# "Math is Cool" Championships – 2005-06

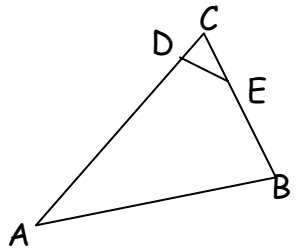
## Sponsored by: Basic American Foods

### $9^{th}$ – $10^{th}$ Grade – October 19, 2005

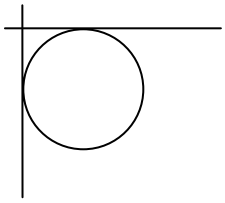
### Individual Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

| | |
|---|---|
| 1 | Evaluate: $92.1 - 4.78$ |
| 2 | Evaluate: $\frac{4}{9} / \frac{8}{3}$ |
| 3 | Evaluate: $2.7 \times 3.14$ |
| 4 | Evaluate: $527 \times 473$ |
| 5 | Evaluate: $987^2$ |
| 6 | Evaluate: $4\frac{1}{3} + 5\frac{3}{4}$ and express your answer as a mixed number. |
| 7 | What is the slope of the line 3x-5y=17? |
| 8 | My favorite number is tripled, the result is then reduced by 34, and then that result is doubled. The final result is 46. What is my favorite number? |
| 9 | What are the coordinates, in (x, y) form, of the y-intercept of the line 3x-2y=7? |
| 10 | What is the length, in centimeters, of the hypotenuse of a right triangle with legs measuring 8 cm and 20 cm? [Give an exact simplified answer.] |
| 11 | When two fair six-sided dice are rolled, what is the probability of rolling a sum of five or six? |
| 12 | If 1327=x mod 17, and 0<x<16, what is the value of x? |
| 13 | A pentagonal pyramid has exactly one of its base vertices (and no other vertices) cut off by a plane cut. How many total vertices are there on the two resulting solids? |
| 14 | What is the surface area of a right rectangular prism with edge lengths of 7, 9, and 3? |
| 15 | What is the volume, in cubic centimeters, of a right circular cone with a base radius measuring 3 cm and a height of 8 cm? |

| | |
|---|---|
| 16 | How many positive two-digit integers consist of two different digits? |
| 17 | Express the base nine number $678_9$ in base ten. |
| 18 | What is the area, in square centimeters, of a rectangle with a perimeter measuring 68 cm and a diagonal measuring 26 cm? |
| 19 | I have 22 coins, each either a nickel or a quarter, with a total value of $2.50. How many nickels are there? |
| 20 | If the probability of event A is $\frac{3}{4}$, the probability of event B is $\frac{2}{3}$, and the probability of both events occurring is $\frac{3}{5}$, what is the probability that at least one of the events occurs? |
| 21 | What is the largest integer satisfying $\|47-3x\|<88$? |
| 22 | What is the sum of the twelve smallest positive multiples of twelve? |
| 23 | A bag contains the six letters of the word "BANANA". If two letters are drawn from the bag, what is the probability that they are the same as one another? |
| 24 | Evaluate: $\begin{vmatrix} 3 & 1 \\ -4 & 2 \end{vmatrix}$ |
| 25 | If $f(x)=4x-x^2$, evaluate $f(f(5))$. |
| 26 | How many positive four-digit numbers are divisible by three but not two? |
| 27 | In the diagram AD=10, CD=2, CE=3 and BE=5, what is the ratio of the area of the quadrilateral ADEB to the area of triangle ABC. Express your answer as m:n where m and n are relatively prime integers.   |
| 28 | What is the sum of an infinite geometric series with a second term of $\frac{4}{3}$ and a common ratio of $\frac{3}{4}$? |
| 29 | If $a_1=1, a_2=2$, and $a_n=3a_{n-2}+a_{n-1}$ evaluate $a_6$. |

# Challenge Questions

| | |
|---|---|
| 30 | What is the maximum value of the function $f(x) = 3x - 2x^2$? |
| 31 | What is the fewest number of triangles that can be formed by five lines in a plane if each line must intersect at least two others and no more than two may be parallel? |
| 32 | Evaluate: $\log_4(128)$ |
| 33 | If (2x+1)=7 mod 13 and x is an integer between 0 and 100 inclusive, what is the sum of all possible values of x? |
| 34 | A set contains the integers from 1 to 1001 inclusive. What is the highest number of elements in a subset such that no two elements from the subset add to a multiple of 5? |
| 35 | A nine term arithmetic sequence $a_1, a_2, \ldots, a_8, a_9$ satisfies $a_5 + a_7 = -17$ and $a_4 + a_6 = 17$. What is the sum of the terms of the sequence? |
| 36 | If $A = \begin{bmatrix} 3 & 0 & 1 \\ -1 & 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ -3 & 1 \\ 0 & -2 \end{bmatrix}$, what is the sum of the elements of AB? |
| 37 | A circle is tangent to two perpendicular lines. There is a point on the circle that is 8 units from one line and 9 units from the other. What is the radius of the circle?  |
| 38 | What is the length of the major axis of the ellipse with equation $4x^2 - 24x + y^2 + 2y = 11$? |
| 39 | A triangle has two sides measuring 2 cm and $\sqrt{2}$ cm with an included angle of 135°. How long, in cm, is the third side? |
| 40 | When four cards are drawn from a standard 52-card deck, what is the probability of getting at most three different ranks (e.g. one King, one seven, and two fours)? |

# "Math is Cool" Championships – 2005-06

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### $9^{th}$ – $10^{th}$ Grade – October 19, 2005

### Individual Multiple Choice Contest

Record only a letter as your answer on the colored sheet.

| | |
|---|---|
| 1 | Evaluate: $4 + 5 \times 6^2 - (7 + 9) \div 8$<br><br>A) 21 B) 182 C) 210 D) 384 E) answer not given |
| 2 | If $2x + 4y + 3z = 27$, $-2x + y + 2z = 18$, and $3x - y - z = -15$, evaluate $4x + y + 3z$.<br><br>A) 11 B) -3 C) 21 D) 8 E) answer not given |
| 3 | A rectangle has an area of 486 square centimeters and a perimeter of 90 cm. What is the length, in centimeters, of one of its diagonals?<br><br>A) $9\sqrt{13}$ B) $25\sqrt{2}$ C)32 D) $4\sqrt{65}$ E) answer not given |
| 4 | What is the area inside the graph of $2x^2 + 2y^2 + 20y - 12x = 220$?<br><br>A) $12\pi$ B) $180\pi$ C) $48\pi$ D) $144\pi$ E) answer not given |
| 5 | What is the least common multiple of 120 and 350?<br><br>A) 1400 B) 3500 C) 4200 D) 21000 E) answer not given |
| 6 | When three fair, six-sided dice are rolled, what is the probability that two of them show the same number and one does not?<br><br>A) $\frac{5}{12}$ B) $\frac{91}{216}$ C) $\frac{23}{54}$ D) $\frac{31}{72}$ E) answer not given |
| 7 | What is the next term of the sequence below?<br><br>2, -3, -5, 8, -12, -13, 15, -18, -22, 23, -25, ____<br><br>A) -26 B) 27 C) -28 D) 28 E) answer not given |
| 8 | When my favorite number is cubed, that result is doubled, and that result is reduced by 4, the final result is -58. What is my favorite number?<br><br>A) -1 B) 1 C) -5 D) -3 E) answer not given |
| 9 | What is the area, in square centimeters, of an isosceles triangle with sides measuring 4 cm, 5 cm and 5 cm?<br><br>A) $\frac{7\sqrt{17}}{4}$ B) $2\sqrt{21}$ C) $2\sqrt{10}$ D) $\frac{5\sqrt{39}}{4}$ E) answer not given |

# "Math is Cool" Championships – 2005-06

## Sponsored by: Basic American Foods

### $9^{th}$ – $10^{th}$ Grade – October 19, 2005

### Team Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

| | |
|---|---|
| 1 | What is the remainder when $13^{15}$ is divided by 17? |
| 2 | How many prime numbers between 100 and 300 have digits that sum to 15? |
| 3 | The line $y = 4x + 2$ intersects the parabola $y = 2x^2 + 15x - 19$ at (a, b) and (c, d). Evaluate $a + b + c + d$ and express as a common fraction. |
| 4 | How many lattice points (points with integer coordinates) lie on the graph of $x^2 - 4x + y^2 + 2y = 80$? |
| 5 | Evaluate: $2^3 \cdot 3^2 \cdot 5 \cdot 7^2 \cdot 11$ |
| 6 | If points (0, 0, 0), (0, 1, 0), and (0, 0, 1) are three of eight vertices of a cube, how many possible locations could be the other vertices of the cube? |
| 7 | How many right circular cones with base radii of 1 and heights of 1 would it take to have the same total volume as a sphere with a radius of 6? |
| 8 | Two real numbers, x and y, are randomly chosen between 0 and ten. What is the probability that $x + y > 10$ and $x^2 + y^2 < 100$? |
| 9 | What is the sum of the ten smallest perfect positive squares that are not perfect cubes? |
| 10 | If $f(x) = \dfrac{2x-3}{3x+2}$ and $g(x) = f(f(x))$, evaluate $g(-1)$. |

# "Math is Cool" Championships – 2005-06

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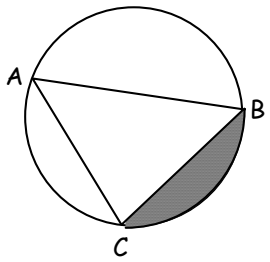
$9^{th}$ – $10^{th}$ Grade – October 19, 2005

Pressure Round Contest

| | |
|---|---|
| 1 | Find the minimum value of $f(x) = 3x^2 - 2x + 8$. |
| 2 | Let n be the total number of positive integer factors of $555^{100}$. How many positive integer factors does n have? |
| 3 | When all the 5-digit numbers that can be made by arranging the digits 2, 9, 7, 8, and 4 (each used exactly once) are listed in increasing order, what will be the 50th number on the list? |
| 4 | Triangle ABC is inscribed in a circle with radius 4 (not drawn to scale). If AB = CB and AC = 8, find the shaded area.   |
| 5 | Find $\prod_{k=2}^{2005}\left(1-\frac{1}{k}\right)$. (The symbol $\Pi$ indicates a product.) |

# "Math is Cool" Championships – 2005-06

## Sponsored by: Basic American Foods

### 9th – 10th Grade – October 19, 2005

### Mental Math Contest

Express all answers as reduced fractions in terms of radicals and π where applicable, unless stated otherwise.

| | | |
|---|---|---|
| **PERSON 1** | | |
| 1.1 | What is the sum of the first 10 positive odd integers? | 100 |
| 1.2 | What is the value of x, if 6 x plus 11 equals 8? | -1/2 |
| 1.3 | What is the measure of each exterior angle in a regular dodecagon, in degrees? | 30 [°] |
| 1.4 | What is the probability of getting exactly two heads when I flip a fair coin 4 times? | 3/8 |
| **PERSON 2** | | |
| 2.1 | What is the least common multiple of 4, 6 and 10? | 60 |
| 2.2 | What is the sum of the solutions to 2 x squared minus 5 x plus 10 equals 0? | 5/2 |
| 2.3 | What is the area of an isosceles triangle with side lengths of 10, 10 and 12? | 48 [un$^2$] |
| 2.4 | Given that I get at least one head on 5 flips of a fair coin, what is the probability that all 5 are heads? | 1/31 |
| **PERSON 3** | | |
| 3.1 | What is the smallest three-digit prime number? | 101 |
| 3.2 | If Y equals 3 and Z equals 5, what is Y cubed times Z squared divided by 9? | 75 |
| 3.3 | If the probability of event A is two-thirds, what is the probability of A intersected with the complement of A? | 0 |
| 3.4 | What is the perimeter of a rhombus with diagonals of 10 and 24? | 52 [un] |
| **PERSON 4** | | |
| 4.1 | What is 25 squared minus 24 squared? | 49 |
| 4.2 | In a two-person race between John and Tom in which there are no ties allowed, the probability that John wins is three-fifths, what are the odds of Tom winning? [Express answer in the form of a colon b.] | 2 : 3 |
| 4.3 | What is the minimum value of the function: f of x equals x squared plus 6 x plus 9? | 0 |
| 4.4 | What is the total surface area of a cone with a base diameter of 8 and a height of 3? | 36π [un$^2$] |

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$9^{th}$ – $10^{th}$ Grade – October 19, 2005

## COLLEGE KNOWLEDGE BOWL ROUND #1

| # | Problem | Answer |
|---|---|---|
| 1 | A 72-degree arc of a circle has a length of 12, what is the area of the circle? | $900 / \pi$ [un$^2$] |
| 2 | The second and third terms of an infinite geometric sequence are 12 and 4, what is the sum of the infinite sequence? | 54 |
| 3 | Luke is dealt a pair of Jacks in a poker game. Three additional cards are turned over. What is the probability that these three cards contain exactly one Jack? | 141 / 1225 |
| 4 | What is the maximum number of points that a circle could intersect with rectangle? | 8 |
| 5 | If Y plus Z equals 10 and Y times Z equals 20, what is Y squared plus Z squared? | 60 |
| 6 | What is the smallest integer greater than 25 that has a remainder of 3 when divided by 5 and a remainder of 7 when divided by 11? | 73 [int] |
| 7 | What is the smallest base that the base 10 number 321 will only have two digits? | [base] 18 |
| | Extra Problem – Only if Needed | |
| 8 | The houses on my block are numbered 103, 107, 111, continuing until 139. What is the sum of the house numbers? | 1210 |

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$9^{th}$ – $10^{th}$ Grade – October 19, 2005

## COLLEGE KNOWLEDGE BOWL ROUND #2

| # | Problem | Answer |
|---|---|---|
| 1 | Anakin was adding the positive integers up to 40, but accidentally skips one, getting total of 799. What number did he skip? | 21 |
| 2 | What is the smallest positive integer, n, for which n squared plus n plus 1 is not prime? | 4 |
| 3 | What is the difference in area between a square of side length 6 and the circle that is inscribed in it? | $36 - 9\pi$ [un$^2$] |
| 4 | What is the smallest solution to x cubed plus 3x squared minus 4x minus 12 equals 0? | -3 |
| 5 | An urn contains 4 red and 5 blue marbles. If I draw out two marbles without replacement, what is the probability that both are the same color? | 4 / 9 |
| 6 | What is the area of an ellipse with a major axis of 12 and a minor axis of 1? | $3\pi$ |
| 7 | How many subsets with three elements can be made from a set containing: Hydrogen, Helium, Nitrogen, Oxygen, Silver, Gold and Uranium? | 35 [subsets] |
| | Extra Problem – Only if Needed | |
| 8 | What is the positive difference between the mean and median of the numbers: 23, 42, 55, 34, and 12? Express answer as a decimal. | 0.8 |

# "Math is Cool" Championships – 2005-06

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9th – 10th Grade – October 19, 2005

## COLLEGE KNOWLEDGE BOWL ROUND #3

| # | Problem | Answer |
|---|---|---|
| 1 | What is the sum of the positive integers less than 100 with exactly three positive integer factors? | 87 |
| 2 | A parabola with an axis of symmetry parallel to the y-axis and vertex at the point 1 comma 8 crosses the axis at the point 6 comma 0. At what other point does it cross the x axis? | (-4,0) |
| 3 | ObiWan knows that the gravitational force between two bodies varies inversely as the square of the distance between them. At 400 thousand feet, the force on his ship is x newtons toward the planet. What will be the force, in newtons, at 100 thousand feet in terms of x? | 16x [newtons] |
| 4 | Expressed in base 10, what is the largest 4 digit number in base 5? | 624 |
| 5 | What is the average speed, in miles per hour, of a car that travels at 50 miles per hour to Spokane and 60 miles per hour back? | 600/11 [mph] |
| 6 | An equilateral triangle has the same area of a 30-60-90 triangle with a hypotenuse of 4 times the square root of 2. What is the side length of the equilateral triangle? | 4 [un] |
| 7 | What is the probability of getting 1, 2 or 3 heads with four flips of a fair coin? | 7/8 |
| | Extra Problem – Only if Needed | |
| 8 | Using pennies, nickels, dimes and quarters, how many ways can you make 37 cents? | 24 [ways] |

# "Math is Cool" Championships – 2005-06

$9^{th}$ – $10^{th}$ Grade – October 19, 2005

Final Score: KEY

First Score

School Name_____Team #_____

Proctor Name_____Room #_____

**STUDENT NAME_____**

## Individual Contest – Score Sheet

## DO NOT WRITE IN SHADED REGIONS

| | Answer | 1 or 0 | 1 or 0 |
|---|---|---|---|
| 1 | 87.32 | | |
| 2 | 1/6 | | |
| 3 | 8.478 | | |
| 4 | 249,271 | | |
| 5 | 974,169 | | |
| 6 | $10\frac{1}{12}$ | | |
| 7 | 3/5 | | |
| 8 | 19 | | |
| 9 | (0, -7/2) | | |
| 10 | $4\sqrt{29}$ [cm] | | |
| 11 | 1 / 4 | | |
| 12 | 1 | | |
| 13 | 12 [vert] | | |
| 14 | 222 [$un^2$] | | |
| 15 | $24\pi$ [$cm^3$] | | |
| 16 | 81 [int] | | |
| 17 | $557_{[10]}$ | | |
| 18 | 240 [$cm^2$] | | |
| 19 | 15 [nickels] | | |
| 20 | 49/60 | | |
| | | | |

| | Answer | 1 or 0 | 1 or 0 |
|---|---|---|---|
| 21 | 44 | | |
| 22 | 936 | | |
| 23 | 4/15 | | |
| 24 | 10 | | |
| 25 | -45 | | |
| 26 | 1500 [num] | | |
| 27 | 15:16 | | |
| 28 | 64/9 | | |
| 29 | 59 | | |
| 30 | 9/8 | | |
| 31 | 0 [triangles] | | |
| 32 | 7/2 | | |
| 33 | 388 | | |
| 34 | 402 [elements] | | |
| 35 | 153/2 | | |
| 36 | -12 | | |
| 37 | 29 [un] | | |
| 38 | $8\sqrt{3}$ [un] | | |
| 39 | $\sqrt{10}$ [cm] | | |
| 40 | $\frac{1349}{4165}$ | | |
| | | | |

# "Math is Cool" Championships – 2005-06

9th – 10th Grade – October 19, 2005

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score

(out of 18)

**STUDENT NAME_____**

## Individual Multiple Choice Contest – Score Sheet

**Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.**

**DO NOT WRITE IN SHADED REGIONS**

| | Answer | -1, 0 or 2 | -1, 0 or 2 |
|---|---|---|---|
| 1 | B | | |
| 2 | A | | |
| 3 | A | | |
| 4 | D | | |
| 5 | C | | |
| 6 | A | | |
| 7 | C | | |
| 8 | D | | |
| 9 | B | | |
| | | | |

# "Math is Cool" Championships – 2005-06

$9^{th}$ – $10^{th}$ Grade – October 19, 2005

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score

(out of 20)

**STUDENT NAME_____**

## Team Contest – Score Sheet

**DO NOT WRITE IN SHADED REGIONS**

| | Answer | 2 or 0 | 2 or 0 |
|---|---|---|---|
| 1 | 4 | | |
| 2 | 0 [primes] | | |
| 3 | $-\frac{47}{2}$ | | |
| 4 | 16 [points] | | |
| 5 | 194,040 | | |
| 6 | 9 [locations] | | |
| 7 | 864 [cones] | | |
| 8 | $\frac{\pi}{4}-\frac{1}{2}$ (or (π-2)/4) | | |
| 9 | 585 | | |
| 10 | $\frac{7}{17}$ | | |
| | | | |

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School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score

**STUDENT NAME_____**

## Pressure Round Answers

| | Answer |
|---|---|
| 1 | 23/3 |
| 2 | 4 [factors] |
| 3 | 72498 |
| 4 | $4\pi - 8$ [$un^2$] |
| 5 | $\frac{1}{2005}$ |