"Math is Cool" Championships -- 2022-23

$7^{th}/8^{th}$ Grade

Mental Math Solutions

	Answer	Solution
1	12	What is the largest factor of twenty-four, besides twenty-four? **12 is the 2nd largest factor of 24**
2	14	What is the next number in the arithmetic sequence starting with two, six, and ten? **2, 6, 10, 14, . . .**
3	10 [minutes]	Alicia runs two-tenths of a mile in two minutes. At this rate, how many minutes does it take her to run a whole mile? **0.2 *5 = 1 mile, and 2 * 5 = 10 minutes**
4	[x * 2y =] 40	If X equals four and Y equals five, what is the value of X times two Y? **4 * 2(5) = 4 * 10 = 40**
5	[A + B =] 8	The probability that it will rain tomorrow is forty percent. As a reduced common fraction the probability that it will not rain tomorrow is A over B. What is the value of A plus B? **100% – 40% = 60% = 3/5, and 3 + 5 = 8**
6	80000	Express seventy-two times ten to the fourth power divided by nine as an integer. **72/9 = 8. 8 x 10^4 = 80000**
7	[A =] 30	A cylindrical tube is open at both ends, has a radius of three centimeters, and is five centimeters long. The external surface area of the tube is A times pi square centimeters. What is the value of A? **SA = 2πrh, so 2π(3)(5) = 30π and A = 30**
8	[A =] 4	Let A and B equal two distinct positive integers. If one-half of A equals two-thirds of B, what is the smallest possible value of A? **(1/2)A = (2/3)B → 3A = 4B → A = 4/3B → A/B = 4/3, so the smallest value of A = 4**

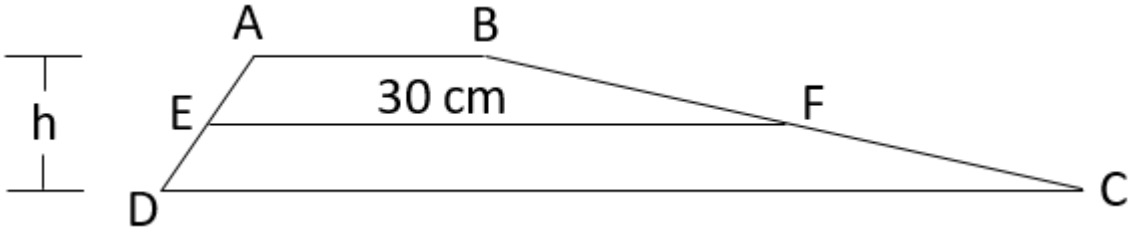
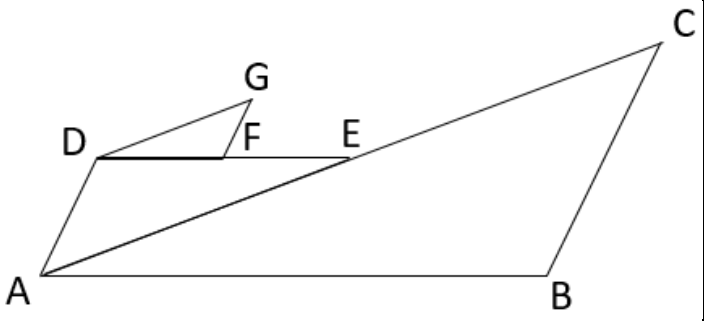
"Math is Cool" Championships -- 2020-21

7th/8th Grade

Individual Test Solutions

	Answer	Solution
1	89	Evaluate: 32 + 57 **32 + 57 = 89**
2	[$] 15 [dollars]	If 24 apples cost $20, what do 18 apples cost, in dollars? **18/24 = x/20, x = 15**
3	[B – A =] 46	Let A = (1)(3)(6) and let B = (2)(4)(8). What is the value of B – A? **A = 18 and B =64, 64 – 18 = 46**
4	4 [cats]	In a line of cats there is one cat in front, one cat in back, and two cats who are neither in front nor in back. How many cats are in the line? **1 + 2 + 1 = 4**
5	66	What is 20% of 330? **0.2 * 330 = 66**
6	[A + B =] 17	My card hand consists of 4 hearts, 4 diamonds, 3 clubs, and 2 spades. As a reduced common fraction, the probability that a card randomly selected from my hand is a heart is A/B. What is the value of A + B? **P(Hearts) = 4/13, and 4 + 13 = 17**
7	10	What is the greatest common factor of 60 and 50? **GCF (60, 50) = 10**
8	21	What is the median value in the following data set? 22, 24, 18, 34, 17, 23, 20, 15 **For 15, 17, 18, 20, 22, 33, 24, 34, the median is (20 + 22)/2 = 21**
9	9 [mph]	Bibi rides her bike for 20 minutes and travels a total of 3 miles. What is her average speed in miles per hour? **3 miles in 20 minutes is equivalent to 9 miles in 60 minutes, or 9 mph**
10	63 [°]	In a right triangle, one of the acute angles has a measure of 27°. What is the number of degrees in the measure of the other acute angle? **180 – 90 – 27 = 63**
11	6 [handshakes]	In a group of 4 friends, each person shakes hands exactly once with each of their friends. How many handshakes occur? **4(3)/2 = 6**
12	432 [in^2]	How many square inches are in the area of a 1-foot by 3-foot rectangle? **1-ft by 3-ft = 12-in by 36-in, and 12 * 36 = 432**
13	[a + b =] 23	As a reduced common fraction, $\frac{7}{6} - \frac{8}{9} = \frac{A}{B}$. What is the value of A + B? **7/6 – 8/9 = 21/18 – 16/18 = 5/18, and 5 + 18 = 23**

14	35	The first three terms of an arithmetic sequence are 5, 10, and 15. What is the 7^{th} term in the sequence? **In the sequence, 5, 10, 15, 20, 25, 30, 35, . . ., 35 is the 7th term**
15	[x =] 14	Solve the following equation for x. 5x – 23 = 47 **5x – 23 = 47 → 5x = 70 → x = 14**
16	2	What is the remainder when 80 is divided by 13? **13 * 6 = 78, so 80/13 = 6r2**
17	216	Evaluate: 6^3 **6^3 = 216**
18	33 [minutes]	Roland has taken 27 minutes to complete 45% of an application. How many additional minutes will he need to finish the application? **45% in 27 minutes is equivalent to 5% in 3 minutes or 55% in 33 minutes**
19	[a/b =] 31	If a + b = 32 and a and b are positive integers, what is the largest possible value of $\frac{a}{b}$? **31 + 1 = 32, and 31/1 = 31**
20	[A + B =] 7	When flipping three coins the probability of not flipping either three heads or three tails as a reduced common fraction is A/B. What is the value of A + B? **P(not 3H or 3T) = 6/8 = 3/4, and 3 + 4 = 7**
21	32 [multiples]	How many multiples of 31 are there between 1 and 1000? **31 * 32 = 992, so there are 32 multiples of 31 before 1000**
22	4	Consider the following data set: 5, 20, 30, 10, 15, 25, 30, 40, 35, 10, 25, 10 Let A be any random number in the data set. If A is replaced by B, such that B = A + 48, by how much does the mean of the data set increase? **Replacing any number with a number that is greater by 48 increases the overall sum of the data set by 48 and so the mean would increase by 48/12 = 4.**
23	[A + B + C =] 10	In the addition problem shown, let A, B, and C each represent distinct single-digit positive integers. What is the value of A + B + C? **There are four possibilities (A, B, C) = (1, 4, 5), (2, 3, 5), (3, 2, 5), or (4, 1, 5), and in each case A + B + C = 10.** A A A + B B A C 5
24	36 [minutes]	On a backpacking trip, while Xiaoyong is moving, he hikes at an average rate of 2.5 miles per hour. During one 10-mile hike, he takes four 30-minute breaks. With the breaks added to his total time, how many minutes on average does it take him to hike one mile during the trip? **2.5 miles in 1 hour = 10 miles in 4 hours and 4 hours + 4 30-minute breaks = 6 hours, so overall he hikes 10 miles in 6 hours = 5/3 miles in 1 hour = 1 mile in 3/5 hour or 36 minutes.**
25	[A + B =] 106	In total there are 10 red, 15 blue, 12 green, and 18 yellow gummy bears in a jar. If two gummy bears are randomly drawn from the jar, the probability as a reduced common fraction that both are blue is A/B. What is the value of A + B? **15/55 * 14/54 = 3/11 * 7/27 = 7/99, and 7 + 99 = 106**

26	7 [cm]	In the trapezoid shown, $\overline{AB}$ is parallel to $\overline{DC}$, and E and F are the midpoints of sides $\overline{AD}$ and $\overline{BC}$. If EF = 30 centimeters and the area of trapezoid ABCD is 210 cm^2, what is the height h of ABCD, in centimeters?  **EF = (AB + CD)/2, and the area is (AB + CD)/2*h, so 30h = 210, and h = 7**
27	[x =] 289	What is the smallest integer solution of x in the following inequality? $\frac{2}{3}x - 12 < \frac{3}{4}x - 36$ **$\frac{2}{3}x - 12 < \frac{3}{4}x - 36 \rightarrow 8x - 144 < 9x - 432 \rightarrow 288 < x$**
28	20 [%]	What percent of 1.1×10^{12} is 220×10^{9} ? **$1.1 \times 10^{12} = 110 \times 10^{10}$ and $220 \times 10^{9} = 22 \times 10^{10}$ and $(22 \times 10^{10})/(110 \times 10^{10}) = 22/110 = 1/5 = 20\%$**
29	14 [values]	A regular polygon has a perimeter of 168 inches. Let N stand for the number of sides in the polygon. If the side lengths must be integers, how many possible values can N have? **There are 8 factor pairs of 168: (1, 168), (2, 84), (3, 56), (4, 42), (6, 28), (7, 24), (8, 21), (12, 14) This would mean 16 values for N, but N cannot be 1 or 2, so the answer is 14.**
30	13	What is the sum of the terms which are integers in the infinite geometric sequence: 7776/2401, 1296/343, 216/49, . . . **The sequence continues as follows: 7776/2401, 1296/343, 216/49, 36/7, 6, 7, 49/6, 343/36, . . . , and 6 + 7 = 13**
31	41 [cm]	In the figure, ΔABC ~ ΔEDA ~ ΔDFG and DF = 3 cm, ED = 6 cm, AB = 12 cm, GF = 2 cm, AE = 8 cm. In centimeters, what is the perimeter of heptagon ABCEFGD? Note: the figure is not to scale.  **The ratio of the three triangles is 3:6:12 or 1:2:4** **If GF = 2, then AD = 4 and BC = 8.** **If AE = 8 then DG = 4 and AC = 16** **The perimeter of the heptagon is AB + BC + CE + EF + FG + GD + DA = 12 + 8 + (16 – 8) + (6 – 3) + 2 + 4 + 4 = 41.**

32 | [A + B =] 74

In a survey about apples, 61 people were asked whether they like Granny Smith, Honeycrisp, both, or neither. Everyone gave a response. There were 11 respondents who like both kinds of apples, 28 who like Honeycrisp, but not Granny Smith, and 9 who like Granny Smith, but not Honeycrisp. As a reduced common fraction, the probability that someone in the survey likes neither Granny Smith nor Honeycrisp is A/B. What is the value of A + B?

From the 2-way table below you can see that P(neither) = 13/61 and 13 + 61 = 74

	GS	not GS	Total
HC	11	28	39
not HC	9	13	22
Total	20	41	61

33 | 24

Let a ¤ b = $2a \cdot \sqrt{\frac{a}{b^2}}$. Evaluate 144 ¤ (36 ¤ 3).

144 ¤ (36 ¤ 3) = 144 ¤ (72 · $\sqrt{\frac{36}{3^2}}$) = 144 ¤ 144 = 288 · $\sqrt{\frac{144}{144^2}}$ = 288 · 1/12 = 24

34 | 18 [palindromes]

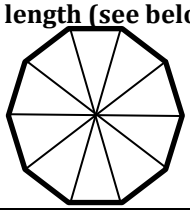
How many 3-digit palindromes are multiples of 4, but do not have all three digits the same?

212, 232, 252, 272, 292, 404, 424, 464, 484, 616, 636, 656, 676, 696, 808, 828, 848, 868 makes 18 palindromes

35 | [A + B =] 8

A regular decagon has multiple diagonals of varying lengths. The reduced common fraction of the total diagonals that have the longest possible length is A/B. What is the value of A + B?

A decagon has 10(7)/2 = 35 total diagonals and 5 of those 35 have the longest possible length (see below), and 5/35 = 1/7, so 1 + 7 = 8



36 | 5 [Friday]

Biff tells lies every Tuesday, Wednesday, and Thursday, and speaks the truth on the other days of the week. Eho tells lies on Fridays, Saturdays, and Sundays, and the other days of the week he speaks the truth. One day, Biff says, "Yesterday I was lying." Then Eho says "I was lying yesterday, too.". What day is it? Your answer should be an integer: 1 = Monday, 2 = Tuesday, 3 = Wednesday, 4 = Thursday, 5 = Friday, 6 = Saturday, 7 = Sunday.

If Biff is telling the truth, it's either Wednesday, Thursday, or Friday, but Friday is the only one of these three days on which Biff tells the truth.
If it's Friday, then Eho's statement, "I was lying yesterday, too", is a lie and this is consistent with him lying on Fridays.

If Biff is lying, it's either Tuesday, Wednesday, Thursday. Wednesday and Thursday are ruled out, because saying he lied yesterday would be a true statement, but he lies on those days. So, it could only be Tuesday.
If it's Tuesday, Eho tells the truth, but the statement "I was lying yesterday, too" would be false, which is a contradiction.

The answer is therefore Friday.

37	[A + B =] 131	An octahedral die has faces numbered 1 through 8 and a dodecahedral die has faces numbered 1 through 12. When an octahedral and a dodecahedral die are rolled together the probability that the sum of the two numbers showing is a prime number is the reduced common fraction A/B. What is the value of A + B? **There are 8 * 12 possible pairs of numbers. The list of pairs that have a prime number as their sum is as follows:** **2 – (1, 1)** **3 – (1, 2), (2, 1)** **5 – (1, 4), (4, 1), (2, 3), (3, 2)** **7 – (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)** **11 – (1, 10), (2, 9), (3, 8), (8, 3), (4, 7), (7, 4), (5, 6), (6, 5)** **13 – (1, 12), (2, 11), (3, 10), (4, 9), (5, 8), (8, 5), (6, 7), (7, 6)** **17 – (5, 12), (6, 11), (7, 10), (8, 9)** **19 – (7, 12), (8, 11)** **There are 35 pairs in this list so the probability is 35/96 and 35 + 96 = 131**
38	$11210_{[5]}$	Evaluate, and express your answer in base-5. Do not include the base 5 in your answer. $27_8 \cdot 38_9$ **27_8 * 38_9 = 23 * 35 = 805 = 1*625 + 1*125 + 2*25 + 1*5 + 0*1 = 11210_5**
39	10 [km]	Two trains are traveling towards each other on the same track. Train 1 has an average speed of 30 km/hr and train 2 has an average speed of 40 km/hr. Exactly when the fronts of the two trains are 14 km from each other a bee begins flying from the front of train 1 to the front of train 2. As soon as the bee reaches the front of train 2, it turns around and flies toward the front of train 1. The bee continues flying back and forth between the two trains until the trains meet. If the bee flies at an average rate of 50 km/hr, how many total kilometers will the bee travel? **The two trains are traveling towards each other at 40 + 30 = 70 km/hr and 14 km is 1/5 of 70, so it will take them 12 minutes to meet. The bee travels at a rate of 50 km/hr for 12 minutes or 1/5 of an hour so the bee will travel 10 km.**
40	486	Six positive integers form a finite geometric sequence whose growth factor is an integer. The mean of the 6 numbers is 364 and the median is 108. What is the fifth number in the sequence? **Let the integer a = the first term.** **Let the integer d = the growth factor** **Then $a(d^2 + d^3) = 108(2)$ and $d^2 + d^3 = 216/a$** **If d = 2 then 4 + 8 = 216/a results in a being 18** **If d = 3 then 9 + 27 = 216/a results in a being 6** **If d = 4 then 16 + 64 = 216/a results in a being a non-integer** **For d = every integer larger than 4, the result is that a is a non-integer** **So, the two possible scenarios are a = 18 and d = 2 or a = 6 and d = 3** **In the first scenario the sequence would be 18, 36, 72, 144, 288, 576, but this does not have a mean of 364** **The second scenario would be 6, 18, 54, 162, 486, 1464 and this has a mean of 364, so the answer is 486.**

"Math is Cool" Championships -- 2020-21

7th/8th Grade

Multiple Choice Solutions

<table>
<tr><th></th><th>Answer</th><th>Solution</th></tr>
<tr><td colspan="3">USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #1 THROUGH #4.
The following table compares minerals and vitamins contained in five leafy greens per 1-cup serving.
<table>
<tr><th></th><th>Calcium (mg)</th><th>Potassium (mg)</th><th>Magnesium (mg)</th><th>Vitamin E (mg)</th><th>Vitamin C (mg)</th></tr>
<tr><td>Kale</td><td>24</td><td>79</td><td>8</td><td>0.3</td><td>19</td></tr>
<tr><td>Spinach</td><td>30</td><td>167</td><td>24</td><td>0.6</td><td>8</td></tr>
<tr><td>Collards</td><td>84</td><td>77</td><td>10</td><td>0.8</td><td>12.7</td></tr>
<tr><td>Arugula</td><td>32</td><td>74</td><td>9</td><td>0.1</td><td>3</td></tr>
<tr><td>Iceberg</td><td>13</td><td>102</td><td>5</td><td>0.1</td><td>2</td></tr>
</table>
*mg = milligrams
Source: 7 Leafy Greens: A Nutritional Comparison</td></tr>
<tr><td>1</td><td>B</td><td>How many times as much magnesium per 1-cup serving does spinach have compared with kale?

A) 2 B) 3 C) 4 D) 6 E) 16
24/8 = 3</td></tr>
<tr><td>2</td><td>E</td><td>Which leafy green in the table has the greatest amount of combined minerals (calcium, potassium, and magnesium) per 1-cup serving?

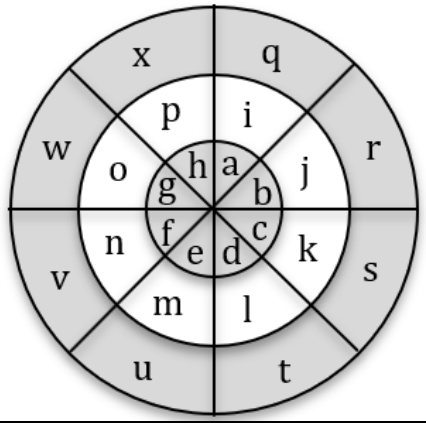
A) arugula B) collards C) iceberg D) kale E) spinach
Spinach has the greatest amount of potassium by 65 and the greatest amount of magnesium by 14, which combined is more than the amount by which is it less in calcium.</td></tr>
<tr><td>3</td><td>A</td><td>Which leafy green in the table has the median value in exactly one of the five columns of minerals and vitamins?

A) arugula B) collards C) iceberg D) kale E) spinach
Arugula's value for magnesium is its only median value. The other four leafy greens either have two median values or zero median values.</td></tr>
</table>

4	D	The Recommended Dietary Allowance (RDA) of Magnesium is 400 mg daily for men. Two cups of collards plus one cup of kale will provide what percent of the RDA for a man? A) 2.5% B) 4.5% C) 5.0% D) 7.0% E) None of these. **2-cups of collards + 1 cup of kale = 28 mg of Magnesium.** **28/400*100 = 7.0%**
USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #5 THROUGH #7. Three friends, Rafa, Novak, and Roger play one round of singles tennis. A round is when Rafa plays one set against Novak, followed by Rafa playing one set against Roger, followed by Novak playing one set against Roger. In every set there is a winner and a loser. There are no ties.		
5	A	If Roger wins two sets during the round, how many sets does he lose? A) 0 B) 1 C) 2 D) 3 E) 4 **Three total sets are played, but each player is involved in two of them. So, if Roger wins both sets he's involved in, then he does not lose any.**
6	C	On a different day, the three friends play two rounds of singles tennis. If, during these two rounds, Novak wins three sets and Roger wins two sets, how many sets does Rafa lose? A) 1 B) 2 C) 3 D) 4 E) 5 **Two rounds consist of a total of 6 sets, so there are 6 total wins and 6 total losses. Each player is involved in 4 sets, so can have a number of wins and losses that add up to 4.** **Since Novak wins 3 and Roger wins 2, Rafa must win 1, because 3 + 2 + 1 = 6. Therefore, Rafa must lose 3, because 4 – 1 = 3.**
7	C	On a third day, the friends play three rounds. What is the minimum number of sets that Roger needs to win so that it is possible for him to be the winner after the three rounds that day. To be the winner for the day means to win more total sets than either of the other two players. A) 2 B) 3 C) 4 D) 5 E) 6 **There are a total of 9 sets played in 3 rounds, so there are 9 total wins and 9 total losses. If Roger wins 5 sets or more, he will definitely be the winner that day. It is also possible for him to be the winner for the day with 4 wins, if the other two players have 3 wins and 2 wins, so 4 is the answer. It is not possible to be the winner for the day with 3 wins, because the other two would not both be able to have fewer than 3 wins.**

USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #8 THROUGH #10.

In the diagram shown here, there are three concentric circles, whose radii are in a ratio of 1:2:3. The four line segments intersect at the center of the circles to form eight 45° angles.



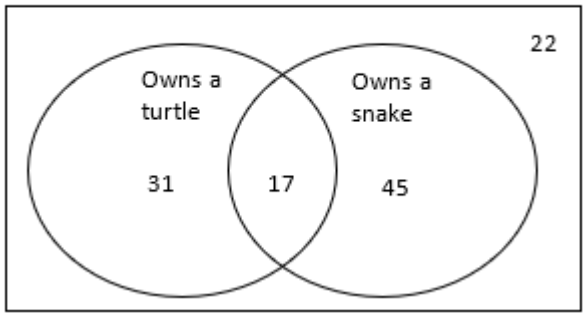
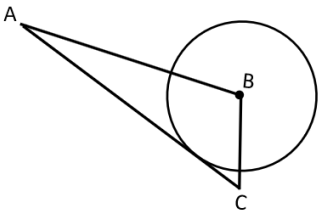
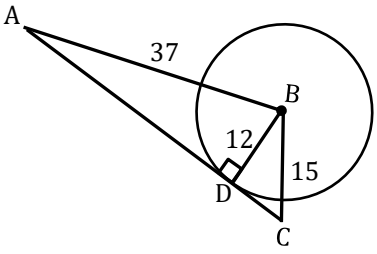
8	B	What fraction of the area of the total figure is contained in the combined sections labeled with the letters e, p, and r? A) 1/16 B) 1/8 C) 1/6 D) 1/5 E) $\frac{1}{4}$ **Since there are 8 45° central angles, this section is 1/8 of the whole figure.**
9	A	What is the ratio of the area of the combined sections labeled with the letters b and i compared to the area of the combined sections labeled with the letters r and d? A) 2:3 B) 3:4 C) 4:5 D) 1:1 E) Answer not given. **b – (1/8)(1/9) = 1/72** **i – (1/8)(1/3) = 1/24** **r – (1/8)(5/9) = 5/72** **d – (1/8)(1/9) = 1/72** **(b + i)/(r + d) = (1/72 + 3/72)/(5/72 + 1/72) = 4/6 = 2:3**
10	D	If AJ throws two darts which are guaranteed to land randomly anywhere on the board, what is the probability that he lands on the letters 'a' and 'j', in any order? A) $\frac{1}{144\pi}$ B) $\frac{1}{288}$ C) $\frac{\pi}{192}$ D) $\frac{1}{864}$ E) $\frac{1}{1728}$ **P(A) = (π/8)/(9π) = 1/72** **P(J) = (3π/8)/(9π) = 1/24** **P(A&J or J&A) = (1/72)(1/24)x2 = 1/864**

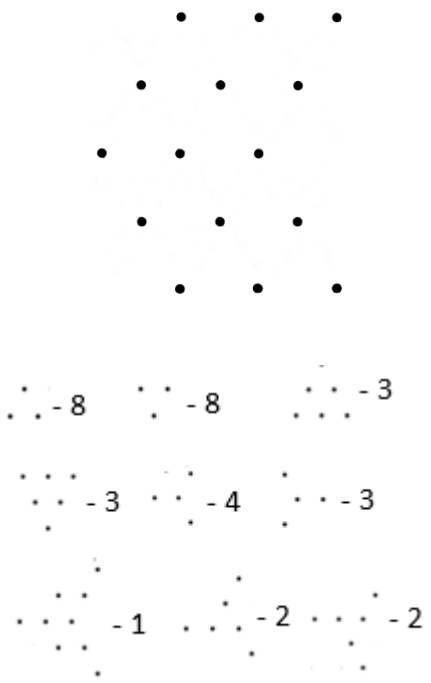
"Math is Cool" Championships -- 2020-21

7th/8th Grade

Team Test Solutions

	Answer	Solution
1	3200 [meters]	How many meters are in 3.2 kilometers? **3.2 * 1000 = 3200**
2	110	A sequence begins as follows. What is the sum of x, y and z? 1, 1, 2, 3, 5, 8, 13, x, y, z, … **21 + 34 + 55**
3	370	What is the arithmetic mean of the three numbers 10, 100, and 1000? **10 + 100 + 1000 = 1110, 1110/3 = 370**
4	[A =] 31	The diameter of a circle is 31 inches. The circumference of the circle is Aπ inches. What is the value of A? **If d = 31, then C = 31π, so A = 31**
5	[A + B =] 8	Let A and B represent two distinct positive prime numbers. What is the smallest possible value of A + B such that A + B is a composite number? **3 + 5 = 8, which is composite, so the answer is 8**
6	294 [cm^2]	What is the number of square centimeters in the surface area of a cube whose volume is 343 cm^3? $\sqrt[3]{343}$ **= 7, so the side length of the cube is 7, and the surface area is 6 * 7^2 = 6 * 49 = 294**
7	7	What is the sum of the integer solutions to the following inequality? -13 < 6x < 26 **-13 < 6x < 26 → -13/6 < x < 26/6 → -2.1666 . . . < x < 4.333 . . ., so the integer solutions are -2, -1, 0, 1, 2, 3, and 4, so the answer is 3 + 4 = 7**

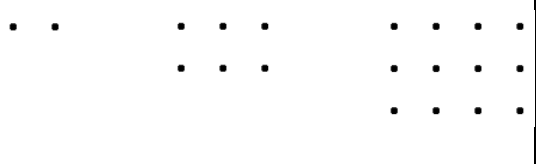
8	[A + B =] 163	Based on the Venn Diagram below, the probability that someone owns a turtle is a reduced fraction A/B. What is the value of A + B? **Survey of 7th graders at Desert Hills Middle School**  **P(turtle) = (31 + 17)/(31 + 17 + 45 + 22) = 48/115, and 48 + 115 = 163**
9	AC = 44 [cm]	In the diagram the radius of circle B is 12 cm, $\overline{AB}$ = 37 cm, $\overline{BC}$ = 15 cm, and $\overline{AC}$ is tangent to circle B. In centimeters, what is the length of $\overline{AC}$?  **Draw in radius BD. This will create two right triangles since a radius is perpendicular to a tangent at the point of tangency. Then do the Pythagorean Theorem to find AD and CD and add them together. CD = 9 (9, 12, 15 Pythagorean triple) and AD = 35 (12, 35, 37 Pythagorean triple), so AC = 9 + 35 = 44.** 

10	34 [triangles]	In the diagram below, assume each dot is equidistant from its closest neighbors in any direction. How many equilateral triangles can be created using any set of three of the dots as the vertices?  **The total is 8 + 8 + 3 + 3 + 4 + 3 + 1 + 2 + 2 = 34**

"Math is Cool" Championships -- 2020-21

6th Grade

Linda Moore Triple Jump Solutions

	Answer	Solution
1	3 [teammates]	On the school basketball team there are 15 players. Four of the players are taller than Catalina and 7 of the players are shorter than Catalina. How many of Catalina's teammates are the same height as her? **15 – 7 – 4 – 1 = 3**
2	12 [%]	What percent of 150 is 18? **18/150 = 36/300 = 12/100**
3	20 [cards]	How many even-numbered cards are in a standard 52-card deck? Note: Aces and face cards are not numbered cards. **4 twos, 4 fours, 4 sixes, 4 eights, and 4 tens makes 20 total**
4	4 [mph]	Rishi takes 10 minutes to ride his bike two-thirds of a mile. What is this rate in miles per hour? **2/3 mile in 10 min = 1 mile in 15 min = 4 miles per hour**
5	2	Evaluate: $15 - 8 \cdot 2 + 9 \div 3$ **$15 - 8 \cdot 2 + 9 \div 3 = 15 - 16 + 3 = -1 + 3 = 2$**
6	43	What is the mode for the data set shown in the following stem-and-leaf plot? $2 \mid 1, 2, 5, 5, 6, 7, 7, 8, 9, 9$ $3 \mid 1, 1, 2, 4, 5, 7, 8, 9, 9, 9$ $4 \mid 1, 3, 3, 3, 3, 5, 7, 7, 8, 9$ **There are more 43s than any other number**
7	92	The first three rectangular numbers are 2, 6, and 12. Visually, they can be represented as shown.  What is the sum of the next three rectangular numbers? **The series continues: 2, 6, 12, 20, 30, 42, . . ., so the sum of the next three terms is 20 + 30 + 42 = 92**

8	[A + B =] 136	A sock drawer contains only 4 argyle, 6 brown, and 5 crimson socks. As a reduced common fraction, the probability that two socks that have been randomly selected from the drawer are matching is A/B. What is the value of A + B? **A – 4, B – 6, C – 5** **AA (4/15 * 3/14 = 2/35), BB (6/15 * 5/14 = 1/7), CC (5/15 * 4/14 = 2/21)** **P(AA, BB, CC) = 2/35 + 1/7 + 2/21 = 31/105, and 31 + 105 = 136**
9	4 [robots]	A group of robots all work at the same rate. A robot takes thirty seconds to complete one ten-inch weld and two minutes to assemble a particular auto component. An individual robot can do both tasks, but not simultaneously. What is the minimum number of robots needed to complete at least 150 ten-inch welds and assemble at least 80 components in less than one hour? **It takes 75 minutes to complete 150 ten-inch welds and 160 minutes to complete 80 component assemblies, for a total of 235 minutes. 235/60 = $3\frac{11}{12}$, so at least 4 robots are needed.** **One possible work distribution is:** **Robot 1: 29 assemblies + 3 welds – 59.5 min** **Robot 2: 29 assemblies + 3 welds – 59.5 min** **Robot 3: 22 assemblies + 31 welds – 59.5 min** **Robot 4: 113 welds – 56.5 min**
10	[A + B =] 3320	Positive integers A and B are in a ratio of 7:13 (A:B). What is the largest possible value of A + B, such that B – A is a three-digit integer? **13x – 7x < 1000** **6x < 1000** **x < 166.$\overline{6}$** **Let x = 166, then the two numbers are 13(166) = 1660 + 498 = 2158 and 7(166) = 700 + 420 + 42 = 1162, and 2158 – 1162 = 996, so the answer is 2158 + 1162 = 3320**

"Math is Cool" Championships -- 2020-21

$7^{th}/8^{th}$ Grade

College Bowl Round #1 Solutions

	Answer	Solution
1	108 [cm]	If a sunflower grows at a constant rate of one point five centimeters per hour, and starts at a height of zero centimeters, how tall with the sunflower be after three days, in centimeters? **1.5 cm/hour * 24 hours/day * 3 days = 108**
2	54 [in^2]	The side lengths of a triangle are nine inches, twelve inches and fifteen inches. In square inches, what is the area of the triangle? **9-12-15 is a right triangle.** **Area = ½(9)(12) = 54**
3	4 [ways]	In how many ways can six be written as the sum of two not necessarily distinct non-negative integers? The order of the integers does not matter. **0+6** **1+5** **2+4** **3+3**
4	[A + B =] 9	Three cards are randomly selected in a row from a standard deck of playing cards. Each time a card is selected, it is replaced in the deck. The probability that all three cards are red can be written as a reduced common fraction A/B. What is A + B? **P(red&red&red) = (1/2)(1/2)(1/2) = 1/8**
5	50	What is the sum of the first five terms of an arithmetic sequence whose first term is four and with a common difference of three? **4, 7, 10, 13, 16** **4+7+10+13+16 = 50**
6	11 [nickels]	Juan has sixteen coins, all of which are nickels or dimes. The total value of the coins is one dollar and five cents. How many nickels does he have? **N + D = 16** **5N + 10D = 105** **Solve the system, or use guess and check.**
7	41 [degrees]	In isosceles triangle DEF, the measure of angle D is ninety-eight degrees. What is the measure of angle E, in degrees? **180 – 98 = 82** **82/2 = 41**

8	1	How many numbers in the following set cannot be the square of a real number, one, four, twenty-five, seven, and negative eight? **Every number except for -8 has a square root that is a real number.**
9	63 [$]	Naveen has one hundred ninety-eight dollars to buy textbooks. He spends seventy-one dollars and seventy cents on a used Calculus textbook, and sixty-three dollars and thirty cents on a used Physics textbook. How much money in dollars does he have left? **198-71.7-63.3=63**
10	41	What is the sum of the first six prime numbers? **2+3+5+7+11+13 = 41**

"Math is Cool" Championships -- 2020-21

7th/8th Grade

College Bowl Round #2 Solutions

	Answer	Solution
1	90 [°]	What is the mean value in degrees of the measure of all internal angles in a quadrilateral? **Sum of angles = 360. 360/4 = 90.**
2	75 [%]	Two fair coins are tossed. What is the probability in percent that at least one of the coins comes up heads? **Outcomes are: HH, HT, TH, TT. ¾ = 75%**
3	304 [meters]	The side length of a square field is seventy-six meters. What length of wire fencing, in meters, will be needed to fence in the field? **76x4 = 304**
4	900 [votes]	Jim and Dwight were the two candidates for school treasurer. Jim got twenty percent of the votes but lost by a margin of five hundred and forty votes. How many total votes were cast? **T = total votes** **J + D = T** **0.2T + (0.2T + 540) = T** **T = 900**
5	8842	Cosmo's PIN code is a four-digit integer. The sum of the four digits is twenty-two. Reading from left to right, the first and second digits are the same. The second digit is twice the third digit. The first digit is four times the fourth digit. What is Cosmo's PIN code? **From the second two clues, the only possibilities are:** **4421** **8842** **Must be 8842 because the digits sum to 22.**
6	81 [roses]	Andrea plants a rose bush in honor of her beloved dog Peanut. At the end of the first week, the bush has one rose, and the number of roses triples every week. How many roses are on the bush at the end of five weeks? **1, 3, 9, 27, 81**

7	22	The average of five numbers is forty-two. If the average of the first two numbers is forty-nine, and the average of the next two numbers is forty-five, what is the value of the last number? **Sum of 5 numbers = 5*42 = 210** **Sum of 1st 2 = 2*49 = 98** **Sum of next 2 = 2*45 = 90** **Fifth = 210 – 98 – 90 = 22**
8	80 [$]	Nine widgets cost thirty-six dollars in total. How many dollars will twenty widgets cost? **9/36 = 20/x** **¼ = 20/x** **X = 20*4 = 80**
9	[A – B – 4 =] -3	If A and B are two consecutive integers, and A is greater than B, then what is the value of A minus B minus four? **(x + 1) – x – 4 = 1 – 4 = -3**
10	6 [packs]	Gatorade bottles are sold in packs of four or ten. Gabe wants to buy exactly forty-eight bottles. What is the smallest number of Gatorade packs that he must buy? **4 packs of 10 = 40** **2 packs of 4 = 8**

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College Bowl Round #3 Solutions

	Answer	Solution
1	[N =] -5	The number zero point zero zero zero zero five three can be written in scientific notation as five point three times ten raised to the power of 'N'. What is the value of N? **$0.000053 = 5.3 \times 10^{-5}$**
2	43 [%]	A survey of six hundred Mathcounts competitors was taken, and it was found that two hundred fifty-eight of the competitors like probability questions, and three hundred forty-two of the competitors dislike probability questions. If one competitor is randomly chosen, what is the probability in percent that they like probability questions? **$258/600 = 43/100 = 43\%$**
3	42 [peanuts]	Chandler eats one more peanut each day than he did on the previous day. If he eats three peanuts on the first day of the week, how many total peanuts does he eat in the entire week? **$3+4+5+6+7+8+9 = 42$**
4	260 [meters]	A square has an area of four thousand two hundred and twenty-five square meters. What is the perimeter of the square, in meters? **Square root of $4225 = 65$.** **$65 \times 4 = 260$**
5	175 [%]	The fraction fifty-six over thirty-two is equivalent to what percentage? **$56/32 = 7/4 = 1.75$** **$1.75 * 100 = 175\%$**
6	20 [years]	Bowen is ten years old, and his mother is four times his age. After how many years will his mother's age be twice Bowen's age? **y = no. of year** **$2(10+y) = 40+y$** **$y = 20$**
7	2 [arrangements]	How many different arrangements of the letters A, B, C and D can be made in which no two adjacent letters are also adjacent letters in the alphabet? For example, no arrangement could include the letters A and B next to each other. **The only possible arrangements are:** **bdac, cadb**

8	66 [DC comic books]	Aaron has two hundred and sixty-four comic books, and they are all either Marvel or DC. He has three times as many Marvel as DC comic books. How many DC comic books does he have? **264/4 = 66**
9	1	What is the mean minus the median of the following data set: two, three, three, three, four, five, five, eight, and twelve? **Mean = 45/9 = 5** **Median = 4** **5-4 = 1**
10	5 [points of intersection]	One circle and two distinct lines are drawn on a whiteboard. What is the largest possible number of points of intersection of these figures? **Each line can intersect the circle in 2 points, and the lines will intersect at the 5th point.**

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College Bowl Round #4 Solutions

	Answer	Solution
1	10 [gerbils]	A pet store has twice as many hamsters as gerbils. If there are thirty total hamsters and gerbils, how many gerbils are there? **20H:10G = 30 total**
2	30	There are two distinct whole numbers I am thinking of. When each of them is multiplied by seven, the two products are whole numbers greater than thirty but less than forty-five. What is the product of the two distinct whole numbers I am thinking of? **Both products must be different multiples of 7, the only possibilities are 35 and 42, therefore the starting numbers are 5 and 6. 5x6=30**
3	80 [pieces]	Kamal bakes a twenty-inch-by-sixteen-inch sheet cake. The sheet cake is cut into pieces that measure two inches by two inches. How many total pieces of cake are there? **There will be 10x8 pieces = 80 pieces.**
4	44 [%]	A furniture store offers a dining room table at thirty percent off the marked price. The next week, the table is marked down an additional twenty percent off the discounted price. What is the total discount, in percent, from the original price? **Assume it costs $100. 30% off = $70. 20% of 70$ = $14. 70 – 14 = $56. Therefore, the total discount is $44 or 44% of the original price.**
5	120 [minutes]	Packard lifted weights for one hour and fifteen minutes each day from Monday through Friday. On the next three days, he lifted weights for one hour and thirty minutes each day. How many minutes would he have to lift weights on the ninth day to average eighty-five minutes of lifting time per day? **75*5 = 375** **90*3 = 270** **Sum for 8 days = 645.** **9*85 = 765** **Needs 765 – 645 = 120**

6	162 [integers]	How many positive integers between ninety-nine and nine hundred ninety-nine contain exactly one zero? **Case 1: 0 is in the middle** **9x1x9 ways = 81** **Case 2: 0 is the last digit** **9x9x1 ways = 81** **81+81 = 162**
7	[C =] 24	The first five terms of a geometric sequence are, three, A, B, C, and forty-eight. What is the value of C? **Multiply by 2 each time.**
8	2122	Shen writes seven numbers on a whiteboard, one of which is two hundred and three. He adds up the seven numbers and gets two thousand and twenty-three. He then substitutes the number three hundred and two for two hundred and three and adds up the seven numbers again. What sum does he get? **2023 – 203 + 302 = 2122**
9	4	How many lines of symmetry does a square have? **Each diagonal, plus through the center both vertically and horizontally.**
10	2 [combinations]	How many different combinations of five-dollar bills and two-dollar bills can be used to make a total of seventeen dollars? **3 $5 + 1 $2, or** **1 $5 + 6 $2**

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College Bowl Round #5 Solutions

	Answer	Solution
1	5	What is the smallest possible mean of four distinct positive even integers? **(2+4+6+8)/4 = 5**
2	729	What is the value of nine cubed. **9x9x9 = 729**
3	36 [cubes]	A solid white cube is painted red on all its faces. It is then cut into one hundred and twenty-five congruent small cubes. How many of the small cubes are painted red on exactly two of their faces? **Each edge will have 3 cubes with 2 painted faces, times 12 edges = 36.**
4	3 [pets]	Mr. Blaser's fourth grade class has some classroom pets. All of the pets except two are hamsters, all of the pets except two are turtles and all of the pets except two are lizards. How many total classroom pets are there? **Interpret each statement:** **Hamsters + 2 others** **Turtles + 2 others** **Lizards + 2 others** **The "2 others" in each case must refer to one each of the other two animals. Therefore there is one pet of each type.**
5	6 [combinations]	Ellie is selling girl scout cookies. She has Thin Mints, Samoas, Tagalongs and Lemon-ups. If a customer chooses two different kinds of cookies, how many different combinations can they get? **4C2 = 6**
6	72 [games]	Julia has played one hundred and thirty-two chess matches, and her ratio of wins to draws to losses is six to three to two. How many games has she won? **From the ratio, she wins 6/11 games.** **6/11 = x/132, x = 72**

7	72 [eggs]	A farmer has twelve chickens. Four of the chickens lay one egg every day. Four of the chickens lay one egg every other day. Four of the chickens lay one egg every four days. What is the maximum possible number of eggs the chickens could produce in ten days? **4 chickens x 1 egg/day * 10 days = 40** **4 chickens x 1 egg/2 days * 10 = 20** **To maximize, the 4 chickens who lay an egg every 4 days, start on day 1, then day 5, then day 9.** **4 chickens x 3 eggs = 12 eggs** **Total = 72**
8	4	The year two thousand two was a palindrome. What is the product of the digits of the next year after two thousand two that is a palindrome? **2112, 2x1x1x2 = 4**
9	360 [cm^2]	Eileen cut a rectangular piece of paper into two congruent triangles with sides nine, forty, and forty-one centimeters. In square centimeters, what is the area of the rectangular piece of paper before it was cut. **9-40-41 is a right triangle, so the dimensions of the rectangle are 9x40, 9x40 = 360**
10	330	Find the sum: two plus four plus six plus eight plus ten plus twenty plus forty plus sixty plus eighty plus one hundred **2 + 4 + 6 + 8 + 10 + 20 + 40 + 60 + 80 + 100 = 330**

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7th/8th Grade

College Bowl Round #6 Solutions

	Answer	Solution
1	1786	What is thirty-eight times forty-seven? **38x47 = 1786**
2	[x =] 18	What is the value of X if one-half times the quantity X plus eleven equals twenty-nine over two. **½(x + 11) = 29/2** **½ x + 11/2 = 29/2** **½ x = 18/2 = 9** **x = 18**
3	9 [cm]	What is the height in centimeters of a right rectangular prism with a length of four cm, a width of three cm and a volume of one hundred and eight cubic centimeters? **V = LWH** **108 = (4)(3)H** **H = 108/12 = 9**
4	170 [gumballs]	At a school party, people took turns guessing how many gumballs were in a large jar. No one guessed correctly, but the closest guesses were one hundred sixty-three, one hundred sixty-nine, and one hundred seventy-two. One of these guesses was off by one, one guess was off by two, and one guess was off by seven. How many gumballs were in the jar? **Draw a number line and indicate the guesses, and the differences between each pair of guesses.** **or** **the three guesses are 163, 169, 172, and one guess is off by 1 and one guess is off by 2, to the correct number must be between 169 and 172, either 170 or 171. The answer is 170, because that is 7 away from the guess of 163.**
5	[month number] 7 [th]	Kai wants to save fifty dollars to buy a new phone case. He saves two dollars the first month, four dollars the second month, six dollars the third month, and so on, saving two dollars more each following month. In what month number will he have enough to buy the phone case? **2+4+6+8+10+12+14 = 56** **In the 7th month**

6	522 [digits]	Hannah writes down all of the integers from one to two hundred and ten, inclusive. How many total digits did Hannah write? **1 digit: 1-9 = 9** **2 digit: 10-99 = 90x2 = 180 digits** **3 digit: 100-210 = 111x3 = 333 digits** **9+180+333 = 522**
7	15 [units]	Starting at the origin of a coordinate plane, an ant crawls one unit to the right, two units up, three units to the right, four units up, five units to the right and six units up. How many units away from the origin is the ant now? **Adds up to 9 units to the right, 12 units up, forms a 9-12-15 right triangle.**
8	154 [words]	Max can type twenty-eight words per minute. At this rate, how many words can Max type in five-and-a-half minutes? **28*5.5 = 154**
9	40 [%]	An urn contains two black marbles and three white marbles. If two marbles are chosen randomly, what is the probability in percent that they are the same color? **P(B&B or W&W) = (2/5)(1/4)+(3/5)(2/4) = 8/20 = 40/100 = 40%**
10	9 [palindromes]	How many four-digit palindromes consist only of the digits one, two, or three, including any combination of these digits? **List them:** **1111** **1221** **1331** **2112** **2222** **2332** **3113** **3223** **3333**