"Math is Cool" Championships -- 2021-22

6th Grade

Mental Math Solutions

	Answer	Solution
1	12 [quarters]	How many quarters have a value of three dollars? 300/25 = 12
2	[A + B =] 8	Kim eats two-fifths of a sandwich. The fraction of the sandwich that is leftover, as a reduced common fraction, is A over B. What is the value of A plus B? 1 – 2/5 = 3/5, and 3 + 5 = 8
3	[A =] 25	The area of a circle with diameter ten centimeters is A times pi square centimeters. What is the value of A? If d = 10, then r = 5 and the area is $5^2\pi = 25\pi$, so A = 25.
4	9 [miles]	How many miles does a bicyclist travel in forty-five minutes, if she travels at an average speed of twelve miles per hour? Forty-five minutes is 3/4 of an hour, so the bicycle travels 3/4 of 12 miles, which is 9 miles.
5	[A + B =] 37	A standard die is rolled twice. As a reduced common fraction, the probability of getting a two on the first roll and a three on the second roll is A over B. What is the value of A + B? (1/6)*(1/6) = 1/36, and 1 + 36 = 37
6	50	What is the mean of all positive two-digit multiples of ten? The list of positive two-digit multiples of 10 is 10, 20, 30, 40, 50, 60, 70, 80, 90, which is an arithmetic sequence, so the mean of is the same as the median, which is 50.
7	[$] 1210 [dollars]	Lilly invests one thousand dollars at ten percent interest compounded at the end of each year. In dollars, what will be the value of this investment after two years? 1000*1.1 = 1100 1100*1.1 = 1210

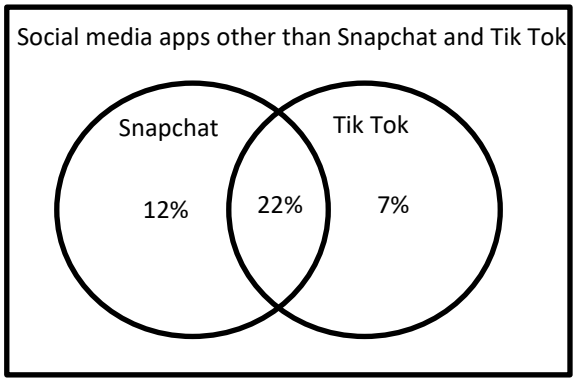
8	1 [triangle]	A triangle has a perimeter of six inches. How many triangles with this perimeter are possible if the side lengths must be integers? Possible values for sides (a, b, c) to consider include (1, 1, 4), (1, 2, 3) and (2, 2, 2). (1, 1, 4) and (1, 2, 3) are not possible because the sum of the two shorter sides must be greater than the length of the 3rd side. So only (2, 2, 2) is possible.

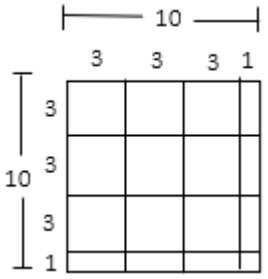
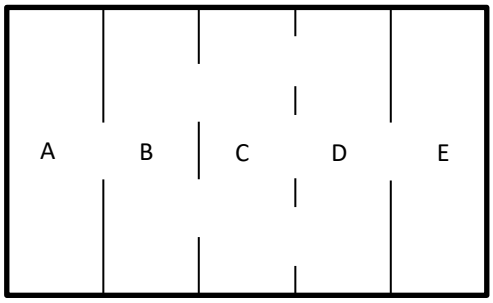
"Math is Cool" Championships -- 2020-21

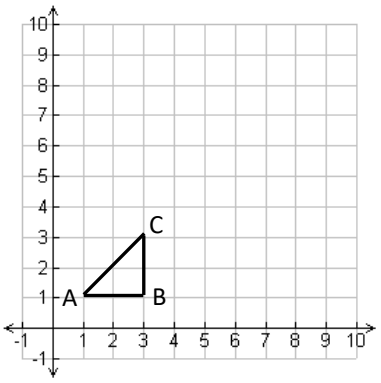
6th Grade

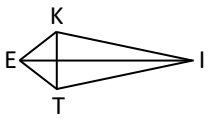
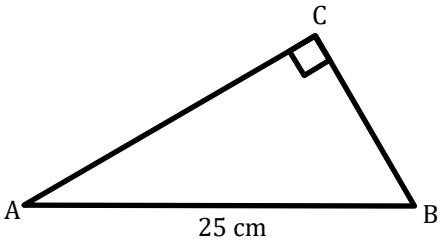
Individual Test Solutions

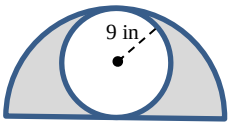
	Answer	Solution
1	25	What is 12 + 13? **12 + 13 = 25**
2	6 [zeros]	How many zeros are in the number 1 million? **1,000,000 has six zeros**
3	13 [years]	Axel is 8 years younger than Bonita and 21 years younger than Cinda. How many years older is Cinda than Bonita? **21 – 8 = 13**
4	112	One hundred is the first positive three-digit integer. What is the thirteenth positive three-digit integer? **100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112 – 13th three-digit integer**
5	27 [right-handed players]	There are 30 participants in a tennis tournament. Ten percent of the players are left-handed, and no one is ambidextrous. How many right-handed players are in the tournament? **10% of 30 is 3 and 30 – 3 = 27**
6	150 [inches]	A rectangle has two sides that are 43 inches long and two sides that are 32 inches long. How many inches are in the perimeter of the rectangle? **2(43) + 2(32) = 150**
7	15 [cents]	Marina buys five bags of chips with a $5 bill. Each bag costs 97 cents. How many cents does Marina receive as change? **500 – 5(97) = 500 – 485 = 15**
8	189	What is the result if you triple 7 three times? **7*3*3*3 = 7*27 = 189**
9	4 [doughnuts]	In a trade at lunch, one Krispy Kreme doughnut is worth five Oreo cookies. How many Krispy Kreme doughnuts are twenty Oreo cookies worth? **20/5 = 4**
10	[A + B =] 7	A drawer contains 4 brown socks and 6 blue socks that are all mixed up. As a reduced common fraction, the probability that a randomly drawn sock is brown is A/B. What is the value of A + B? **4/10 = 2/5, and 2 + 5 = 7**
11	20	What is the mean of the numbers in the following data set: {10, 14, 18, 22, 26, 30} **(10 + 14 + 18 + 22 + 26 + 30)/6 = 120/6 = 20**

12	[B – A =] 4500	Let A = 5 · 102 and B = 5 · 103. As a 4-digit integer, what is the value of B – A? **5000 – 500 = 4500**
13	7 [units]	On a coordinate plane, how many units to the right of the y-axis is the point with coordinates (7, 11)? **(7, 11) is 7 units to the right of the y-axis**
14	[x =] 23	Solve the following equation for x: 34 = 5x – 81 **34 = 5x – 81 → 115 = 5x → 23 = x**
15	960 [km]	A rocket travels at an average speed of 8 kilometers per second for two minutes. How many kilometers does it travel during these two minutes? **2 minutes = 120 seconds and 120 * 8 = 960**
16	[A =] 3600	In terms of π, the area of a circle with radius 60 inches is Aπ in2, where A is a four-digit integer. What is the value of A? **$60^2\pi = 3600\pi$, so A = 3600**
17	65 [%]	The probability of rain on Tuesday is 35%. As a percentage, what is the probability that it will not rain on Tuesday? **100 – 35 = 65**
18	2200 [grams]	How many grams are equivalent to 2.2 kilograms? **2.2 * 1000 = 2200**
19	1	If today is Friday, what day will it be 10 days from now? Answer as an integer: Monday = 1, Tuesday = 2, Wednesday = 3, Thursday = 4, Friday = 5, Saturday = 6, Sunday = 7 **7 days from now would also be Friday, then Saturday, Sunday, Monday, so Monday is the answer**
20	80 [%]	Sue has made 5 out of 7 free throws. If she makes the next three in a row, what will her 'free throws made' percentage be? **(5 + 3)/(7 + 3) = 8/10 or 80%**
21	59 [%]	The Venn diagram below shows the percentage of U.S. teens who use Snapchat, Tik Tok, or both. What percent of U.S. teens use neither one? **100 – 12 – 22 – 7 = 59** **Social Media Apps Used by U.S. Teens** 

22	[A + B =] 31	Let P = 30 times 40 and let Q equal 25 times 45. As a reduced common fraction, Q divided by P equals A/B. What is the value of A + B? **(25*45)/(30*40) = (5*9)/(6*8) = (5*3)/(2*8) = 15/16, and 15 + 16 = 31**
23	4 [integers]	How many positive two-digit integers have a tens digit that is two times the ones digit? **21, 42, 63, and 84**
24	9 [squares]	How many complete 3-inch-by-3-inch squares can be cut from a 10-inch-by-10-inch square piece of paper? Squares do not count as complete if they are made up of smaller pieces that are combined. **There are nine 3 x 3 squares in the figure shown**. 
25	6 [pathways]	In the diagram below, how many possible pathways are there from rectangle A to rectangle E through the openings shown, assuming you may not pass through any rectangle more than once? **1 * 3 * 2 * 1 = 6** 

26	16 [square units]	Triangle ABC is shown on the coordinate grid below. To locate the vertices of ΔA'B'C', all coordinates of points A, B, and C, are each multiplied by 3. What is the positive difference, in square units, between the area of ΔA'B'C' and the area of ΔABC? **The area of ΔABC is 2(2)/2 = 2 and the area of ΔA'B'C' is 32, or 9 times the area of ΔABC, which is 18. 18 – 2 = 16** 
27	[A + B =] 7	Adeem eats one-third of a pizza. Baeza then eats three-fourths of what is left. Finally, Cam'ron eats the rest. As a reduced common fraction, the fraction of the original pizza that Cam'ron eats is A/B. What is the value of A + B? **1 – 1/3 – (3/4)(2/3) = 1 – 1/3 – 1/2 = 1 – 2/6 – 3/6 = 1/6, and 1 + 6 = 7**
28	375	Simplify the following expression to a three-digit integer: $$\frac{15 \cdot 10^{19}}{4 \cdot 10^{17}}$$ $\frac{15 \cdot 10^{19}}{4 \cdot 10^{17}}$ **= 3.75 · 10^2 = 375**
29	5 [friends]	A group of twenty friends is planning to rent a hall for a party. If they split the rental cost evenly then each person pays $60. How many additional friends would need to be part of the group so that each person only needs to pay $48? **20 * 60 = 1200** **1200/48 = 25** **25 – 20 = 5**
30	[x =] -6	A line has a slope of -3/8 and passes through the point with coordinates (10, -3). A second point on the line has coordinates (x, 3). What is the value of x? **A slope of -3/8 means down 3 and right 8, or up 3 and left 8, or proportional rises and runs. Here the y-coordinate goes up by 6 (double 3), so the x-coordinate should go left by 16 (double 8). The x-coordinate is 10 – 16 = -6**

31	30 [cm]	The area of KITE is 1350 square centimeters. How many centimeters are in KT if IE = 3KT? **One way to calculate the area of a kite is to use the formula $A = d_1d_2/2$** **Let KT = x and IE = 3x** **$x(3x)/2 = 1350 \rightarrow 3x^2 = 2700 \rightarrow x^2 = 900 \rightarrow x = 30$, so KT = 30** 
32	10816	Let $a \lozenge b = a^2 - b^2$ and let $a \square b = a^2 + b^2$. Evaluate the given expression: $(3 \lozenge 7) \square (5 \lozenge 11)$ **$(3 \lozenge 7) \square (5 \lozenge 11) = (9 - 49) \square (25 - 121) = -40 \square -96 = 1600 + 9216 = 10816$**
33	2	There are three people (Alexa, Ben and Cody), one of whom is a knight, one a knave, and one a spy. The knight always tells the truth, the knave always lies, and the spy can either lie or tell the truth. Alexa says: "Cody is a knave." Ben says: "Alexa is a knight." Cody says: "I am the spy." Who is the spy? Answer as an integer: Alexa = 1, Ben = 2, Cody = 3. **Ben can't be the knight, because he'd be telling the truth when he says, "Alexa is a knight" and there would be two knights. So, either Alexa or Cody is the knight. If Cody is the knight, there would be a contradiction when he says, "I am the spy", because if he is the knight, this statement cannot be true. Therefore, Alexa is the knight and since she says, "Cody is a knave", then Cody is the knave, which leaves Ben as the spy.**
34	15 [cm]	Side $\overline{AB}$ is 25 cm long and the area of ΔABC is 150 cm^2. What is the length of the shorter side $\overline{BC}$? **The height of the triangle is 12 since the base is 25 and the area is 150, $25(12)/2 = 150$. So, AC * BC must also equal 25(12) or 300. Since it's a right triangle $AC^2 + BC^2 = 25^2$ and by guess and check, two numbers that multiply to make 300 that also satisfy the Pythagorean equation are 15 and 20.** **To prove it: let AC = a and BC = b and solve the system of equations: $a^2 + b^2 = 25^2$ and $ab = 300$. To solve square $a + b$, so $(a + b)^2 = a^2 + 2ab + b^2 = a^2 + b^2 + 2ab = 625 + 2(300) = 1225$ and $a + b = 35$. Since $ab = 300$, $a = 300/b$, substitute to get $300/b + b = 35$. Multiply by b to get $300 + b^2 = 35b$ or $b^2 - 35b + 300 = 0$ or $(b - 15)(b - 20) = 0$, so b equals 15 and a equals 20, or vice versa. In this triangle BC is the shorter length, so the answer is 15.** 

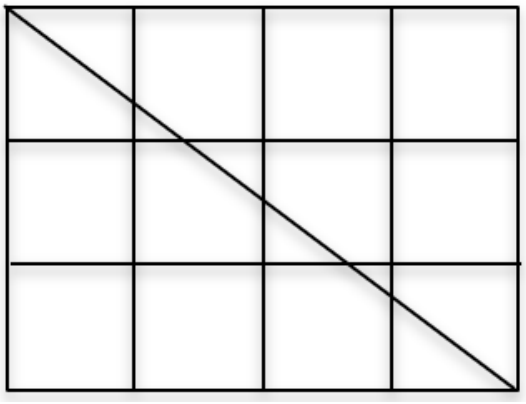
35	26	A set of five positive integers has a mean, median, and unique mode that all equal 10. What is the largest possible difference between the largest and the smallest of the five integers? **To make the mean equal 10, the five integers must add up to 50. To make the median equal 10, the middle number must be 10 when they are in order. To make the unique mode equal 10, there must be more 10s than any other number. To maximize the difference between the largest and smallest numbers, the two numbers smaller than 10 must be as small as possible, without repeating, because 10 is a unique mode. The set that meets all these conditions is {1,2,10,10,27}, and 27 – 1 = 26**
36	8 [ways]	How many ways are there to add three distinct positive two-digit integers, such that the sum is exactly 40? The order of the integers is not important. **{10,11,19}, {10,12,18}, {10,13,17}, {10,14,16}** **{11,12,17}, {11,13,16}, {11,14,15}** **{12,13,15}** **4 + 3 + 1 = 8**
37	18 [inches]	A circle with radius 9 inches is inscribed in a semicircle as shown, creating two shaded regions that are congruent to each other. What is the positive difference between the number of inches in the perimeter of one of these two shaded regions and the number of inches in the circumference of the inscribed circle? **The circumference of the inscribed circle is 18π. The radius of the semicircle is double the radius of the inscribed circle, or 18 inches. The diameter of the semicircle is 36 inches and the length of the arc of the semicircle is 36π/2 = 18π inches, so the perimeter of the semicircle is 18π + 36. The perimeter of one of the two shaded regions is half of the perimeter of the semicircle plus half the circumference of the inscribed circle, or 9π + 18 + 9π = 18π + 18. The difference between this and the circumference of the inscribed circle is 18π + 18 - 18π = 18.** 

38	22 [times]	Jim runs clockwise around a quarter-mile oval track at an average rate of 5 miles per hour. Pam runs counterclockwise around the same track at an average rate of 6 miles per hour. Jim and Pam begin at the same starting point, and they both run for 31 minutes. After they begin running, how many times will they pass each other on the track, not counting when they start at the same position? **Pam runs 6 miles in 60 min, which is 6/60 or 1/10 mi per min. Jim runs 5 miles in 60 min, which is 5/60 or 1/12 mi per min. Together, they run (1/10 + 1/12) = 22/120 = 11/60 mi per min. The length of the track is 1/4 = 15/60 mi, so it takes them $\frac{15/60}{11/60}$ = 15/11 min to run one complete lap. Then $\frac{31}{15/11}$ = 31*11/15 = 341/15 = 22 $11/15$ means they will pass each other 22 times.**
39	[A + B =] 967	Chinmayi draws three cards at random without replacement from a standard deck. As a reduced common fraction, the probability that exactly two of them are hearts is A/B. What is the value of A + B? **There are $_3C_2$ or 3 ways for two of three cards to be hearts and the probability of each of those ways is $_{13}C_2/_{52}C_2$ * $_{39}C_1/_{50}C_1$ or 13/52*12/51*39/50 = 39/850, so the answer is 3 * 39/850 = 117/850, and 117 + 850 = 967.**
40	966	What is the largest three-digit integer whose prime factorization is in the form a · b · c · d, where a, b, c, and d are distinct prime numbers? **Make a list of four primes starting with the three smallest primes and the largest possible 4th prime. Adjust the list by increasing the 3rd largest prime. It is possible to increase this prime only twice. Then adjust the list by increasing the 2nd largest prime. It is only possible to do this once. Increasing the smallest prime results in a 4-digit number.** **2 · 3 · 5 · 31 = 930, (2 · 3 · 5 · 37 > 1000)** **2 · 3 · 7 · 23 = 966, (2 · 3 · 7 · 29 > 1000)** **2 · 3 · 11 · 13 = 858, (2 · 3 · 11 · 17 > 1000)** **2 · 5 · 7 · 13 = 910, (2 · 5 · 7 · 17 > 1000)** **(3 · 5 · 7 · 11 > 1000)** **So there are only four results to compare and the largest one is 966.**

"Math is Cool" Championships -- 2020-21

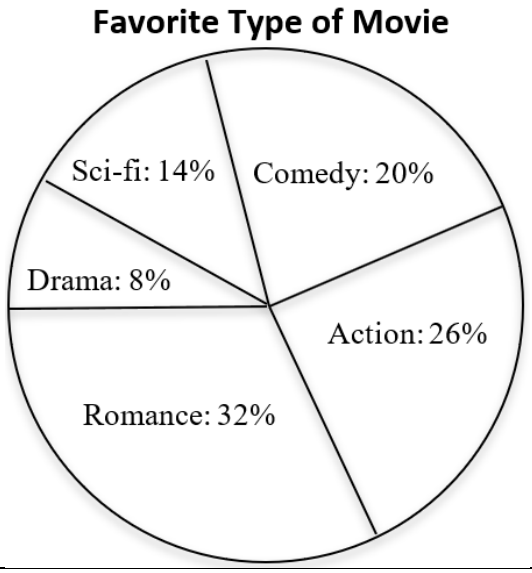
6th Grade

Multiple Choice Solutions

	Answer	Solution
USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #1 THROUGH #4. The figure shows a 3 cm by 4 cm rectangle that is divided into 12 square-centimeter grid squares and has a diagonal drawn from the upper left vertex to the lower right vertex. 		
1	E	How many grid squares are completely untouched by the diagonal in the figure shown? A) 2 B) 3 C) 4 D) 5 E) 6 **The diagonal passes through 6 squares and does not touch the other 6.**
2	C	What is the length of the diagonal that is shown in the figure? A) 3 cm B) 4 cm C) 5 cm D) 6 cm E) 7 cm **$3^2 + 4^2 = 5^2$, so the length is 5**
3	B	How many triangles which are contained within a single grid square are in the figure shown? A) 5 B) 6 C) 7 D) 8 E) 9 **There are 6 triangles**
4	C	What is the area of the largest of the triangles described in problem 3? A) 2/7 cm^2 B) 1/3 cm^2 C) 3/8 cm^2 D) 2/5 cm^2 E) 4/9 cm^2 **The sides are in a ratio of 3:4, so the base and the height of the triangle are 3/4 and 4/4. Therefore, the area is 3/4(1)/2 = 3/8.**

USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #5 THROUGH #7.

The pie chart shows the results of a poll given to 50 people about their favorite type of movie. The names of the five categories and the percentage of respondents who selected each category are shown. Assume each person responded and gave only one choice of movie type.



5	D	How many respondents chose comedy as their favorite type of movie? A) 7 B) 8 C) 9 D) 10 E) 20 **20% of 50 = 0.2*50 = 10**
6	A	What is the probability that a randomly selected point on the chart is in either the Comedy or the Action sector of the pie chart? A) 23/50 B) 47/100 C) 12/25 D) 49/100 E) $\frac{1}{2}$ **20% + 26% = 46% = 46/100 = 23/50**

7	E (4)	If the poll were given to a different group of 50 people, the results could be different from what is shown in the pie chart. Imagine the poll is given to an additional 19 different groups of 50 people. As a result of this process, how many distinct sets of five percentages are possible in which the order of least popular to most popular matches the order of the pie chart, and where the five percentages form an increasing arithmetic sequence (including the set of percentages in the pie chart)? Assume that in each of the 20 total polls that all 50 respondents give exactly 1 preference. A) 8 B) 9 C) 10 D) 18 E) Answer not given **To form an arithmetic sequence of five numbers that add up to 100, the middle number must be 20. In addition, all numbers in the sequences must be even, to account for an integral number of survey respondents out of the 50. Since the sequences must be increasing, the possible sequences are:** **4, 12, 20, 28, 36** **8, 14, 20, 26, 32** **12, 16, 20, 24, 28** **16, 18, 20, 22, 24** **Therefore, there are 4 distinct sets of five percentages that are possible.**

USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #8 THROUGH #10.

Base-2 (binary) numbers consist entirely of 1s and 0s. For example, the number 101_2 means $1*2^2 + 0*2^1 + 1*2^0 = 1*4 + 0*2 + 1*1 = 4 + 0 + 1 = 5$ as a base-10 number.

Base-4 numbers consist entirely of 3s, 2s, 1s, and 0s. For example, the number 3210_4 means $3*4^3 + 2*4^2 + 1*4^1 + 0*4^0 = 3*64 + 2*16 + 1*4 + 0*1 = 192 + 32 + 4 + 0 = 228$ as a base-10 number.

Since $4 = 2^2$, there is a special relationship between base-4 and binary numbers as described below:

Consider the base-four number 3210_4 from above.

- The first digit, 3, means $3*4^3 = 1*4^3 + 2*4^3 = 1*2^6 + 2*4^3 = 1*2^6 + 1*2^1*2^6 = 1*2^6 + 1*2^7 = 1*2^7 + 1*2^6$. So, in general, when you see a 3 in base-4 it is replaced with 11 when written in binary.

- The second digit is 2, which means $2*4^2 = 2*2^4 = 1*2^5 + 0*2^4$. So, in general, when you see a 2 in base-4 it is replaced with 10 when written in binary.

- The third digit is 1, which means $1*4^1 = 1*2^2 = 0*2^3 + 1*2^2$. So, in general, when you see a 1 in base-4 it is replaced with 01 when written in binary. The only exception to this would be when 1 is the left-most digit in the base-4 number, in which case it is replaced with 1, not 01.

- The fourth digit is 0, which means $0*4^0 = 0*2^0 = 0*2^1 + 0*2^0$. So, in general, when you see a 0 in base-4 it is replaced with 00 when written in binary.

This means that the binary number that equals 3210_4 is 11-10-01-00 or 11100100.

8	A	Which binary number is equal to 2301_4? A) 10110001 B) 10110010 C) 10110100 D) 10110101 E) 10110110 **2301_4 = 10-11-00-01 = 10110001**
9	D	Which binary number is equal to the largest three-digit base-4 number? A) 11111 B) 111101 C) 111110 D) 111111 E) 1111111 **The largest three-digit base-4 number is 333_4 = 11-11-11 = 111111**

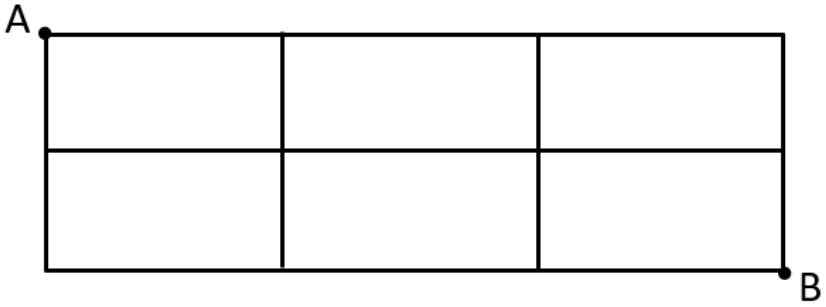
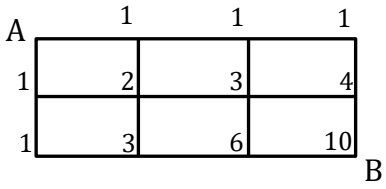
10	*C*	Since $9 = 3^2$, base-9 and base-3 numbers have a similar relationship to that of base-4 and binary numbers, where each digit in a base-9 number can always be replaced by the same two digits when rewritten in base-3. In a base-9 number, what two digits would replace a 7 when rewriting it in base-3? A) 23 B) 22 C) 21 D) 20 E) Answer not given **Rewrite a base-9 number that has a digit of 7 in base-3 to check. For example, $75_9 = 7*9^1 + 5*9^0 = 7*3^2 + 5*3^0 = (6*3^2 + 1*3^2) + (3*3^0 + 2*3^0) = (2*3*3^2 + 1*3^2) + (1*3^1 + 2*30) = (2*3^3 + 1*3^2) + (1*3^1 + 2*3^0) = 2112_3$, so the 7 is replaced with 21.**

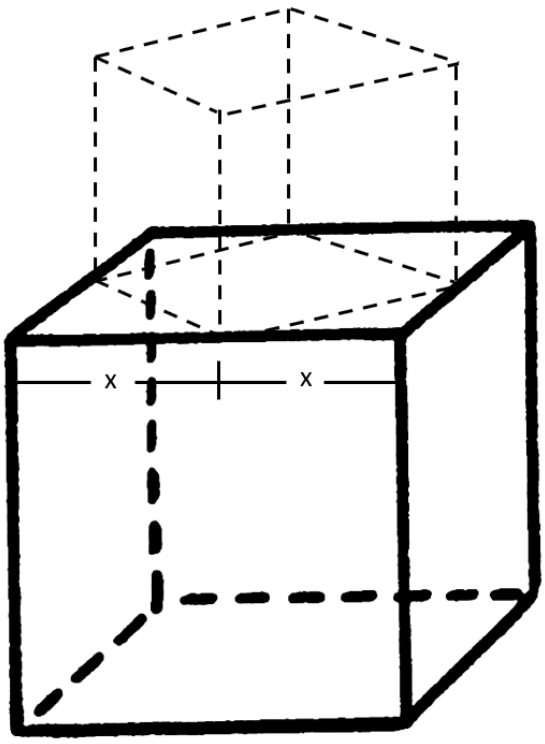
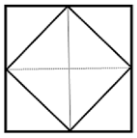
"Math is Cool" Championships -- 2020-21

6th Grade

Team Test Solutions

	Answer	Solution
1	3 [minions]	Minions have three fingers on each of their two hands. A group of minions has eighteen fingers total. How many minions are in the group? **18/3 =6, so there are 6 hands and three minions.**
2	[A + B + C =] 9	A cookie recipe calls for 3/4 teaspoon of vanilla. As a decimal to the nearest hundredth, the number of teaspoons of vanilla needed when the recipe is tripled is A.BC, where A, B, and C are each single-digit integers. What is the value of A + B + C? **3*3/4 = 9/4 = 2.25 and 2 + 2 + 5 = 9**
3	12 [whatchamacallits]	Three whatchamacallits equal 5 doohickies. Ten doohickies equal 9 thingamajigs. How many whatchamacallits are equal to 18 thingamajigs? **3w = 5d → 6w = 10d** **10d = 9t → 6w = 9t → 12w = 18t**
4	600 [kph]	The distance by plane from Reno to Seattle is 900 kilometers and the flight takes 1.5 hours. In kilometers per hour, what is the average speed of the plane during this flight? **900/1.5 = 1800/3 = 600**
5	228	What is the product of the greatest prime number less than 20 and the smallest composite number greater than 10? **19 * 12 = 228**
6	282[°]	The measures of two angles in a certain rhombus are each 39°. In degrees, what is the sum of the measures of the other two angles in the rhombus? **A rhombus has two pairs of congruent angles that add up to 360°, so 360 – 2(39) = 282.**

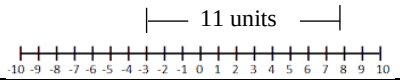
7	10 [pathways]	In the diagram below, how many pathways are there that lead from point A to point B, if you must either move right or down along the solid lines from any junction point?  **In the diagram, the numbers indicate how many pathways there are to each junction, so the answer is 10.** 
8	[P + Q =] 41	Let A = {1, 2, 3, 4, 5} and B = {11, 12, 13, 14, 15}. As a reduced common fraction, the probability that multiplying a randomly selected number from set A times a randomly selected number from set B results in an even number is P/Q. What is the value of P + Q? **P(odd) = 3/5 * 3/5 = 9/25 and P(even) = 1 – P(odd) = 1 – 9/25 = 16/25 – or – P(even) = 3/5 * 2/5 + 2/5 * 5/5 = 6/25 + 10/25 = 16/25, and 16 + 25 = 41.**

9	[A + B =] 3	A smaller cube fits on top of a larger cube such that each of its bottom four vertices coincide with the midpoints of the four horizontal edges of the top face of the larger cube, as shown. As a reduced common fraction, the ratio of the entire surface area of the smaller cube to the entire surface area of the larger cube is A/B. What is the value of A + B?  **Let the edge lengths of the larger cube be 2x. The surface area of the larger cube is then $2x(2x)*6 = 24x^2$.** **The edge length of the smaller cube would be $x\sqrt{2}$. The surface area of the smaller cube is then $(x\sqrt{2})(x\sqrt{2})*6 = 12x^2$.** **The ratio of the smaller surface area to the larger surface area is $12x^2/24x^2 = 1/2$, and 1 + 2 = 3.** **Or divide the top face into 8 congruent triangles to see that the area of the bottom face of the smaller cube is half the area of the top face of the larger cube. Since all the faces on a cube are congruent, this must be the ratio for all 6 pairs of faces, and so must be the ratio of the surface areas.** 
10	3840 [numbers]	How many positive three-digit hexadecimal (base-16) numbers are there, assuming the left-hand-most digit may not be a zero? **The ones and sixteens digits can be anything from 0 to 15, so 16 possibilities each. The two-hundred-fifty-sixes digit can be anything from 1 to 15, so 15 possibilities. Therefore, there are 16 * 16 * 15 = 3840 possible numbers.**

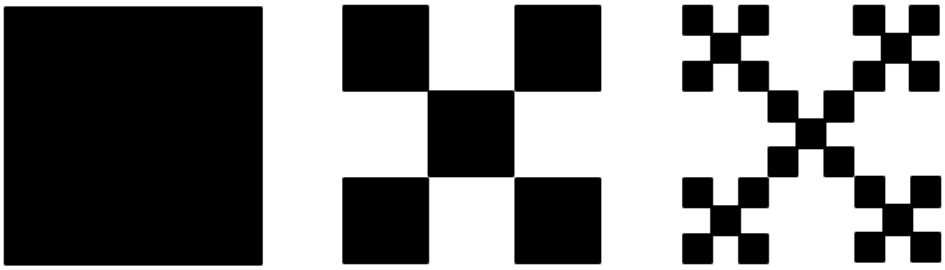
"Math is Cool" Championships -- 2020-21

6th Grade

Linda Moore Triple Jump Solutions

	Answer	Solution
1	11 [units]	What is the distance between -3 and 8 on a number line? **From -3 to 8 is 11 units** 
2	5	Follow the steps below. Step 1: Multiply the largest single-digit perfect square number by 10 Step 2: Divide the result of Step 1 by four Step 3: Subtract 17.5 from the result of Step 2 What is the result of Step 3? **1. 9 * 10 = 90** **2. 90/4 = 22.5** **3. 22.5 – 17.5 = 5**
3	60[°]	What is the number of degrees in the smaller of the two angles formed by the hands of an analog clock at 10:00 pm? **360/12 = 30, so there are 30 degrees between each pair of consecutive numbers, and from 10 to 12 there are two consecutive numbers, so the angle is 2*30 = 60.**
4	19 [integers]	How many integer values of x satisfy the inequality? $-7.2 < x < 11.3$ **There are 7 to the left of 0, 11 to the right of 0, and 0, so 7 + 11 + 1 = 19.**

5	216 [blocks]	A cardboard box has dimensions 12 inches by 24 inches by 6 inches as shown. What is the maximum number of 2 inch by 2 inch by 2 inch wooden blocks that the box can hold so that the blocks do not extend above the sides of the box?  **Divide each dimension in half since 2 goes into 12 six times, goes into 24 twelve times, and goes into 6 three times. Then multiply together, 6 * 12 * 3 = 216.**
6	[A + B =] 73	One number is selected at random from the set {4, 6, 8, 10, 11, 21, 23, 25}. Then a second number is also selected at random from the same set forming an ordered pair (x, y). The first and the second number may be the same number. The probability as a reduced common fraction that the sum of the two selected numbers is greater than 40 is A/B. What is the value of A + B? **There are 8*8 = 64 ways to select the two numbers. The 9 pairs that add up to more than 40 are (25, 25), (25, 23), (25, 21), (23, 25), (23, 23), (23, 21), (21, 25), (21, 23), and (21, 21). So, the probability is 9/64, and 9 + 64 = 73.**
7	64 [days]	In the United States, Labor Day is always the first Monday in September and Veteran's Day is always on November 11. What is the fewest possible number of days in any given calendar year between Labor Day and Veteran's Day (not including Labor Day or Veteran's Day themselves)? **The latest Labor Day could be is Monday, September 7. This leaves 23 days in September, 31 days in October, and 10 days in November before Veteran's Day, so 23 + 31 + 10 = 64.**
8	[100r=] 15	In a running race one time around a 400-meter track, Dalilah gets a head start and Allyson starts second. Allyson finishes the race at the exact same time as Dalilah. If Dalilah's head start is 1.5 seconds and her average rate is 6.25 meters per second, then Allyson runs 'r' meters per second faster than Dalilah. What is 100 times r? **Dalilah takes 400/6.25 = 1600/25 = 64 seconds to complete the race, so Allyson takes 64 – 1.5 = 62.5 seconds to complete the race. Her rate would be 400/62.5 = 800/125 = 160/25 = 6.4 meters per second. Then 6.4 – 6.25 = 0.15 mps faster, and 100(0.15) = 15**

9	500 [cm]	Figure 1 is formed by removing four congruent squares that are each one ninth of the area of the Figure 0, leaving five congruent shaded squares as shown. This process is then repeated on each of the five shaded squares in Figure 1 to get Figure 2 and again to get Figure 3 (not shown). The perimeter of any figure is the sum of the perimeters of the shaded squares. If the perimeter of Figure 0 is 108 cm, then what is the number of centimeters in the perimeter of Figure 3?  Figure 0 Figure 1 Figure 2 **The perimeter of Figure 0 = 108 = 4(27) = $4*3^3*5^0$** **Perimeter of Figure 1 = 180 = 4(9)*5 = $4*3^2*5^1$ = 108(5/3)** **Perimeter of Figure 2 = 300 = 4(3)*25 = $4*3^1*5^2$ = $108(5/3)^2$** **So, the perimeter of Figure 3 will be $4*3^0/5^3$ = $108(5/3)^3$ = 500**

10	4 [solutions]	For the given equation, how many solutions are ordered pairs of integers (x, y), where both integers are between -5 and 5? $$\left(\frac{6}{x}+\frac{4}{y}\right)^2+\left(\frac{6}{x}-\frac{4}{y}\right)^2=16$$ **Simplify the equation as shown here.** $\left(\frac{6}{x}+\frac{4}{y}\right)^2+\left(\frac{6}{x}-\frac{4}{y}\right)^2=16$ $\frac{36}{x^2}+\frac{24}{xy}+\frac{16}{y^2}+\frac{36}{x^2}-\frac{24}{xy}+\frac{16}{y^2}=16$ $2\left(\frac{36}{x^2}+\frac{16}{y^2}\right)=16$ $\frac{36}{x^2}+\frac{16}{y^2}=8$ **Since x and y must be integers between -5 and 5, you need to look for squares of these integers that are factors of 36. This includes $\pm1^2=1$, $\pm2^2=4$, and $\pm3^2=9$, and does not include $\pm4^2=16$, and $\pm5^2=25$.** **You can rule out ±1, because $36/1=36$, meaning $16/y^2$ would have to be negative, which is impossible. You can rule out ±2 for the same reason. So, check ±3.** **$36/3^2=36/(-3)^2=4$, so $16/y^2$ must equal 4 as well, and it does if $y=\pm2$.** **So, there are four solutions: (3, 2), (-3, 2), (3, -2), (-3, -2)** **You could also reason out these solutions without simplifying the original equation.**

"Math is Cool" Championships -- 2020-21

6th Grade

College Bowl Round #1 Solutions

	Answer	Solution
1	40 [hours]	Ryan is at work from nine AM to five PM every day. How many hours does he spend at work in five days? $8*5 = 40$
2	[x =] 7	Solve the equation for X: thirteen X plus five equals ninety-six $13x + 5 = 96$ $13x = 91$ $x = 7$
3	36 [boards]	A wall is ten feet tall, fifteen feet wide, and is covered with vertical wooden boards. Each board is ten feet tall and five inches wide, with no gaps between boards. How many boards are there? $15*12 = 180$ and $180/5 = 36$
4	5040 [ways]	How many ways are there to arrange the letters A-M-O-N-G-U-S? $7! = 5040$
5	49	The sum of two numbers is one hundred and eighteen and their positive difference is twenty. What is the smaller of the two numbers? $x + y = 118$ $x - y = 20$ $2x = 138$ $x = 69, y = 49$
6	2 [minutes]	It takes Biff three minutes to paint a fence, and Eho six minutes to paint the same fence. How many minutes would it take to paint the fence if Biff and Eho worked together? $1/3 + 1/6 = 1/x \rightarrow 2x + x = 6 \rightarrow 3x = 6 \rightarrow x = 2$ minutes
7	6 [inches]	A cone has a base with diameter two inches and a volume of two pi cubic inches. What is the height of the cone, in inches? $\frac{1}{3} * \pi r^2 * h = 2\pi$ $\frac{1}{3} * \pi 1 * h = 2\pi$ $\pi 1 * h = 6\pi$ $h = 6$
8	9999	What is the product of one hundred one and ninety-nine? $101*99 = 9999$

9	12421	A palindrome is a number that reads the same forwards and backwards. What is the smallest palindrome larger than twelve thousand three hundred and forty-five? **Since it's bigger than 12345, the middle digit must be a 4, so it is 12421.**
10	1 [prime number]	How many prime numbers between one and one thousand are multiples of three? **3 is a prime number, but every multiple after that is not a prime number**

"Math is Cool" Championships -- 2020-21

6th Grade

College Bowl Round #2 Solutions

	Answer	Solution
1	9 [odd numbers]	How many odd numbers are between eight and twenty-six? **9, 11, 13, 15, 17, 19, 21, 23, 25 makes 9 odd numbers**
2	235 [years]	The Industrial Revolution started in the year seventeen sixty. The book Industrial Society and its Future was published in nineteen ninety-five. How many years after the start of the Industrial Revolution was the book published? **1995 – 1760 = 235**
3	16	What is the mean of the first seven positive multiples of four? **The first 7 multiples of 4 are: 4,8,12,16,20,24,28,21** **The mean is the same as the median in an arithmetic sequence, so it's 16.**
4	50 [minutes]	Five people each write math problems at the same rate, and together they write eighty questions per hour. If two people leave, how many minutes will it take the remaining three people to write forty questions? **80/5 = 16 problems per hour per person, so three people will write 48 problems in 60 minutes, which is equivalent to 40 problems in 50 minutes.**
5	175 [miles]	Two trains leave a station at the same time, one going due north and the other going due west. The first train averages thirty miles an hour, and the second train averages forty miles an hour. How many miles apart are they after three point five hours? **$30^2 + 40^2 = 50^2$, so they're 50 miles apart after 1 hour** **50*3.5 = 175 miles**
6	500 [ft^3]	A road is twelve feet wide, and the pavement is five inches deep. How many cubic feet of pavement are in a one-hundred-foot stretch of this road? **5 inches is 5/12 ft, so 12*5/12*100 = 500**
7	504	What is nine factorial divided by six factorial? **9!/6! = 9*8*7 = 504**
8	15	What is the greatest common factor of seventy-five and ninety? **75 = 5*15** **90 = 6*15, so 15 is the GCF**
9	2 [meters]	A regular octagon has a side length of twenty-five centimeters. In meters, what is the perimeter of the octagon? **8*25/100 = 2**

10	3069 [cents]	Biff and Eho go out to dinner. Their bill is twenty-seven dollars and ninety cents, after tax. If they tip the waiter ten percent of this number, how much did they spend altogether, in cents? **2790*0.10 = 279, so 2790 + 279 = 3069**

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College Bowl Round #3 Solutions

	Answer	Solution
1	340	What is the sum of twelve squared and fourteen squared? **$12^2 + 14^2 = 144 + 196 = 340$**
2	75 [pounds]	A balanced scale has nine hundred pillows on one side. On the other side there are fifty pounds of steel plus three hundred pillows. In pounds, how much do the nine hundred pillows weigh together? **50 lbs of steel is equivalent to 600 pillows so 900 pillows would be 1.5(50) = 75 lbs.**
3	400 [minutes]	Alice's cat sleeps from ten-fifty AM to five-thirty PM on Monday. How many minutes does Alice's cat sleep during this time? **10:50 AM to 5:30 PM is 400 minutes**
4	[A + B =] 10	A standard deck of cards has all tens, jacks, queens, and kings removed. As a reduced common fraction, the probability of drawing a seven from the remaining cards is A over B. What is the value of A + B? **52-16= 36 4/36=1/9, and 1 + 9 = 10**
5	25 [minutes]	Arlene runs one and a half times as fast as Emily. If Arlene runs six miles in twenty minutes, how many minutes will it take Emily to run five miles? **Emily runs 2/3 as fast as Arlene, so she can run 4 miles in 20 minutes, or 1 mile in 5 min, or 5 miles in 25 minutes**
6	27000 [messages]	To prevent spam, a popular messaging app only allows users to send five messages every two seconds. What is the maximum number of messages three users can send in an hour? **5/2*60=150** **150*60=9000** **9000*3=27000**
7	6 [factors]	How many positive factors does ninety-six have that are not divisible by three? **1, 2, 4, 8, 16, 32**
8	210	What is the product of all positive prime numbers smaller than ten? **2*3*5*7 = 210**

9	8 [marbles]	There are five red marbles, three teal marbles, ten white marbles, and one grey marble in a bag. Without looking, what is the smallest number of marbles you need to pull out to guarantee at least three marbles of the same color? **The first 4 could all be different, then the next 3 could be a 2nd red, teal, and white marble, then on the 8th draw you would have to get the third of one of the colors.**
10	81	What is the sum of the first nine odd counting numbers? **1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 = 81, or 9^2 = 81**

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College Bowl Round #4 Solutions

	Answer	Solution
1	3	What is the remainder when one hundred ninety-two is divided by nine? **192/9 = 21r3**
2	57 [photos]	On a four-day trip, Diane and Mindy together took four hundred and fifty-six photos. If Diane took exactly half of the photos, how many photos per day on average did Mindy take? **456/2 = 228** **228/4 = 57**
3	[A =] 324	A circle has a circumference of thirty-six pi centimeters. The area of this circle is A times pi square centimeters. What is the value of A? **d = 36, r = 18, A = $18^2\pi = 324\pi$, so A = 324**
4	200 [minutes]	Bob's dog runs away from home at an average rate of five miles per hour and leaves at five AM. Bob starts running at seven AM the same day and uses a tracker to follow his dog. If he runs at an average rate of eight miles per hour, how many minutes will it take him to catch up with his dog? **Dog gets a 2-hour head start at 5 mph, so is 10 miles ahead. Bob makes up 3 miles each hour, so he will catch up in 3 and 1/3 hours or 200 minutes.**
5	3 [dollars]	At a hardware store, one person buys seven cans of paint and two brushes for forty-eight dollars. Another person buys one can of paint and two brushes for twelve dollars. In dollars how much does one brush cost? **7p + 2b = 48** **1p + 2b = 12** **6p = 36, so p = 6 and b = 3**
6	35 (ways)	Biff owns three identical copies of the game "Fence Painting Simulator" and four identical copies of the game "Mystery Marble Bag". In how many distinct ways can Biff arrange these games on a shelf, if each game is indistinguishable from its copies? **7!/(3!*4!) = 35**

7	21 [minutes]	A train is moving at fifteen miles per hour, and the front of the train is at the entrance of a five-mile tunnel. The train is a quarter of a mile long. How many minutes does it take the train to completely exit the tunnel? **In total, the train must travel 5.25 miles to exit the tunnel. t = d/r = 5.25/15 = 0.35 hours * 60 minutes/hr = 21 min.**
8	300 [ft^2]	A trapezoid has bases of length fourteen feet and ten feet and a height of twenty-five feet. What is the area of the trapezoid, in square feet? **(14 + 10)25/2 = 300**
9	74 [legs]	An animal pen on a spaceship contains cows and chickens. There are eleven cows and fifteen chickens. How many legs are there? **11*4+15*2**
10	8	What is the sum of twelve plus seven plus negative eleven? **12 + 7 + -11 = 19 – 11 = 8**

"Math is Cool" Championships -- 2020-21

6th Grade

College Bowl Round #6 Solutions

	Answer	Solution
1	35	What is the product of fifteen, one-third, and seven? **15* 1/3 * 7**
2	32 [days]	Alice's cat eats a half cup of food each day. How many days would it take Alice's cat to eat a gallon of food? **16 cups in a gallon, so 16/0.5 = 32 days**
3	83	The hit video game Among Them received review scores of eighty-four from Metacritic, seventy-five from Rotten Tomatoes, and ninety from IGN. What is the mean review score of the video game from these three sources? **(84 + 75 + 90)/3 = 83**
4	15 [percent]	The price of a pair of Airpods dropped from one hundred and fifty dollars to one hundred and twenty-seven dollars and fifty cents. What is the absolute value of the percent decrease in the price of this item? **150 – 127.50 = 22.50, and 22.50/150 = 225/1500 = 9/60 = 3/20 = 0.15 or 15%**
5	7 [pounds]	Potatoes cost seventy cents per pound and carrots cost forty cents per pound. Nathan buys fifteen pounds of potatoes and carrots combined at a total cost of eight dollars and forty cents. How many pounds of carrots does Nathan buy? **70p + 40c = 840** **p + c = 15** **40p + 40c = 600** **30p =240** **p = 8 and c = 7**
6	[A + B =] 14	A bag has five white marbles and five black marbles. Two marbles are taken out of the bag, without replacement. As a reduced common fraction, the probability that the two marbles are different colors is A over B. What is the value of A + B? **P(W, then B) = 5/10*5/9 = 5/18** **P(B, then W) = 5/10*5/9 = 5/18** **5/18 + 5/18 = 10/18 = 5/9, and 5 + 9 = 14**

7	32 [seconds]	Thelma and Louise are racing on the same circular track in the same direction. Thelma runs ten meters per second while Louise runs twelve meters per second. Thelma has a head start of sixty-four meters. How many seconds will it take Louise to catch up to Thelma? **Louise runs 2 meters per second faster than Thelma, so 64/2 = 32**
8	240 [minutes]	The Math is Cool problem writing team has four hours left to write the questions before the contest begins. How many minutes does the team have left? **4*60 = 240**
9	276 [hours]	Micah plays Minecraft all ninety-two days of summer for three hours each day. How many hours of Minecraft does he play throughout the summer? **92*3 = 276**
10	0 [%]	A standard twenty-sided die and a standard six-sided die are each rolled once. What is the probability in percent that each die shows a seven? **Impossible, because one of the two dice is a standard six-sided die.**