"Math is Cool" Championships – 2005-06

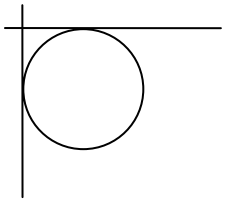
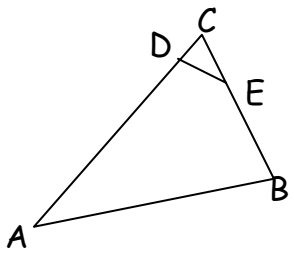
Sponsored by: Western Polymer Corporation

11^{th} – 12^{th} Grade – October 19, 2005

Individual Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

1	Evaluate: 2.7×3.14
2	Evaluate: 987^2
3	Evaluate: $4\frac{1}{3} + 5\frac{3}{4}$ and express your answer as a mixed number.
4	Evaluate: $2\frac{1}{4} / 1\frac{7}{8}$
5	What is the slope of the line 3x-5y=17?
6	My favorite number is tripled, the result is then reduced by 34, and then that result is doubled. The final result is 46. What is my favorite number?
7	What is the length, in centimeters, of the hypotenuse of a right triangle with legs measuring 8 cm and 20 cm? [Give an exact simplified answer.]
8	When two fair six-sided dice are rolled, what is the probability of rolling a sum of five or six?
9	If 1327=x mod 17, and 0<x<16, what is the value of x?
10	A pentagonal pyramid has exactly one of its base vertices (and no other vertices) cut off by a plane cut. How many total vertices are there on the two resulting solids?
11	How many positive two-digit integers consist of two different digits?
12	Evaluate: $\begin{vmatrix} 3 & 1 \\ -4 & 2 \end{vmatrix}$
13	If $f(x) = 4x - x^2$, evaluate $f(f(5))$.
14	Evaluate: $\csc(5\pi/3)$
15	A line through P intersects circle O at points A and B, and another line passes through P and is tangent to circle O at C. If AB=48 and BP=6, what is the length of PC?
16	What is the sum of the twelve smallest positive multiples of twelve?
17	A bag contains the six letters of the word "BANANA". If two letters are drawn from the bag, what is the probability that they are the same as one another?

18	What is the sum of an infinite geometric series with a second term of $4/3$ and a common ratio of $3/4$?
19	What is the maximum value of the function $f(x) = 3x - 2x^2$?
20	If the probability of event A is $3/4$, the probability of event B is $2/3$, and the probability of both events occurring is $3/5$, what is the probability that at least one of the events occurs?
21	What is the largest integer satisfying $\|47-3x\|<88$?
22	Evaluate: $\log_4(128)$
23	A set contains the integers from 1 to 1001 inclusive. What is the highest number of elements in a subset such that no two elements from the subset add to a multiple of 5?
24	A circle is tangent to two perpendicular lines. There is a point on the circle that is 8 units from one line and 9 units from the other. What is the radius of the circle? 
25	If (2x+1)=7 mod 13 and x is an integer between 0 and 100 inclusive, what is the sum of all possible values of x?
26	If $A = \begin{bmatrix} 3 & 0 & 1 \\ -1 & 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ -3 & 1 \\ 0 & -2 \end{bmatrix}$, what is the sum of the elements of AB?
27	If $a_1 = 1, a_2 = 2$, and $a_n = 3a_{n-2} + a_{n-1}$ evaluate a_6.
28	In the diagram AD=10, CD=2, CE=3 and BE=5, what is the ratio of the area of the quadrilateral ADEB to the area of triangle ABC. Express your answer as m:n where m and n are relatively prime integers. 
29	A nine term arithmetic sequence $a_1, a_2, \ldots, a_8, a_9$ satisfies $a_5 + a_7 = -17$ and $a_4 + a_6 = 17$. What is the sum of the terms of the sequence?

Challenge Questions

30	What is the fewest number of triangles that can be formed by five lines in a plane if each line must intersect at least two others and no more than two may be parallel?
31	What is the length of the major axis of the ellipse with equation $4x^2 - 24x + y^2 + 2y = 11$?
32	What is the perimeter, in centimeters, of a regular polygon with interior angles measuring 162 degrees and sides measuring 162 cm?
33	If $\log_3(18) = a$, evaluate $\log_3(72)$ in terms of a.
34	How many positive four-digit numbers are divisible by three but not two?
35	The graph of $y = \frac{3x^3 + x^2 + 4}{x^2 - 48}$ in the Cartesian plane has asymptotes $x = a, x = b$, and $y = cx + d$. Evaluate $abcd$.
36	What is the period of the following function? f(x) = 3sin(3x/4) + 5cos(5x-3)
37	Consider the six orthogonal unit vectors North ($\vec{N}$), East ($\vec{E}$), South ($\vec{S}$), West($\vec{W}$), Up ($\vec{U}$), and Down ($\vec{D}$) in my apartment building. If Jorve is 30 meters away in the $\vec{N} \times \vec{U}$ direction, Anya is 40 meters away in the $\vec{W} \times \vec{S}$ direction, and Pradeep is 24 meters away in the $\vec{N} \times \vec{E}$ direction, what is the area, in square meters, of the triangle formed by these three individuals?
38	A triangle has two sides measuring 2 cm and $\sqrt{2}$ cm with an included angle of 135°. How long, in cm, is the third side?
39	What is the largest value of x (in radians) less than π satisfying cosx = cos(6x)?
40	When four cards are drawn from a standard 52-card deck, what is the probability of getting at most three different ranks (e.g. one King, one seven, and two fours)?

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Individual Multiple Choice Contest

Record only a letter as your answer on the colored sheet.

1	Evaluate: $4+5\times 6^2-(7+9)\div 8$ A) 21 B) 182 C) 210 D) 384 E) answer not given
2	If $2x+4y+3z=27$, $-2x+y+2z=18$, and $3x-y-z=-15$, evaluate $4x+y+3z$. A) 11 B) -3 C) 21 D) 8 E) answer not given
3	A rectangle has an area of 486 square centimeters and a perimeter of 90 cm. What is the length, in centimeters, of one of its diagonals? A) $9\sqrt{13}$ B) $25\sqrt{2}$ C)32 D) $4\sqrt{65}$ E) answer not given
4	What is the area inside the graph of $2x^2+2y^2+20y-12x=220$? A) 12π B) 180π C) 48π D) 144π E) answer not given
5	If $\sin q=\frac{3}{8}$, what is the product of all possible values of $\tan q$? A) 1 B) $-\frac{1}{2}$ C) $-\frac{\sqrt{5}}{11}$ D) $-\frac{9}{55}$ E) answer not given
6	When three fair, six-sided dice are rolled, what is the probability that two of them show the same number and one does not? A) $\frac{5}{12}$ B) $\frac{91}{216}$ C) $\frac{23}{54}$ D) $\frac{31}{72}$ E) answer not given
7	What is the next term of the sequence below? 2, -3, -5, 8, -12, -13, 15, -18, -22, 23, -25, ____ A) -26 B) 27 C) -28 D) 28 E) answer not given
8	How many integer values of x between 100 and 200 inclusive satisfy $3x+5\equiv 7 \bmod 11$? A) 3 B) 34 C) 9 D) 33 E) answer not given
9	What is the smallest possible area, in square centimeters, of an isosceles triangle with sides measuring 4 cm and 5 cm? A) $2\sqrt{21}$ B) $3\sqrt{8}$ C) $\frac{5\sqrt{39}}{4}$ D) $2\sqrt{10}$ E) answer not given

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11^{th} – 12^{th} Grade – October 19, 2005

Team Contest

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1	What is the remainder when 13^{15} is divided by 17?
2	How many prime numbers between 100 and 300 have digits that sum to 15?
3	The line $y = 4x + 2$ intersects the parabola $y = 2x^2 + 15x - 19$ at (a, b) and (c, d). Evaluate $a + b + c + d$ and express as a common fraction.
4	How many lattice points (points with integer coordinates) lie on the graph of $x^2 - 4x + y^2 + 2y = 80$?
5	What is the sum of the values of x, $0 \le x < 2\pi$, that satisfy $\sec x = 4\sin x$?
6	If points (0, 0, 0), (0, 1, 0), and (0, 0, 1) are three of eight vertices of a cube, how many possible locations could be the other vertices of the cube?
7	Evaluate: $\begin{vmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 4 & -3 \\ 1 & 2 & 3 & 1 \\ 2 & -5 & 0 & 3 \end{vmatrix}$
8	Two real numbers, x and y, are randomly chosen between 0 and ten. What is the probability that $x + y > 10$ and $x^2 + y^2 < 100$?
9	What is the sum of the ten smallest perfect positive squares that are not perfect cubes?
10	Evaluate: $\sin\left(\frac{\pi}{12}\right) - \cos\left(\frac{\pi}{6}\right) + \tan\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right) - \cos\left(\frac{5\pi}{12}\right)$

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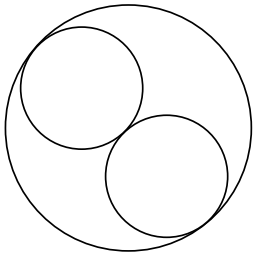
Pressure Round Contest

1	Find the minimum value of $f(x) = 3x^2 - 2x + 8$.
2	Let n be the total number of positive integer factors of 555^{100}. How many positive integer factors does n have?
3	What will be the median of all the 5-digit numbers that can be made by arranging the digits 2, 9, 7, 8 and 4 (each used exactly once) expressed as a decimal?
4	A circle of radius 12 cm has two congruent circles internally tangent to it and externally tangent to each other, as shown. The centers of these 3 circles are all collinear. A fourth, smaller circle C is tangent to all three of these circles. Find the radius, in cm, of circle C. 
5	For $n \geq 1$, $x_n^2 - (x_{n-1})(x_{n+1}) = (-2)^n$. Given that $x_0 = x_1 = 1$, find x_4.

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11^{th} – 12^{th} Grade – October 19, 2005

Mental Math Contest

Express all answers as reduced fractions in terms of radicals and π where applicable, unless stated otherwise.

PERSON 1		
1.1	What is the sum of the first 10 positive odd integers?	100
1.2	What is the value of x, if 6 x plus 11 equals 8?	-1/2
1.3	What is the measure of each exterior angle in a regular dodecagon, in degrees?	30 [°]
1.4	For what negative value of x is the logarithm base 2 of the quantity x squared minus 1 equal to 3?	-3
PERSON 2		
2.1	What is the least common multiple of 4, 6 and 10?	60
2.2	What is the sum of the solutions to 2 x squared minus 5 x plus 10 equals 0?	5/2
2.3	What is the area of an isosceles triangle with side lengths of 10, 10 and 12?	48 [un^2]
2.4	What is the tangent of 30 degrees times the tangent of 60 degrees?	1
PERSON 3		
3.1	What is the smallest three-digit prime number?	101
3.2	If Y equals 3 and Z equals 5, what is Y cubed times Z squared divided by 9?	75
3.3	If the probability of event A is two-thirds, what is the probability of A intersected with the complement of A?	0
3.4	What is the inverse sine of the sine of 100 degrees, in degrees?	80 [°]
PERSON 4		
4.1	What is 25 squared minus 24 squared?	49
4.2	In a two-person race between John and Tom in which there are no ties allowed, the probability that John wins is three-fifths, what are the odds of Tom winning? [Express answer in the form of a colon b.]	2 : 3
4.3	What is the minimum value of the function: f of x equals x squared plus 6 x plus 9?	0
4.4	What is the logarithm base 27 of 81?	4/3

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COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	What is the imaginary number, i, raised to the 43rd power?	-i
2	How many ways can you permute the letters in the word: B-A-N-A-N-A so that no two vowels are together?	12 [ways]
3	What is the remainder when six hundred twelve thousand seven hundred thirteen is divided by thirteen?	10
4	Of the solutions to x cubed minus three x squared minus eight x equals five, how many are both real and positive?	1 [sol]
5	What is the distance from the point 1 comma 2 to the line given by 3x plus 4y equals 1?	2
6	For what value of m greater than 4 is the quantity 3m plus 8 divided by the quantity m minus 1, an integer?	12
7	What is quantity log base 4 of one-third plus the quantity log base 4 of 24?	3/2
	Extra Problem – Only if Needed	
8	What is the product of the values of k that make x squared plus kx plus 49 a perfect square trinomial?	-196

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COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	What is the ninth digit after the decimal point in the decimal representation of one-thirteenth?	6
2	Express the ratio of eighteen one-hundredths to two and four tenths as a common fraction.	3 / 40
3	What is the smaller angle, in degrees, between the hands of an analog clock at 2:20pm?	50 [°]
4	The mean of three positive integers is greater than 12 and less than 18, what is the greatest possible value for the largest integer?	51
5	If i stands for the square root of negative 1, what is the quantity 1 plus i raised to the eighth power?	16
6	What is the limit as x approaches zero of the quantity 2x squared minus 5x plus 5 divided by the quantity 3x squared plus x minus 2?	-5 / 2
7	What is the sum of reciprocals of the solutions to the equation five x squared plus four x minus two equals zero?	2
	Extra Problem – Only if Needed	
8	True or False: In probability, if A and B are two disjoint events then they are independent.	False

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COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	Find the reciprocal of the complex number 5 plus 12i in the form of the quantity a plus bi divided by c where a, b and c are integers. What is a plus b plus c?	162
2	Given the set containing the numbers: 1, 3, 5, 7, 9, and 11; how many subsets of size three are there where the sum of the elements is greater than 20?	7 [subsets]
3	If f of x is an even function and g of x is an odd function; is f plus g of x necessarily even, necessarily odd or neither of these?	Neither
4	What is the mean of all the 5-digit numbers that can be formed using the digits 1, 2, 3, 4, and 5 exactly once each?	33333
5	When the point 5 comma 4 is reflected through the point 1, 1; what are coordinates of the new point?	(-3, -2)
6	I roll a fair 6-sided die 5 times, what is the probability that the last two rolls are both sixes?	1 / 36
7	In the circle described by x squared plus y squared equals 25, a tangent line is drawn at the point 3 comma 4. What is the y-coordinate of the y-intercept of the tangent line?	25 / 4
	Extra Problem – Only if Needed	
8	If a coin is weighted so that the probability of getting a head is 6/13, what is the probability of getting equal numbers of heads and tails on 5 flips?	0

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Final Score:
KEY

First Score

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	8.478		
2	974,169		
3	$10\frac{1}{12}$		
4	6/5		
5	3/5		
6	19		
7	$4\sqrt{29}$ [cm]		
8	1 /4		
9	1		
10	12 [vert]		
11	81 [int]		
12	10		
13	-45		
14	$\frac{-2\sqrt{3}}{3}$		
15	18 [un]		
16	936		
17	4/15		
18	64/9		
19	9/8		
20	$49/60$		

	Answer	1 or 0	1 or 0
21	44		
22	7/2		
23	402 [elements]		
24	29 [un]		
25	388		
26	-12		
27	59		
28	15:16		
29	153/2		
30	0 [triangles]		
31	$8\sqrt{3}$ [un]		
32	3240 [cm]		
33	3a-4		
34	1500 [num]		
35	-144		
36	8π		
37	960 [m^2]		
38	$\sqrt{10}$ [cm]		
39	6 π/7 [radians]		
40	$\frac{1349}{4165}$		

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School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score

(out of 18)

STUDENT NAME_____

Individual Multiple Choice Contest – Score Sheet

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer	-1, 0 or 2	-1, 0 or 2
1	B		
2	A		
3	A		
4	D		
5	D		
6	A		
7	C		
8	C		
9	C		

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School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score

(out of 10)

STUDENT NAME_____

Team Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	2 or 0	2 or 0
1	4		
2	0 [primes]		
3	$-\frac{47}{2}$		
4	16 [points]		
5	3π		
6	9 [locations]		
7	153		
8	$\frac{\pi}{4}-\frac{1}{2}$ (or (π-2)/4)		
9	585		
10	$1-\sqrt{3}$		

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School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Final Score:
KEY

First Score

Pressure Round Answers

	Answer
1	23/3
2	4 [factors]
3	76615.5
4	4 [cm]
5	11