | Each car burns one gallon of gas at 30 mph and then one gallon at 60 mph. Which car completes the test in the shortest amount of time?<br>A) Albert B) Hawking C) Tycho D) Gabe E) All cars are equal |
| 7 | The Tycho is driven from Spokane to Seattle (240 miles) at 60 mph, then it is driven in Seattle for 7 hours at 30 mph, and finally starts on its way back to Spokane at 60 mph. How far, in miles, from Spokane will the Tycho be when it runs out of gas?<br>A) 40 B) 200 C) 150 D) 90 E) 120 |
| 8 | Both the Hawking and the Gabe follow a linear model for fuel efficiency with respect to speed. Write an equation for the difference (D) in fuel efficiency (E) between the Hawking and the Gabe, in terms of speed (S).<br>A) D=S/2 B) D=S/3-15 C) D=S/3-10 D) D=S+10 E) Answer not given |
| 9 | Write an expression in terms of "g" and "m" that would calculate how many minutes it would take to burn "g" gallons of gas at 30 mph with a car that gets "m" miles per gallon.<br>A) 2mg B) mg C) $(\frac{1}{2})$mg D) 4mg E) Answer not given |

# "Math is Cool" Master's – 2004-05

7th & 8th Grade – November 20, 2004

Final Score: KEY

First Score 7th/8th

School Name_____Team #_____

Proctor Name_____Room #_____

Division: _____

**STUDENT NAME_____**

**Individual Multiple Choice Contest – Score Sheet**

**Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.**

**DO NOT WRITE IN SHADED REGIONS**

| Answer | | -1, 0 or 2 | -1, 0 or 2 |
|---|---|---|---|
| 1 | B | | |
| 2 | D | | |
| 3 | B | | |
| 4 | A | | |
| 5 | D | | |
| 6 | A | | |
| 7 | D | | |
| 8 | B | | |
| 9 | A | | |
| | | | |

# "Math is Cool" Championships – 2004-05

$9^{th}$ – $10^{th}$ Grade – October 13, 2004

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score

(out of 10)

**STUDENT NAME_____**

## Team Contest – Score Sheet

**DO NOT WRITE IN SHADED REGIONS**

| | Answer | 1 or 0 | 1 or 0 |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| | | | |

# "Math is Cool" Master's – 2004-05

## 8th Grade – November 20, 2004

### Team Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

| | |
|---|---|
| 1 | You have twelve one-foot-square tiles to tile a three foot by four foot stoop. If nine tiles are white, two are black, and one is red, how many distinguishable tilings are possible? |
| 2 | Raskin & Bobbin's Cookie Parlor offers 100 different varieties of cookies. If 72 varieties contain chocolate, 39 contain nuts, 16 contain oatmeal, 13 contain both chocolate and oatmeal, 10 contain both oatmeal and nuts, 21 contain both chocolate and nuts, and 9 contain all three ingredients, how many varieties contain none of these three ingredients? |
| 3 | You are hired for a job that pays only one cent the first day, but the amount you are paid triples each day. After ten days, how much money, in dollars, will you have earned, to the nearest cent? |
| 4 | Given a regular hexagon, a second hexagon is created by joining the midpoints of the sides of the first hexagon, then a third hexagon is created by joining the midpoints of the sides of the second hexagon. What is the ratio of the area of the third hexagon to the area of the first hexagon? |
| 5 | Drew's age is three times Adam's age. In five years, Drew's age will be twice Adam's age. How old, in months, will Adam be in ten years? |
| 6 | What is the greatest common factor of 5,082 and 18,837? |
| 7 | What is the area of the triangle bounded by the coordinate axes and the line $7x - 5y = 105$? |
| 8 | Black Box A takes a positive number as input, squares it, subtracts three times the input number, and adds 12 to produce its output. Black Box B takes a positive number as input, divides it by five, squares the result, and subtracts 9 to produce its output. My sister input a number to Black Box A, then took the output from Black Box A and input it into Black Box B. If the output of Black Box B was 55, what number did my sister input to Black Box A? |
| 9 | A bag contains three red, five green, and two blue marbles. When two marbles are chosen, what is the probability they are of different colors? |
| 10 | What is the sum of all numbers between fifty and one-hundred (inclusive) that are not multiples of 3? |

# "Math is Cool" Master's – 2004-05

8th Grade – November 20, 2004

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score: KEY

First Score 8th

**STUDENT NAME_____**

**Team Contest – Score Sheet**

**DO NOT WRITE IN SHADED REGIONS**

| | Answer | 1 or 0 | 1 or 0 |
|---|---|---|---|
| 1 | 660 [tilings] | | |
| 2 | 8 [varieties] | | |
| 3 | [$] 295.24 | | |
| 4 | $\frac{9}{16}$ | | |
| 5 | 180 [months] | | |
| 6 | 21 | | |
| 7 | $\frac{315}{2}$ | | |
| 8 | 7 | | |
| 9 | $\frac{31}{45}$ | | |
| 10 | 2550 | | |
| | | | |

# "Math is Cool" Master's – 2004-05

## 8th Grade – November 20, 2004

### Pressure Round Contest

| | |
|---|---|
| 1 | Find the arithmetic mean of all prime numbers between 100 and 150 which have two digits the same. |
| 2 | A cube of volume 27 cubic units is removed from one corner of a solid cube of volume 216 cubic units. What is the total surface area, in square units, of the solid figure that remains? |
| 3 | If $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}<1$, where a, b, and c are distinct positive integers, find the smallest possible value for a + b + c. |
| 4 | Letters are given numbers values according to their positions in the alphabet: A = 1, B = 2, etc. The value of a word is the sum of the values of its letters. Which of the following words has a total value closest to 100: TRUSS, TRUST, RUSTY, RUSTS, CRUSTS, or CRUSTY? |
| 5 | In a certain dice game, a player rolls two dice and wins if he rolls a sum of 7 or a sum of 11, or if he rolls any doubles except 1-1. Before each roll, Joe Chump pays "the house" $1 to play this game. For Joe to break even, on average, how much would "the house" have to pay Joe each time he wins? Answer in dollars, rounded to the nearest cent. |

# "Math is Cool" Master's – 2004-05

8th Grade – November 20, 2004

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score
8th

**STUDENT NAME_____**

## Pressure Round Answers

| | Answer |
|---|---|
| 1 | 115 |
| 2 | 216 [sq un] |
| 3 | 11 |
| 4 | CRUSTS |
| 5 | [$] 2.77 |

# "Math is Cool" Master's – 2004-05

## $7^{th}$ & $8^{th}$ Grade – November 20, 2004

## Mental Math Contest

Express all answers as reduced fractions in terms of radicals and π where applicable, unless stated otherwise.

| | | |
|---|---|---|
| **PERSON 1 Name:** | | |
| 1.1 | What is the area of a triangle whose side lengths are 6, 8 and 10? | 24 |
| 1.2 | What is the arithmetic mean of 2, 3, 8, 10, 13 and 6? | 7 |
| 1.3 | Jon hosts a party with 7 of his friends. If they all shake hands with each other once, how many handshakes will occur? | 28 [handshakes] |
| 1.4 | If 7 lollipops cost 50 cents, how much, in dollars, do 84 lollipops cost? | [$] 6 |
| **PERSON 2 Name:** | | |
| 2.1 | What is the area of a trapezoid whose bases are 5 and 7 and whose height is 8? | 48 |
| 2.2 | Evaluate for x = -7: $2x^2$ | 98 |
| 2.3 | Evaluate 5 factorial plus 4 factorial. | 144 |
| 2.4 | If Brandt is bungee jumping off a 135-foot bridge and falls at a rate of 15 feet per second, how many seconds will it take for him to reach the water below the bridge? | 9 [sec] |
| **PERSON 3 Name:** | | |
| 3.1 | How many ways can the letters in the word 'MOON' be arranged? | 12 [ways] |
| 3.2 | Evaluate: 27 to the two thirds power | 9 |
| 3.3 | What is the area of the circle inscribed in a square with side length of 12? | 36π |
| 3.4 | There are three-legged chickens and four-legged ants at a local scientific lab. If there are 15 heads and 52 legs, how many ants are there? | 7 |
| **PERSON 4 Name:** | | |
| 4.1 | The length of a rectangle is twice its width and the perimeter is 18. What is the area of the rectangle? | 18 |
| 4.2 | In a twenty-one story building on which floor would you be if the walk to the $21^{st}$ floor would be twice as far as the walk to the first floor? | 7 [th floor] |
| 4.3 | When the repeating decimal 0.151515… is expressed as a common fraction in lowest terms, what is the sum of its numerator and denominator? | 38 |
| 4.4 | An isosceles triangle has base angles each measuring x degrees and the other angle is of measure 2x degrees. What is the measure of one of the base angles in degrees? | 45 [°] |

# "Math is Cool" Master's – 2004-05

**7th & 8th Grade** – November 20, 2004

## COLLEGE KNOWLEDGE BOWL ROUND #1

| # | Problem | Answer |
|---|---|---|
| 1 | Sarah is reading a book. When she multiplies the two pages she sees in front of her, she gets the number 2,756. What is the sum of the page numbers? | 105 |
| 2 | What is the area of an equilateral triangle with side length 4? | $4\sqrt{3}$ |
| 3 | How many ways can 9 people be seated around a circular table? | 40,320 [ways] |
| 4 | If n% equals seven eighths percent, what is the value of n. | 87.5 |
| 5 | What is the probability of rolling a sum of five when you toss two fair six-sided dice? | 1/9 |
| 6 | How many integers between 200 and 500 do not contain an even digit? | 25 [integers] |
| 7 | Solve for x: 15 times the quantity x minus four equals 3x plus 24. | 7 |
| | Extra Problem – Only if Needed | |
| 8 | The average height of the players on a basketball team is 75 inches. After a player who is 72 inches tall is added to the team, the average changes to 74 and five-eighths inches. How many players were on the team before the new player was added? | 7 [players] |

# "Math is Cool" Master's – 2004-05

## 7th & 8th Grade – November 20, 2004

## COLLEGE KNOWLEDGE BOWL ROUND #2

| # | Problem | Answer |
|---|---|---|
| 1 | How many prime factors does 54 have? | 2 |
| 2 | Billy wants to buy paper and pencils. Pencils cost 40 cents per package and paper costs 5 cents per package of 220 sheets. If Billy has $2.35 and he wants as many pencils as he can buy, how many sheets of paper would he need to buy to spend all his money? | 1540 [sheets] |
| 3 | Compute: 8 factorial divided by the quantity 3 factorial times 5 factorial. | 56 |
| 4 | What is the sum of the first 15 positive even integers? | 240 |
| 5 | What is the volume of a right circular cylinder with radius 3 and height 8? | $72\pi$ |
| 6 | The sum of two numbers is 96 and their difference is 92. Find their product. | 188 |
| 7 | A palindrome is a number that is read the same backwards as forwards. What must be added to 19991 to form the smallest palindrome greater than 19991? | 11 |
| | Extra Problem – Only if Needed | |
| 8 | John is examining the houses in his neighborhood. There are twelve houses with each having one light on their porch. How many different lighting patterns could be created by these 12 lights? | 4096 [patterns] |

# "Math is Cool" Master's – 2004-05

## 7th & 8th Grade – November 20, 2004

## COLLEGE KNOWLEDGE BOWL ROUND #3

| # | Problem | Answer |
|---|---|---|
| 1 | What is the maximum number of intersections a graph of a square and a circle can have? | 8 [intersections] |
| 2 | What number must be added to the following set of numbers to make the arithmetic mean 6?<br>5, 3, 8, 7, and 2 | 11 |
| 3 | What is the remainder when 147 is divided by the solution to $3x + 14 = x + 20$? | 0 |
| 4 | Find the radius of a circle whose area is $144\pi$? | 12 |
| 5 | If nine fifths x plus 32 equals y, what is y when x is 200? | 392 |
| 6 | How many distinct positive integer factors does 192 have? | 14 [factors] |
| 7 | What is the length of the segment joining the points (3,4) and (13,28)? | 26 |
| | Extra Problem – Only if Needed | |
| 8 | Two trains leave from New York and Seattle at the same time. If the distance between New York and Seattle is 3,720 miles and one train is traveling 30 miles per hour and the other train is traveling at 90 miles per hour, how long, in hours, will it take for the trains to cross each other? | 31 [hours] |