# "Math is Cool" Championships – 2004-05

## Sponsored by: Western Polymer Corporation

### $9^{th}$ – $10^{th}$ Grade – October 13, 2004

### Individual Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

| | |
|---|---|
| 1 | Compute $17 + 48 - 3\left(2 - \frac{7}{2}\right)^2 - 4!$ |
| 2 | What quadrant is the point (-3,-2) located? |
| 3 | What is the x-coordinate of the x-intercept of 3x + 4y = 12? |
| 4 | Find the midpoint of the line segment joining the points (7,8) and (5,2). |
| 5 | What is the slope of the line perpendicular to $y = x\sqrt{2} + 7\sqrt{3}$? |
| 6 | What is the largest number less than 100 with exactly 12 factors? |
| 7 | How many more arrangements of the word 'squares' are there than of the word 'square'? |
| 8 | Simplify and reduce: $\frac{12^3}{2^4 \cdot 3^2}$ |
| 9 | A circle is inscribed in a square with perimeter 16, what is the circumference of the circle? |
| 10 | What is the prime factorization of 180? |
| 11 | The functions $f(x) = 3x^2+4$ and $g(x) = 2x+5$. Find $f(x) + g(x)$. |
| 12 | If z varies directly with the sum of x and y, and z=12 when x=1 and y=5, what is z equal to when x=7 and y=-12? |
| 13 | How many positive palindromes less than 10,000 have at least three identical digits? |
| 14 | What is the smallest prime number greater than 200? |
| 15 | Convert $2817_9$ to base 3. |
| 16 | If x is the sum of the measures of the external angles of a regular nonagon, what is x/9? |

| | |
|---|---|
| 17 | A farmer's barn is populated with cows, chickens and spiders (at least one of each). If there are 31 heads and 92 legs present, at most, how many chickens could there be? |
| 18 | A cube is constructed such that its volume in cubic centimeters is half its surface area measured in square centimeters, what is the space diagonal of the cube, in centimeters? |
| 19 | How many positive numbers less than 24 are relatively prime to 24? |
| 20 | What is the length of the altitude to the hypotenuse for a 7-24-25 right triangle? |
| 21 | After the 2004 'Math is Cool Masters' competition, a small party took place. If the 14 people there all shook hands with each other, how many handshakes took place? |
| 22 | Evaluate the infinite expression: $2\sqrt{3+2\sqrt{3+2\sqrt{3+\ldots}}} = ?$ |
| 23 | If the surface area of a sphere is $\frac{25\pi}{4}$, what is its volume? |
| 24 | How many positive numbers less than 100 have a remainder of 3 when divided by 5, and a remainder of 4 when divided by 7? |
| 25 | What is the sum of the roots of the polynomial: $3x^3 + 2x^2 - 5x + 7$ ? |
| 26 | Jack is currently running laps around a track defined by the equation $x^2 + y^2 = 16$, while Sierra is walking along a path defined by the parabola $y = x^2 - 4$. If Sierra stops and waits for Jack to come by each time she crosses the track, at what point(s) will they meet? |
| 27 | What is the maximum value of the function $f(x) = \cos(x) - x^2 + 4\pi x + 7$? |
| 28 | A fish is randomly swimming around in a fishbowl in the shape of a pyramid with a square base of side length 8, and a height of 14. At any given time, what is the probability that the fish is within 7 vertical units of the base of the pyramid? |
| 29 | If x is a randomly chosen point on the interval between 0 and 1, what is the expected value of $f(x) = 3x + 9/2$? |

# Challenge Questions

| | |
|---|---|
| 30 | What is the sum of the values of x such that $\left(x^2-9x+14\right)^{\left(x^2-25x+46\right)}=0$? |
| 31 | One afternoon five friends meet up to play ultimate frisbee, all bringing their own Frisbee. In how many ways can all but one person leave with someone else's Frisbee? |
| 32 | What is the sum of the following numbers in base (-2): $10111_{-2}+11010_{-2}$ ? |
| 33 | What is the remainder when $2^{101}$ is divided by 13? |
| 34 | A certain right rectangular prism has surface area equal to 148 and its side lengths form an arithmetic series with difference 1 between terms. What is the volume of the shape? |
| 35 | A pyramid has height 8 and a hexagonal base inscribed in a circle with radius 6. What is the surface area of the pyramid? |
| 36 | What is the largest possible area of a triangle inscribed in a circle with area $25\pi$? |
| 37 | For n>4, what is the probability that there exists a prime between n and 2n ? |
| 38 | If x is a randomly chosen point on the interval $[-1,1]$, what is the expected value of $f(x)=\sqrt{1-x^2}$ ? |
| 39 | If the harmonic mean of x and y is $\frac{1}{2}$, and the harmonic mean of $x^2$ and $y^2$ is $\frac{1}{5}$, what is the harmonic mean of $x^3$ and $y^3$? |
| 40 | What is the maximum number of terminal zeros when the function $f(x)=x^4+4x^3-12x^2$, for integral values of $x\geq 2$, is evaluated and expressed in base x? |

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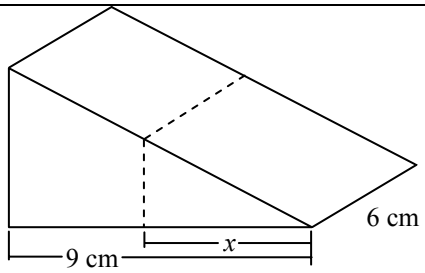
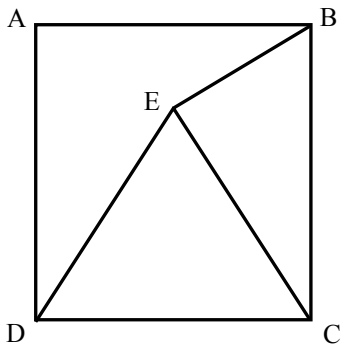
### $9^{th}$ – $10^{th}$ Grade – October 13, 2004

### Individual Multiple Choice Contest

Record only a letter as your answer on the colored sheet.

| | |
|---|---|
| 1 | Simplify: $(a^2bc^{-2})^3(a^{-2}b^5c^3)^2$<br><br>A) $a^2b^{-13}$ B) $a^{-2}b^{-13}$ C) $a^{-2}b^{-13}$ D) $a^2b^{13}$ E) Answer not given |
| 2 | A wedge (a right triangular prism) of cheese is cut with a single vertical cut to remove a smaller wedge, leaving 2/3 of the original volume of cheese. What is length **x**, in cm?<br>  <br><br>A) 3 B) $4\frac{1}{2}$ C) $3\sqrt{3}$ D) $3^{5/3}$ E) answer not given |
| 3 | Tweedledum and Tweedledee stand next to each other, each holding up a card with a different 2-digit positive integer written on it. If you read the two cards together as one 4-digit number, it will be a palindrome. What is the largest value possible for the greatest common factor of Tweedledum's number and Tweedledee's number?<br><br>A) 3 B) 4 C) 6 D) 12 E) answer not given |
| 4 | Simplify $(0.05^2)/(0.5^5)$.<br><br>A) 0.08 B) 0.125 C) 0.25 D) 0.1 E) answer not given |
| 5 | You find the "digital product" of a natural number by multiplying its digits. If you repeat this process on the digital product, then on that product, and so on, eventually you will reach a 1-digit integer, the "product root". Which of the following numbers takes the least number of steps to reach a product root?<br><br>A) 379 B) 77 C) 5171 D) 8888 E) 68 |
| 6 | Math Team members sit in a single ring of 172 chairs around a big round table. |

| | |
|---|---|
| | Tony is feeling anti-social and doesn't want to sit next to anybody else. He comes in late as usual and sees that there is nowhere he can sit that is not next to a chair already occupied. What is the smallest number of Math Team members who could already be seated before Tony sits down?<br><br>A) 171 B) 63 C) 86 D) 57 E) answer not given |
| 7 | Let $M = 2^{2004}$, $A = 3^{1503}$, $T = 5^{1002}$, and $H = 7^{501}$. When these four quantities are listed in order from least to greatest, the order is:<br><br>A) HMTA B) HTAM C) HMAT D) MATH E) answer not given |
| 8 | In the figure at right, ABCD is a square and CDE is an equilateral triangle. What is the measure of angle CBE?<br><br>A) 45° B) 80° C) 60°<br>D) 75° E) answer not given<br><br>  |
| 9 | The string of digits 456454**d**, where **d** stands for an unknown digit, can be arranged in exactly 140 ways. What digit does **d** stand for?<br><br>A) 4 B) 5 C) 6 D) 0, 1, 2, 3, 7, 8, or 9 E) not enough information to tell |

# "Math is Cool" Championships – 2004-05

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### $9^{th}$ – $10^{th}$ Grade – October 13, 2004

### Team Contest

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Record all answers on the colored cover sheet.

| | |
|---|---|
| 1 | What is the area of the region satisfying $x - y \geq -1$, $6x + y \leq 22$, and $x + 6y \geq -8$? |
| 2 | How many ways can you make change for \$100 using only \$1, \$5, \$10, and \$20 bills? Note: you are not required to use each type of bill in each way of making change; i.e. 100 \$1 bills is an acceptable way to make change for \$100. |
| 3 | An equilateral triangle is circumscribed about a regular hexagon with two-centimeter sides such that three of the sides of the hexagon are coincident with portions of the sides of the triangle. What is the area of the triangle, in square centimeters? |
| 4 | The Iccanobif sequence is defined recursively as $I_1 = I_2 = I_3 = 1$ and $I_n = I_{n-2} + I_{n-3}$ for $n > 3$, beginning 1, 1, 1, 2, 2, 3, 4, 5, 7, 9, 12, … . Determine the value of $I_{25}$. |
| 5 | Selling at her lemonade stand, Jenni has found that if she charges $c$ cents per cup, she can sell $100 - 2c$ cups each day. How many (integer) cents should she charge per cup to maximize her daily revenue? |
| 6 | What is the smallest number greater than 1000 that leaves a remainder of 4 when divided by 15 and a remainder of 3 when divided by 4? |
| 7 | A sphere and a right circular cylinder have the same volume, but the radius of the sphere is three times that of the cylinder. If the height of the cylinder is 90 centimeters, what is the radius of the sphere, in centimeters? |
| 8 | In order to randomly choose a day of the week, I roll two dice and make my decision based on the sum of the numbers on their upper faces. If the sum is 4, 5, 6, 7, 8, 9, or 10, I choose Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, or Saturday respectively. If any other sum is rolled, I ignore that sum and re-roll the dice, continuing to re-roll as necessary until I get a sum between 4 and 10 inclusive. What is the probability that I end up choosing Tuesday? |
| 9 | Determine the ordered triple, $(a,b,c)$, which satisfies the equations $4a + 2b - c = -6$, $3a - 3b - 2c = -11$, and $-a + b + 4c = -3$. |
| 10 | In square ABCD, line segments $\overline{BE}$ and $\overline{CF}$ are drawn, with F being the midpoint of side $\overline{AD}$ and E lying on $\overline{AD}$ such that $AE = 2ED$. If G is the point of intersection of $\overline{BE}$ and $\overline{CF}$, what is the ratio of the area of triangle EFG to that of square ABCD? |

# "Math is Cool" Championships – 2004-05

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### $9^{th}$ – $10^{th}$ Grade – October 13, 2004

### Pressure Round Contest

| | |
|---|---|
| 1 | Julie can shampoo 5/6 of a canary in 4/5 of an hour of work, and Gregg can shampoo 7/8 of a canary in each 6/7 of an hour of work. At these rates, how many hours would it take the two of them working together to shampoo 66 canaries? |
| 2 | I'm thinking of a positive integer that is a multiple of 99. If every digit of my number is a 5, what is the smallest number of digits my number could have? |
| 3 | The sum of three numbers is 147. The ratio of the first to the second is 2/3, and the ratio of the second to the third is 5/8. What is the second number? |
| 4 | How many different integers from 1 through 100 inclusive are either divisible by 4 or contain a digit divisible by 4 (or both)? |
| 5 | A spherical steel ball of radius 3.5 mm just fits inside a hollow cylindrical drinking straw, open at each end. Tom cuts a length of this straw such that the outer surface area of the piece of straw is exactly equal to the surface area of the ball inside it. By how many mm does the length of the piece of straw exceed the diameter of the ball? |

# "Math is Cool" Championships – 2004-05

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### $9^{th}$ – $10^{th}$ Grade – October 13, 2004

### Mental Math Contest

Express all answers as reduced fractions in terms of radicals and π where applicable, unless stated otherwise.

| | | |
|---|---|---|
| **PERSON 1** | | |
| 1 | What is the maximum product that can be obtained from two real numbers that sum to one-hundred? | 2500 |
| 2 | How many integers are greater than or equal to negative thirty-seven and less than or equal to forty-two? | 80 |
| 3 | I toss a coin three times; given that there is at least one tail, what is the probability that I toss exactly two heads? | 3/7 |
| 4 | What is the slope of the line with the equation y=3x-5? | 3 |
| **PERSON 2** | | |
| 1 | What is the sum of the first one-hundred positive integers? | 5050 |
| 2 | Solve for "x", if four to the x power equals one-half. | $-\frac{1}{2}$ or -0.5 |
| 3 | What is the lateral surface area, in square centimeters, of a right circular cone with a base diameter of six centimeters and a height of four centimeters? | $15\pi$ [$cm^2$] |
| 4 | What is the area of a circle with diameter 10? | $25\pi$ |
| **PERSON 3** | | |
| 1 | If a car traveled at 60 mph for 3 hours, how many miles did it travel? | 180 [miles] |
| 2 | How many diagonals can be drawn in a regular nine-sided polygon? | 27 [diag] |
| 3 | A circle of radius five centimeters is inscribed in a triangle that has a perimeter of sixty centimeters. What is the area of the triangle, in square centimeters? | 150 [$cm^2$] |
| 4 | What is the result when the largest four-digit base five number is expressed in base ten? | 624 |
| **PERSON 4** | | |
| 1 | What is the arithmetic mean of 81 and 17? | 49 |
| 2 | Three students are chosen out of eight to make a team. In how many ways can this be done? | 56 [ways] |
| 3 | One leg of a right triangle is nine centimeters and the other leg and hypotenuse differ by one centimeter. What is the length of the hypotenuse, in centimeters? | 41 |
| 4 | How many zeros does twenty factorial end in? | 4 |

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### $9^{th}$ – $10^{th}$ Grade – October 13, 2004

## COLLEGE KNOWLEDGE BOWL ROUND #1

| # | Problem | Answer |
|---|---|---|
| 1 | Let n be the probability of obtaining first a head and then a tail among two flips of a fair coin. Find the value of n and express your answer as a decimal. | 0.25 |
| 2 | Let f of x equal 3 times x-cubed minus five times x-squared plus four x minus one. What is f of the quantity x plus 2? | $3x^3+13x^2+20x+11$ |
| 3 | How many pairs of twin prime numbers, that is, primes separated by two, exist where both primes are less than fifty? | 6 |
| 4 | What is the remainder when three hundred thirty-three thousand three hundred thirty-five is divided by thirteen? | 2 |
| 5 | How many ways can seven people be seated around a circular dinner table? | 720 [ways] |
| 6 | Anna and Andrew are having a race. Anna can run at three meters per second, while Andrew runs at four meters per second. If Anna gets a ten meter head start, how many meters ahead will Andrew be at the end of the 60 meter race? | 5 |
| 7 | How many square posts are needed to fence off a 20 foot by 20 foot plot of land if the posts are spaced one foot apart? | 80 [posts] |
| | Extra Problem – Only if Needed | |
| 8 | What is the sum of the factors of 36, if 1 and 36 are included? | 91 |

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## COLLEGE KNOWLEDGE BOWL ROUND #2

| # | Problem | Answer |
|---|---|---|
| 1 | How many ways are there to choose three people for a committee out of five candidates? | 10 [ways] |
| 2 | How many positive integers less than 100 are divisible by 3 or 5? | 46 [integers] |
| 3 | A cylinder has radius 2 and height 6. The radius of the cylinder is doubled and the height is halved. What is the ratio of the old surface area to the new surface area? | 4:7 or 4/7 or 4 to 7 |
| 4 | The sum of two numbers is 3 and their product is 2. What is the sum of the squares of the two numbers? | 5 |
| 5 | Willy has three blue socks, four orange socks, and two brown socks in his drawer. How many socks must he draw to ensure that he has at least one non-brown pair? | 5 [socks] |
| 6 | What is the probability that Jerrad obtains two or more heads when he flips a fair coin five times? | 13/16 |
| 7 | Find the inverse of the following function defined for non-negative x: y equals three times x-squared plus four. | $y=\sqrt{\frac{x-4}{3}}$ |
| 7.1 | Extra Problem – Only if Needed | |
| 8 | How many diagonals does a decagon have? | 35 [diag] |

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$9^{th}$ – $10^{th}$ Grade – October 13, 2004

## COLLEGE KNOWLEDGE BOWL ROUND #3

| # | Problem | Answer |
|---|---|---|
| 1 | What is the product of all possible values of j in the equation: two j-cubed plus three j-squared minus j plus four equals zero? | -2 |
| 2 | Determine the median of the following set of data: $\left\{\frac{3}{2}, 3, 5, 4.3, 1, \frac{3}{4}, 7\right\}$ | 3 |
| 3 | A pentagon has interior angles of A, 2A, 3A, 4A, and 5A. What is the value of 15A, in degrees? | 540 [$^0$] |
| 4 | Triangle ABC has sides a and b of lengths 4 and 6, angle C, which is in between sides a and b, has a measure of 45 degrees. What is the area of this triangle? | $6\sqrt{2}$ |
| 5 | How many zeros are at the end of the product of the first ten non-negative integers? | 1 |
| 6 | If there are seven people, out of which three people have the flu, five people have the hiccups, and two people have the measles, what is the largest possible number of people who could have none of the above? | 2 [people] |
| 7 | How many ways are there to rearrange the letters in the word MOOSE if the two O's must be kept together? | 24 [ways] |
| | Extra Problem – Only if Needed | |
| 8 | How many prime numbers are there between 90 and 100? | 1 |

# "Math is Cool" Championships – 2004-05

$9^{th}$ – $10^{th}$ Grade – October 13, 2004

Final Score: KEY

First Score

School Name_____Team #_____

Proctor Name_____Room #_____

**STUDENT NAME_____**

## Individual Contest – Score Sheet

## DO NOT WRITE IN SHADED REGIONS

| | Answer | 1 or 0 | 1 or 0 |
|---|---|---|---|
| 1 | $\frac{137}{4}$ | | |
| 2 | III (or 3 or $3^{rd}$) | | |
| 3 | 4 | | |
| 4 | (6,5) | | |
| 5 | $\frac{-\sqrt{2}}{2}$ | | |
| 6 | 96 | | |
| 7 | 1800 | | |
| 8 | 12 | | |
| 9 | $4\pi$ | | |
| 10 | $2^2 \bullet 3^2 \bullet 5$ | | |
| 11 | $3x^2 + 2x + 1$ | | |
| 12 | [z=] -10 | | |
| 13 | 18 [palindromes] | | |
| 14 | 211 | | |
| 15 | $2220121_{(3)}$ | | |
| 16 | 40 | | |
| 17 | 24 [chickens] | | |
| 18 | $3\sqrt{3}$ [cm] | | |
| 19 | 8 [numbers] | | |
| 20 | $168/25$ | | |
| | | | |

| | Answer | 1 or 0 | 1 or 0 |
|---|---|---|---|
| 21 | 91 [handshakes] | | |
| 22 | 6 | | |
| 23 | $\frac{125\pi}{48}$ | | |
| 24 | 3 | | |
| 25 | $-2/3$ | | |
| 26 | $(\sqrt{7},3),(-\sqrt{7},3),(0,-4)$ | | |
| 27 | $8 + 4\pi^2$ | | |
| 28 | 7/8 | | |
| 29 | 6 | | |
| 30 | 7 | | |
| 31 | 45 [ways] | | |
| 32 | $1101001_{(-2)}$ or $-111011_{(-2)}$ | | |
| 33 | 6 | | |
| 34 | 120 | | |
| 35 | $18\sqrt{91} + 54\sqrt{3}$ | | |
| 36 | $\frac{75\sqrt{3}}{4}$ | | |
| 37 | 1 | | |
| 38 | $\pi/4$ | | |
| 39 | $1/14$ | | |
| 40 | 4 | | |
| | | | |

# "Math is Cool" Championships – 2004-05

9th – 10th Grade – October 13, 2004

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score

(out of 18)

**STUDENT NAME_____**

## Individual Multiple Choice Contest – Score Sheet

**Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.**

**DO NOT WRITE IN SHADED REGIONS**

| | Answer | -1, 0 or 2 | -1, 0 or 2 |
|---|---|---|---|
| 1 | D | | |
| 2 | C | | |
| 3 | D | | |
| 4 | A | | |
| 5 | D | | |
| 6 | E (58) | | |
| 7 | A | | |
| 8 | D | | |
| 9 | B | | |
| | | | |

# "Math is Cool" Championships – 2004-05

$9^{th}$ – $10^{th}$ Grade – October 13, 2004

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score

(out of 10)

**STUDENT NAME_____**

## Team Contest – Score Sheet

**DO NOT WRITE IN SHADED REGIONS**

| | Answer | 1 or 0 | 1 or 0 |
|---|---|---|---|
| 1 | $\frac{35}{2}$, $17\frac{1}{2}$, or 17.5 | | |
| 2 | 286 [ways] | | |
| 3 | $9\sqrt{3}$ | | |
| 4 | 616 | | |
| 5 | 25 [cents] | | |
| 6 | 1039 | | |
| 7 | $\frac{15}{2}$ [cm] | | |
| 8 | $\frac{1}{6}$ | | |
| 9 | $(-3,2,-2)$ (must be as an ordered triple) | | |
| 10 | $\frac{1}{84}$ | | |
| | | | |

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School Name_____Team #_____

Proctor Name_____Room #_____

**STUDENT NAME_____**

Final Score:
KEY

First Score

## Pressure Round Answers

| | Answer |
|---|---|
| 1 | 32 [hours] |
| 2 | 18 |
| 3 | 45 |
| 4 | 54 |
| 5 | 0 [mm] |