"Math is Cool" Master's – 2004-05

Sponsored by: Lukins & Annis

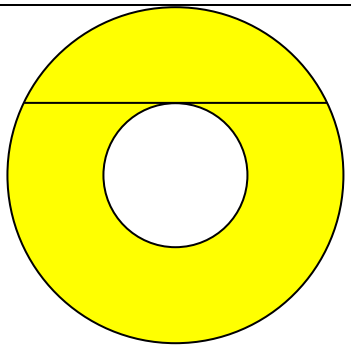
9^{th} – 10^{th} Grade – November 20, 2004

Individual Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

1	What is the slope of the line perpendicular to the line passing through (-1,-4) and (0,7)?
2	Evaluate: 5 - 2(3-4)-8(6-11)
3	What is the perimeter of a regular hexagon with a distance from the center to a vertex of 7?
4	Reduce the following fraction and write as an improper fraction. (-81/-54)
5	What is the least common multiple of 14, 15, and 21?
6	Solve for all values of x: $(x+3)^2 = 1$
7	What is the area of the triangle formed by the lines y=2, y=x+1, and x=6?
8	Two lighthouses start pointing their beams in the same direction at 11:11 PM. One completes its rotation every 18 minutes and the other every 28 minutes. What is the next time they will both be pointing in the same direction? (Express your answer in terms of AM or PM.)
9	Simplify $\frac{x^{2r} \cdot x^{r+3}}{x^{3r}}$ to the form x^a. What is the value of a?
10	What is the vertex, expressed in the form of (x,y), of the parabola: $f(x) = 3x^2 - 6x + 11$?
11	Solve for x in base 7: $41_6 + 10011_5 + x_7 = 1961_{10}$
12	Joe decides his first-born child must have a four-letter name using the letters J, O, E, and one other letter from the English alphabet. No letter may be used twice. How many ways can Joe name his child?
13	The measure of the interior angles of a pentagon are $(x+3)^\circ$, $(2x-8)^\circ$, $(6x+1)^\circ$, $(4x-3)^\circ$, and $(x+9)^\circ$. Solve for x. Write your answer as an improper fraction.
14	What is the square root of the product of the first five positive perfect squares?

15	Write $6x^2 - 11x - 10$ as the product of two linear factors with integer coefficients. (In other words, factor completely over the set of integers.)
16	A politician figures he should spend 70% of his speech on extraneous information and 30% of his speech on the issues. If he has spent 10 minutes on extraneous information and 15 minutes on the issues thus far, how long, in minutes, will his speech be if he spends the rest of the time on extraneous information and meets his goals?
17	Frankie can mow a lawn by himself in 150 minutes and Robert can mow that same lawn alone in x hours. When they start at the same time and work together at their normal rates, they finish the lawn in 1 hour 20 minutes. Find x and express answer as an improper fraction.
18	How many times does the graph of the equation $f(x) = x^5 - 3x^4 + 9x^3 - 19x^2 + 18x - 6$ cross the x-axis?
19	Jarret's license plate has 4 digits and 2 letters, with the digits all before the letters. Exactly three of the four digits are the same. The two letters are identical. Knowing all this, how many possible license plates are there for Jarret?
20	What is the surface area of an ice cream cone, which is made of a hemisphere placed on top of a cone with equal radii, with a total height of 8 and a radius of 2?
21	How many positive integers satisfy the following conditions: divisible by 12, perfect square, and less than 2500?
22	How many positive integers less than 1200 have an odd number of distinct factors?
23	A group of 6 friends went to the movies. Colin and Megan insist on sitting next to each other and Abe wants to sit next to them as well. Lee, Dani, and Libbey also want to sit together. How many ways can they do this?
24	Combine the following rational expressions. Reduce to lowest terms. $\frac{4x}{2x+6} - \frac{3}{x+3}$
25	Colin has a $\frac{1}{2}$ probability of waking up each time his alarm rings in the morning. His alarm continues going off at regular intervals. How many times must his alarm ring for his mom to figure there is at least an 85% probability that he is awake?
26	A mad soda mixer wants a drink that is 30% cranberry juice, 50% club soda, and 20% orange juice. He currently has a 24 ounce drink that is 50% cranberry juice and 50% orange juice. If his assistant spilled all the orange juice so all he has left to add is more cranberry juice and some club soda to the mix, how many additional ounces of fluid does he need to add?
27	What is the area of the shaded region of the concentric circles?

	 Line tangent to inner circle has length 16.
28	Solve the following absolute value inequality: $3\|2x-1\| \le x+4$
29	In a math class where everyone owns at least one graphing calculator of the Texas Instruments persuasion, 19 students own a TI-83, 8 own a TI-86, 7 own a TI-89, and 2 own a TI-92. Exactly 24 students own only one calculator, and only one student owns all four types. The rest have two or three calculators each. What is the minimum number of students in the class?

Challenge Questions

30	When all the 5-digit numbers that can be made by arranging the digits 2, 9, 7, 8, and 4 (each used exactly once) are listed in increasing order, what will be the 50th number on the list?
31	What is the sum of x, y, and z in the solution to this system of equations? $3x + 2y - 4z = 15$ $-6x + y + 2z = -16$ $x - 4y - 8z = -5$
32	Evaluate $\sum_{n=2}^{5}\left(\frac{n^2}{n!}-\frac{(n-1)^2}{(n-1)!}\right)$
33	Evaluate the finite geometric series: $\frac{2}{3}+\frac{4}{9}+\frac{8}{27}+\frac{16}{81}+\frac{32}{243}$
34	Mathitis is a disease that affects 10% of the population. There is a simple test for mathitis. This test results in a false positive 10% of the time and 1% of the people with the disease will test negative for it. What is the probability that someone who tests positive for mathitis actually has the disease?
35	What is the sum of the squares of the fourth roots of unity?
36	A gas tank formed by hemispheres attached to each end of a right cylinder turned on its side is filled to 2/3 capacity. The surface area of the entire tank is 84π square feet and the total length of the tank is 14 feet. What is the volume of gas, in cubic feet, currently contained in the tank?
37	What is the largest possible sum of two relatively prime factors of 4680 (not including 1 and 4680)?

38	A Pythagorean triple is a set of 3 positive integers that could form the sides of a right triangle. How many Pythagorean triples with all terms less than 50 exist such that at least two of the terms of the triple differ by exactly one?
39	The polynomial $x^3 + ax^2 + bx + 192 = 0$ has three real roots in geometric sequence. Determine the value of $\frac{b}{a}$.
40	Find three positive integral values for x that make $4^x + 4^{51} + 4^{54}$ a perfect square.

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Individual Multiple Choice Contest

Record only a letter as your answer on the colored sheet.

1	If **j** = $0.\overline{012}$, **k** = $0.0\overline{12}$, and **m**= $0.01\overline{2}$, what is the maximum difference between any two of **j**, **k**, and **m**? A) 0 B) 7/33,300 C) (12/990) – (12/999) D) 1/9900 E) 0.0002
2	Curve A is the set of points (**x**, **y**) such that **x** = **m** + 1 and **y** = -2**m** + 3. Curve B is the set of points (**x**, **y**) such that **x** = -2**n** + 2, **y** = 4**n** + 1. Both **m** and **n** are real numbers. How many points do curves A and B have in common? A) 0 B) 1 C) 2 D) infinitely many E) answer not given
3	For triangle ABC, the point equidistant from vertices A, B, and C is A) the point at which the 3 altitudes intersect B) the point at which the 3 perpendicular bisectors of the sides intersect C) the point at which the 3 angle bisectors intersect D) the point at which the 3 medians intersect E) none of the above
4	Biff can climb up 14 steps of a 65-step stairway in one minute. He rests for a minute after each minute of climbing. During each minute of rest, Eho pushes Biff down **n** steps, where **n** is a whole number. If Biff first reaches the top of the stairway sometime during the 15^{th} minute after he starts to climb, find **n**. A) 5 B) 6 C) 7 D) 8 E) 9 F) answer not given
5	Which of the following could not be the measure of an interior angle of a regular polygon? A) 171° B) 173° C) 175° D) 177° E) 179° F) answer not given

6	What is the value of x so that the line passing through (x, 5) and (4, 11) has a slope of 6? A) 0 B) 1 C) 2 D)3 E) answer not given
7	What is the slope of a line perpendicular a line with a slope of 2/3? A) 0 B) -2/3 C) 3/2 D) 4 E) answer not given
8	$y = x^3$ is symmetric to the A) x - axis B) y- axis C) origin D) the line y = 2x + 3
9	The table on the right shows the result of the operation r £ c, where r is the row entry and c is the column entry. For example, D £ B = A. What is A £ B £ C £ D £ E? A) A B) B C) C D) D E) E F) answer not given

£	A	B	C	D	E
A	B	C	D	E	A
B	C	D	E	A	B
C	D	E	A	B	C
D	E	A	B	C	D
E	A	B	C	D	E

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9th – 10th Grade – November 20, 2004

Team Contest

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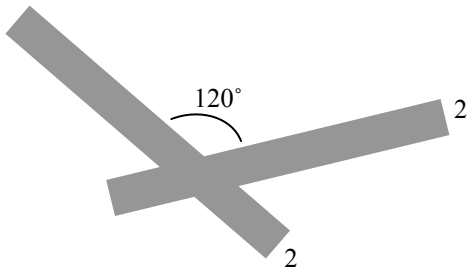
1	When two distinct numbers are chosen from the first ten positive integers, what is the probability that 4 is the smaller of the two numbers?
2	Two 8 by 2 inch rectangles are overlapped as shown, such that the obtuse angle between the two is 120°. What is the total shaded area, in square inches?
3	Find the vertex of the following parabola: $y = -2x^2 - 8x - 1$ Write your answer as an ordered pair (x,y).
4	Evaluate: $125^{(1/3)} + 8^{(2/3)} - 4096^{(1/4)} + 16^{1.25}$
5	The sides of a triangle are of integral lengths with two sides of length 6 and 10. What is the sum of the possible lengths for the third side of the triangle?
6	Simplify (positive exponents only): $\left(\frac{2^2 x^{(-4)}(3xy^2)^{(-1)}}{2^{(-3)} x^2 y^{(-3)}(45x^{(-2)})^0}\right)^{(-1)}$
7	What is the largest possible value of A in the cryptarithm shown, where each instance of a particular letter represents the same digit (0-9) and different letters represent different digits? $\begin{array}{r} ABC \\ +CAB \\ \hline BBDA \end{array}$
8	A game is played using two special cubical dice: one die has faces labeled 1, 1, 1, 2, 2, 3; the other die has faces labeled 1, 2, 3, 5, 5, 6. When these dice are rolled, what is the expected value of the sum of the numbers shown on their upper faces?
9	What is the sum of the 25 smallest perfect squares which are not perfect cubes?
10	What is the largest distance between any two of the following points? (5, 1), (-3, -4), (1, -2), (-2, 4), (3, 3), (-4, 1), (0, -3), (6, 0), (-1, 3), (2, -2)



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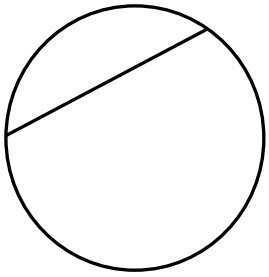
9^{th} – 10^{th} Grade – November 20, 2004

Pressure Round Contest

1	Of 100 students, 84 play basketball, 77 speak two languages, 67 have a job, and 96 are on the Math Team. At least **x** but not more than **y** Math Team members in this group have a job, speak two languages, and also play basketball. Find **x** and **y**, and give your answer as an ordered pair (**x**, **y**).
2	The sum of the digits of a three-digit number is 20. The tens digit exceeds twice the units digit by 1. The hundreds digit is one less than twice the units digit. Find the number.
3	Find the product of x and y if: $8^{(3x+2)} \cdot 3^{2y} = 2^{x} \cdot 27^{(y+1)}$
4	For how many 2-digit primes are both the sum of the digits and the positive difference between the digits also prime?
5	One chord cuts a circle into two regions, as shown. Find the median of all possible numbers of regions that could be created by 5 chords, no two collinear. (It may be possible to create a given number of regions in more than one way. In calculating median, ignore these duplicates and consider only the distinct numbers of regions possible.) 

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9^{th} – 10^{th} Grade – November 20, 2004

Mental Math Contest

Express all answers as reduced fractions in terms of radicals and π where applicable, unless stated otherwise.

PERSON 1		
1	Evaluate the expression "twice 'a' minus the square root of 'b'" given that a = 17 and b = 121.	23
2	During a season in a certain baseball league, every team plays every other team ten times. If there are five teams in the league, how many games are played in one season?	100 [games]
3	A right triangle has integral side lengths x and x+2 and a hypotenuse of 2x-2, all measured in centimeters. What is x?	6
4	There is enough candy in a bag to give twelve pieces of candy to each of my nieces. If five of my nieces do not want candy, sixteen pieces of candy can be given to each of my remaining nieces. How many nieces do I have?	20 [nieces]
PERSON 2		
1	Evaluate the expression "the difference between twice 'a' factorial and the cube root of 'b'" if a = 5 and b = 1000.	230
2	The sum of the interior angles of a polygon is 540 degrees. How many sides does the polygon have?	5 [sides]
3	If the area of an equilateral triangle is nine root three square centimeters, what is the side length of the triangle, in centimeters?	6 [units]
4	Josh travels from home to school. If he walks three-fourths of the way, jogs one-ninth of the way, and rides his hoverboard the remaining ten miles of his trip, how far is it from Josh's home to his school, in miles?	72 [miles]
PERSON 3		
1	Evaluate the expression given that a = 6, b = 4, and c = 3. Twice "a" cubed minus "b" squared plus "c".	419
2	An isosceles triangle has base angles each measuring x degrees and the other angle is of measure 2x degrees. What is the measure of one of the base angles in degrees?	45 [°]
3	The perimeter of a rectangle is eight "c". If one side has length one-half "c", what is the area of the rectangle?	$7c^2/4$
4	What is the volume of a cone of height 6 and base radius of 3?	18π
PERSON 4		
1	Evaluate the expression given that a = 25 and b = 35. Find the sum of "a" squared and "b" squared.	1850
2	A rectangle has a side length of measure 5 and diagonals of length 13. What is the measure of the other side?	12
3	A circle is inscribed in a square that is inscribed in a circle. What is the ratio of the area of the larger circle to that of the smaller circle?	2:1 [or 2 to 1]
4	What is the surface area of a sphere of radius 5?	100π

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COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	Solve for x: Eight minus 3 times the quantity five minus x equals 11.	6
2	A right triangle has sides of length x and x+1. The hypotenuse has a length of x+2. What is the value of x?	3
3	Write the equation 6x - 3y = 21 in slope intercept form.	Y=2x-7
4	How many four digit numbers have four distinct digits, the first three of which are prime, and the last of which is a multiple of 4?	48 [numbers]
5	What is the sum of the reciprocals of the roots of the equation y equals twenty x-squared minus x minus 1?	-1
6	A total of 925 tickets were sold for a total of $1150. If adult tickets sold for $2.00 each and children's tickets sold for $1.00 each, how many adult tickets were sold?	225 [tickets]
7	Evaluate: 125 raised to the negative two-thirds power. Write as a reduced fraction.	1/25
	Extra Problem – Only if Needed	
8	What is the interquartile range for this set of data: 13, 15, 25, 22, 18, 19	7

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COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	Evaluate seven choose three, plus six choose three.	55
2	What is the larger angle, in degrees, between the hour hand and the minute hand on a standard twelve-hour analog clock at three forty-eight PM?	186 [deg]
3	The sum of the interior angles of a polygon is 540 degrees. How many sides does the polygon have?	5 [sides]
4	Solve the following inequality for x: Four times the quantity x minus 1 is greater than 3x plus 6 minus 2x.	X>10/3 [x is greater than 10/3]
5	z varies jointly with x and the square root of y. If z is 15 when x is 5 and y is 36, find z when x is 2 and y is 25.	5
6	Find the equation of the line perpendicular to y equals negative one half x plus two and passing through the point (2 comma 7) in slope intercept form.	Y=2x+3
7	What is the units digit in the expansion of 7 raised to the 111 power?	3
	Extra Problem – Only if Needed	
8	Given the hypotenuse of a 30-60-90 triangle is 16, what is the length of the side opposite the 60 degree angle?	$8\sqrt{3}$

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COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	Find the range for this set of data: 17, 16, 12, 18, 23	11
2	What is the remainder when four x cubed minus five x squared plus six x minus eight is divided by the quantity x minus 1 ?	-3
3	Tealah likes to go shopping at American Eagle. She wants to know how much, in dollars, she will pay the cashier for a pair of pants that costs $53.20 in a town with 8% sales tax?	[$] 57.46
4	Use the equation y equals negative three times two to the x power to find the value of x when y = -48	4
5	If a letter is selected at random from the English alphabet, what is the probability that it is a letter from the word MATHEMATICS? Express as a reduced fraction.	4/13
6	Find the radius of a sphere with a surface area of 144B.	6
7	Find the number you would add to both the numerator and the denominator of 8/11 so the result would be 6/7.	10
	Extra Problem – Only if Needed	
8	Find the circumradius, in centimeters, of a triangle with a six-centimeter side opposite an angle measuring $\frac{\pi}{4}$ radians.	$3\sqrt{2}$

"Math is Cool" Master's – 2004-05

9^{th} – 10^{th} Grade – November 20, 2004

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score
9^{th}/10th

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	-1/11		
2	47		
3	42		
4	3/2		
5	210		
6	[x=] -2, -4		
7	25/2		
8	3:23 AM		
9	3		
10	(1,8)		
11	$31_{[7]}$		
12	552 [ways]		
13	269/7		
14	120		
15	(3x+2)(2x-5)		
16	50 [min]		
17	20/7 [hours]		
18	1 [time]		
19	9360 [lic plates]		
20	$(8+4\sqrt{10})\pi$		

	Answer	1 or 0	1 or 0
21	8 [integers]		
22	34 [integers]		
23	48 [ways]		
24	(2x-3)/(x+3)		
25	3 [times]		
26	36 [ounces]		
27	64π		
28	$-1/7 \le x \le 7/5$ or [-1/7,7/5]		
29	28 [students]		
30	72498		
31	21/4		
32	-19/24		
33	$\frac{422}{243}$		
34	11/21		
35	0		
36	72π [ft^3]		
37	941		
38	5		
39	$4\sqrt[3]{3}$		
40	X= {47, 53, 56}		

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9th – 10th Grade – November 20, 2004

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score
9th/10th

STUDENT NAME_____

Individual Multiple Choice Contest – Score Sheet

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

Answer		-1, 0 or 2	-1, 0 or 2
1	B		
2	D		
3	B		
4	B		
5	B		
6	D		
7	E		
8	C		
9	E		

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9^{th} – 10^{th} Grade – November 20, 2004

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Final Score:
KEY

First Score
$9^{th}/10^{th}$

Team Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	$\frac{2}{15}$		
2	$32 - \frac{8\sqrt{3}}{3}$		
3	(-2,7)		
4	33		
5	110		
6	$\frac{3x^7}{32y}$		
7	6		
8	$\frac{16}{3}$		
9	6920		
10	$\sqrt{101}$		

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School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Final Score:
KEY

First Score
9th/10th

Pressure Round Answers

	Answer
1	(24, 67)
2	794
3	9/4
4	5
5	11